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Home Energy Management based on Optimal Production Control Scheduling with Unknown Regime Switching

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***Abstract*— We propose a novel home energy management framework to intelligently schedule the distributed energy storage (DES) for the cost reduction of customers in this paper. The proposed optimal production control technique determines the action policy (e.g., charging or discharging) and the power allocation policy of the DES to provide DES power at proper time with lower price than that of the utility grid, resulting in the reduction of the long term financial cost. Specifically, we first formulate the optimal decision problem for home energy systems with solar and energy storage devices, when the demand, renewable energy, electricity purchase from grid are all subject to Brownian motions. Both drift and variance parameters are modulated by a continuous-time Markov chain that represents the regime of electricity price. In particular, we set up a mean-variance problem where the cost function is both the running cost of diesel generator and deviation from the target State of Charge (SOC) of batteries. We assume the regime information follows a *Hidden Markov Model* (HMM), and then estimate the state by change of measure based on the *Girsanov's theorem*. Finally, the problem boils down to solving a *stochastic differential equation (SDE)*, which we provide both the explicit and numerical solutions to this specific SDE. An example is provided to illustrate the effectiveness of our proposed approach. Moreover, we compare it with the traditional *Model Predictive Control* (MPC) technique, and show it outperforms MPC.**

I. INTRODUCTION

There are different types of conventional and non-conventional energy sources used to generate electricity. Green (solar and wind in particular) energy production is supposed to increase significantly in the next years [1]. Unlike conventional generation sources, green energy brings in a lot of uncertainty and instability into existing power grids, and has significantly complicated energy system management for microgrids [2], [3], [4]. In other words, renewable energy utilization brings great challenges for the traditional power system operation due to uncertain/intermittent renewable generation and inelastic demand. On one hand, even a small error in renewable resource forecasting may result in great uncertainties for real-time operations of a microgrid given its limited scale and size [5]. On the other hand, in microgrids, the customers play a more important role by managing the controllable loads and local energy storage devices. Therefore, effective energy management of microgrids is one of the key components to adapt uncertain operating conditions of renewable resources and to achieve flexible and economic operation with various resources are among the challenges in energy management and optimization of microgrids [6].

The general objective of energy management of a micro grid is to solve the following problem: given a stochastic renewable generation, how to minimize the operating costs such as fuel, maintenance, and the purchased cost of exchange power from the main grid. Traditional deterministic microgrid operations are not sufficient to deal with this complicated energy system management. Meanwhile, the stochastic modeling and energy management methods that have been studied in transmission-level energy management such as [7], [3] demonstrate promising results in capturing the uncertainty associated with renewable energy resources considering worst-case scenarios.

Seeking to address the uncertain issues in energy optimization, two different approaches (direct/indirect) have been reported in the literature. The *direct* approach is to apply stochastic optimization, and probability statistical methods as [8], [9], to name a few. The indirect alternative approach is to aggregate more controllable nonrenewable generators to make the power system robust to the uncertainties. Different hybrid renewable energy systems (HRES) with grid integration were introduced to overcome the drawback of being unpredictable in nature [10]. It should be mentioned that in the hardware level, the robustness issue of voltage source converters has been addressed by two time-scale separation redesign technique [11].

More recently, [12] borrowed a probabilistic framework from real options in financial theory to assess the value of a portfolio of demand response customers under both operational (short-term) and planning (long-term) uncertainties. The financial theory framework will help reduce the exposure to electricity price risk caused by very high and volatile prices, particularly for small commercial and domestic customers in demand response based on real-time pricing. However, detailed home energy devices such as battery, back-up generator are not considered in their model, no mention of critical constraints of these components.

This paper is a continuous work of our previous paper

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[13], where we model the users' demand and day-ahead market bidding as stochastic dynamical systems with regime switching and generate the optimal demand response in a stochastic way. As proposed in [13], we use the optimal management technique to solve the problem in the real-time case. However, [13] requires additional regime information to be known. It should be mentioned that the regime here is more general than the existing peak/off-peak regime defined by the utility. Although the utility usually has a contract with the customers and sets the peak and off-peak tariffs, customers can still define their own more complicated regimes to maximize their benefits based on real-time market price and demand. Especially when more demand response or promotion based strategy has been involved in the whole operation. A predefined peak/off-peak regime may not provide correct incentive to obtain desired load shape or not responsive to unexpected fluctuation from renewable resources. Therefore, an adaptive regime which is related to real-time market can better regulate the power grid, which makes it meaningful to study this regime.

The paper is organized as follows. We begin with the preliminary modeling framework and problem formulation in Section II. Main results of deriving the optimal control for both regime known and unknown cases are given in Section III. This is followed by an illustrative example in Section IV. Finally we draw our conclusion in Section V.

II. Preliminaries: Notation and Problem Formulation

Assume that the cumulative demand of a customer follows a Brownian motion with drift modulated by a continuous-time Markov chain that alternates between two regimes. We allow both the drift and the diffusion to be affected by this continuous-time Markov chain, which represents the regime (or the state) of the economy. One regime may represent an off-peak period with a low demand rate and the other may represent a peak period with a high demand rate. The objective of management is to maintain the battery level as close as possible to a fixed target level; there is a penalty associated with the deviation of the battery's SOC from its target value. In addition to that, the customer may also want to maintain a production rate that is as close as possible to a fixed target rate to obtain the best efficiency of the generator. In this paper, we follow the same notation and formulation introduced in [13].

A. Notation

Throughout the paper ω, ω_i etc. will be used to denote random variables. We denote random variables with upper case letters, and their realization with lower case letters, e.g., for a random variable X, x denotes its realization.

We assume X and D are adapted stochastic processes defined as the following notation:

We consider a continuous-time Markov chain $\Delta(t), t \geq 0$, that can take values in two regimes $\varphi = \{1, 2\}$. We denote the amount of time the electricity price remains in off-peak (or peak) regime is exponentially distributed with rate $\lambda_1(\lambda_2)$.

TABLE I
Table of Notation

X_t	$\triangleq$	battery state of charge (SOC) at time t,
G_t	$\triangleq$	amount (Kwh) purchased (sold) from (to) grid at time t,
Y_t	$\triangleq$	cumulative PV generation (Kwh) up to time t,
D_t	$\triangleq$	cumulative demand (Kwh) up to time t,
p_t	$\triangleq$	diesel generator production rate (Kw) at time t,
$\Delta(t)$	$\triangleq$	regime of the electricity price at time t.
		Regime switching Variables of two-state model
$\Delta(t)$	$=$	$\begin{cases} 1, & \text{electricty is in off-peak price} \\ 2, & \text{electricty is in peak price} \end{cases}$

This can be explained from the practical application's aspect. Specifically, depending on whether the electricity price is *off-peak (peak)* i.e. belonging to regime $1(2)$, the cumulative consumer demand follows a Brownian motion ω with a drift $\mu_1(\mu_2)$ and a variance $\sigma_1^2(\sigma_2^2)$. We also assume that ϵ and ω are independent.

B. Model and problem formulation

The dynamic system can be modeled by a controlled regime-switching diffusion process. The switching action reflects system structural changes, which is exemplified by scheduled or emergency maintenance of solar modules, failure of a battery cell, addition of super-capacitor banks and tap changes in transformer. In this home energy management system, we consider the *regime switching to be electricity price*.

To maintain grid functionality, smooth operations and reduce waste, it is desirable that **(generation - consumption)** disparity in transient be kept as small as possible. The problem is thus naturally formulated as a mean-variance control problem.

The objective is to minimize the risk (measured by variance) of the terminal battery storage subject to a given expected terminal storage level.

1) Load (demand) model: We consider that the *cumulative demand* satisfies the dynamics

$$dD_t = \mu_{\Delta(t)} dt + \sigma_{\Delta(t)} d\omega_t. \tag{1}$$

where, at any time t, the demand rate $\mu_{\Delta(t)}$ and the diffusion $\sigma_{\Delta(t)}$ depend on the regime $\Delta(t)$, $d\omega(t)$ represents the Brownian Motion.

In reality, the *drift term* $\mu_{p(\Delta(t))}$ represents *average* solar radiation values for each time step (the smooth curve for maximum output); and *diffusion term* $\sigma_{p(\Delta(t))}$ represents solar radiation *fluctuations* which are commonly caused by many factors such as wind, clouds, many other uncertain weather conditions.

Similarly, we can define the stochastic generation models for the PV panels by Y_t, and diesel generator by G_t, respectively [13].

2) Power balance: In order to achieve the energy management, it is necessary to satisfy the power balance in the grid. Therefore, the battery storage level X has to satisfy the following equation with notations defined in Table. I

$$X_t = x_0 + \int_0^t p_s ds \quad - \quad D_t + Y_t + G_t \tag{2}$$

X_t can be computed by $\Downarrow$

$$\begin{aligned} X_t &= x_0 + \int_0^t (p_s - \underbrace{(\mu_\Delta(s) - \mu_{p(\Delta(s))})}_{\mu_{\lambda(\Delta(s))}})ds \\ &- \int_0^t \underbrace{(\sigma_\Delta(s) + \sigma_{p(\Delta(s))})}_{\sigma_{\lambda(\Delta(s))}} d\omega_s + G_t. \end{aligned} \tag{3}$$

As mentioned in [13], we could generalize to larger system, which involves many more components in real-life. As long as we could describe each component in a stochastic way, we are able to wrap up similar terms to get an augmented formulation as shown in the last line of (3).

The aggregator operates the power grid and aims to minimize the long-term time-averaged system cost by jointly managing supply, demand, and storage units. With increased integration of renewable generation and energy storage, business models of power system operators and electricity markets are constantly evolving. If we consider each residential home user as an individual agent or utility, we can formulate our problem as follows.

C. Cost function

Problem 2.1: Each agent wants to select an optimal production rate $p : [0, \infty) \times \Omega \mapsto (-\infty, \infty)$ for the local CG that minimizes the functional J

$$\begin{aligned} &J(p) := \\ &E\Bigg[\int_0^\infty \Gamma_t \left\{\tau_{\Delta(t)}(X_t - \Xi_{\Delta(t)})^2 + \upsilon_\Delta(t)(p_t - \mathscr{P}_{\Delta(t)})^2\right\} \\ &+ G_t^2 + p_t^2 dt\Bigg] \end{aligned} \tag{4}$$

where $\Gamma_t = exp\left\{-\int_0^t \kappa_{\epsilon(u)} du\right\}$, and $\kappa_i \in (0, \infty), \tau_i \in (0, \infty), \upsilon_i \in (0.\infty), \Xi_i \in (-\infty, \infty), \mathscr{P}_i \in (-\infty, \infty)$ are constants for each regime $\Delta(t) = i\,(i \in \{1, 2\})$.

The cost functional defined in (4) represents not only the cost from diesel generator and grid but also the running cost incurred by deviating from the target battery SOC Ξ_i and from the diesel generator production target rate $\mathscr{P}_i$.

Both of the target rates Ξ_i and $\mathscr{P}_i$ help the battery and diesel generator pick the most effective working strategy according to the regime of electricity price. An appropriate range of the SOC of the battery should be guaranteed to prevent the battery from being over- or under-charged. For example, when the electricity is in peak price, the battery may pick a small value of Ξ_i sacrifice to the life-cycle by almost depleting itself. In contrast, it would pick a large value to store as much as possible when the electricity price is low.

Remark 2.2: It's worth mentioning that, we are trying to minimize the diesel generator production, therefore the production target rate $\mathscr{P}_i$ are ideally set to 0 all the time. For a more general framework, we could take advantage of our switching framework by setting it as:

$$\mathscr{P}_i = \begin{cases} Pr_{min}, & \text{if } \Delta(t) = 1 \quad \text{(off-peak price)}, \\ Pr_{max}, & \text{if } \Delta(t) = 2 \quad \text{(peak price)}, \end{cases}$$

where Pr_{min} and Pr_{max} represent the minimum and maximum production rate of the range with the best efficiency.

In the market with an aggregator involved, most of the trading decisions are made ahead of time. Then our local components (battery, diesel generator and PV panel) are working as supplementary power sources to supply the demand fluctuations. For simplicity, we will focus on the case with fixed 24-hour ahead bid strategy such that the consumer has made the decision for purchasing how much electricity in advance. By observing a large historical data of the purchase for each time period, we could fit the trade transaction into a stochastic model similar as load and PV models:

$$dG_t = \mu_{G(\Delta(t))} dt + \sigma_{G(\Delta(t))} d\omega_t. \tag{5}$$

Substituting into (3) yields:

$$\begin{aligned} X_t &= x_0 + \int_0^t (p_s - \underbrace{(\mu_\Delta(s) - \mu_{p(\Delta(s))} - \mu_{G(\Delta(s))})}_{\mu_{\lambda(\Delta(s))}})ds \\ &- \int_0^t \underbrace{(\sigma_\Delta(s) + \sigma_{p(\Delta(s))} + \sigma_{G(\Delta(s))})}_{\sigma_{\lambda(\Delta(s))}} d\omega_s. \end{aligned} \tag{6}$$

Therefore, the cost function can be rewritten as:

$$J(p) := E\left[\int_0^\infty R_t \left\{\tau_{\Delta(t)}(X_t - \Xi_{\Delta(t)})^2 + \upsilon_\Delta(t)p_t^2\right\} dt\right]$$

III. Main results

Following the models and formulation introduced in the last section, we will present the main results of this paper which is to derive an optimal production rule for the stochastic home energy management problem.

If we assume the regime is known to each customer (agent), the explicit solution and simulation result were provided in [13]. However, explicit solutions are provided in this paper for a more general infrastructure, where each individual residential customer has few information about how his/her neighbors are using.

Under this situation, the regime state $\Delta(t)$ is unknown to us, we could only observe the demand D_t.

Let $\left\{\mathscr{F}_t^D, t < \infty\right\}$ be the filtration generated by the demand process $\{D_t, t < \infty\}$. We now require that the production process p be adapted to the filtration $\left\{\mathscr{F}_t^D, t < \infty\right\}$.

If we follow the procedure to define necessary constants, we are guaranteed to obtain the complete square with all the other terms vanished.

Theorem 3.1: [14] The optimal production rate $p^{(l)}$ with limited information regarding to the state of the regime is given by:

$$p_s^{(l)} = -\frac{\tau}{\upsilon} X_s^{(l)} + E[-\frac{1}{2\upsilon} f_{\epsilon(s)} | \mathscr{F}_s^D], \tag{7}$$

which can be written as

$$\begin{aligned} p_s^{(l)} = & -\tfrac{\tau}{\upsilon} X_s^{(l)} - \tfrac{1}{2\upsilon} f_1 P\left\{\epsilon(s) = 1 | \mathscr{F}_s^D\right\} \\ & -\tfrac{1}{2\upsilon} f_2 P\left\{\epsilon(s) = 2 | \mathscr{F}_s^D\right\}. \end{aligned} \tag{8}$$

where l denotes limited information.

Proof: The proof is based on the completing squares method, and is omitted due to space limit. However, it should be remarked that the expression for optimal production is much more complicated due to the fact that we only have limited information available. In order to make (8,) computable, we must calculate the conditional probabilities $P\left\{\epsilon(s) = 1 | \mathscr{F}_s^D\right\}$ and $P\left\{\epsilon(s) = 2 | \mathscr{F}_s^D\right\}$ first.

This objective can be achieved by utilizing the tools from optimal estimation for hidden Markov model (HMM) [15] if we denote the conditional probabilities as a two-dimensional process $\Delta(t)(t) = (\theta_1(t), \theta_2(t))$. ■

A. Optimal estimation for HMM

The above conditional probabilities in (8) can be characterized in the following HMM

$$\theta(t) = e_1 \mathbb{1}_{\Delta(t)=1} + e_2 \mathbb{1}_{\Delta(t)=2}, \tag{9}$$

where $\theta(t) = \begin{pmatrix} \theta_1(t) \\ \theta_2(t) \end{pmatrix}$ and $\theta_1(t) + \theta_2(t) = 1$.

Further, this can be denoted as conditional expectation

$$\hat{\theta}(t) = E\left[\theta(t) | \mathscr{F}_t^D\right]. \tag{10}$$

Consequently, we only need to compute the estimate $\hat{\theta}$ since

$$\mathbb{P}\{\theta(s) = 1 | \mathscr{F}_s^D\} = \hat{\theta}_1(s), \tag{11}$$

$$\text{and} \quad \mathbb{P}\{\theta(s) = 2 | \mathscr{F}_s^D\} = \hat{\theta}_2(s). \tag{12}$$

This turns into an optimal estimation problem which has been solved in [15]. Let H denotes the generator,

$$H = \begin{bmatrix} -\lambda_1 & \lambda_1 \\ \lambda_2 & -\lambda_2 \end{bmatrix} \tag{13}$$

Both the dynamics of hidden states and observation process follow the decomposition:

$$\theta(t) = \theta(0) + \int_0^t H^T \theta(s) ds + V_t, \tag{14}$$

$$\frac{1}{\sigma} D_t = \frac{1}{\sigma} D_0 + \int_0^t \frac{1}{\sigma}(\mu_1 \theta_1(s) + \mu_2 \theta_2(s)) ds + \omega_t. \tag{15}$$

Define Λ_t as in [14], [15], we construct a new probability measure $\bar{P}$ satisfying $\frac{d\bar{P}}{dP} = \Lambda_t$.

Remark 3.2: By Girsanov's theorem [16] the process $\{(1/\sigma)(D_s - D_0);\ s \le t\}$ is a standard Brownian motion under the new probability measure $\bar{P}$.

Simple application of Bayes' theorem [17] leads to

$$\hat{\theta}(t) = E\left[\theta(t) | \mathscr{F}_t^D\right] = \frac{\bar{E}\left[\hat{\Lambda}_t \theta(t) | \mathscr{F}_t^D\right]}{\bar{E}\left[\hat{\Lambda}_t | \mathscr{F}_t^D\right]}. \tag{16}$$

Therefore, it can be divided into two parts to compute (16) - the numerator and denumerator of (16). In the following, we first characterize the numerator, then computation of the denumerator will follow easily.

Denote $\pi(t) = \begin{bmatrix} \pi_1(t) \\ \pi_2(t) \end{bmatrix} = \bar{E}\left[\hat{\Lambda}_t \theta(t) | \mathscr{F}_t^D\right]$, then following [15], we can rewrite $\pi(t)$ as

$$\pi(t) = \pi(0) + \int_0^t H^T \pi(s) ds + \int_0^t G \pi(s) dDs, \tag{17}$$

where $G = \frac{1}{\sigma^2} \begin{bmatrix} \mu_1 & \\ & \mu_2 \end{bmatrix}$ with initial condition defined as

$$\pi(0) = E(\theta(0)) = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}. \tag{18}$$

Then the denumerator follows $\bar{E}\left[\hat{\Lambda}_t | \mathscr{F}_t^D\right] = \pi_1(t) + \pi_2(t)$.

Remark 3.3: We have converted the problem from original conditional probabilities to the computation of $\hat{\theta}(t)$, then further rewritten in $\pi(t)$ which are the solutions to the stochastic differential equation (SDE) (17):

$$\hat{\theta}_i(t) = \frac{\pi_i(t)}{\pi_1(t) + \pi_2(t)}, \quad i = 1, 2. \tag{19}$$

Therefore, the only remaining problem is to solve for $\pi(t)$ in (17).

By observation, we see this SDE arises, for example, as an asset price model in financial mathematics [18]. (Indeed, it yields the same form as the well-known *Black-Scholes* equation.) We will not proceed as in [14] to write the SDE as a system of linear ODE, instead, we will solve all the SDE's numerically by applying the *Euler-Maruyama* (EM) method [19].

It is known that the exact solution to the Black-Scholes like SDE (16) is:

$$\pi(t) = \pi(0) exp((H - \frac{1}{2} G^2) t + G D(t)). \tag{20}$$

Therefore, we can solve for $\hat{\theta}$ explicitly by solving "*Black - Scholes like*" equation (17).

As mentioned before, for simulation, we pick the EM method which approximates real solutions as follows:

$$\begin{aligned} X_j &= X_{j-1} + f(X_{j-1}) \delta t \\ &+ g(X_{j-1})(W(\tau_j)) - W(\tau_{j-1}),\ j = 1, 2, \cdots, L. \end{aligned}$$

which comes from the integral

$$\begin{aligned} X(\tau_j) &= X(\tau_{j-1}) + \int_{\tau_{j-1}}^{\tau_j} f(X(s)) ds \\ &+ \int_{\tau_{j-1}}^{\tau_j} g(X(s)) dW(s). \end{aligned} \tag{21}$$

Finally, we substitute into the optimal production rate equation to get the following theorem.

Theorem 3.4: The optimal production rate $p^{(l)}$ with limited information can be computed by

$$\begin{aligned} p_s^{(l)} &= -\frac{\tau}{\upsilon}\left(X_s^{(l)} - \Xi_1\hat{\theta}_1(s) - \Xi_2\hat{\theta}_2(s)\right) \\ &+ \frac{1}{\upsilon\gamma}\Big\{(\lambda_2 a + \tau\hat{\theta}_1(s))\mu_1 \\ &+ (\lambda_1 a + \tau\hat{\theta}_2(s))\mu_2 + \tau\lambda_1\hat{\theta}_1(s)(\Xi_2 - \Xi_1) \\ &+ \tau\lambda_2\hat{\theta}_2(s)(\Xi_1 - \Xi_2)\Big\}. \end{aligned} \quad (22)$$

Proof: This follows by plugging the conditional probabilities into Theorem 3.1. ■

IV. Numerical results

In this section, we first present a stochastic scenario to demonstrate the efficiency of the controller presented in this paper. As claimed in this paper, the regime information is not required to make the optimal decision, therefore this enables us to compare with the traditional MPC solution under the same information structure. Basically, this would tell us if we use the same information as MPC, whether we would achieve a better result with much less computational cost. We will briefly introduce the MPC algorithm for a simplified deterministic version of the same home energy management problem, and then show its simulation results. Finally, some preliminary comparison results are given due to the space limit here. We use the same example as given in [13].

A. Simulation with unknown regime

The time intervals for peak hours are assumed to be "$7:00 \sim 8:00$", "$12:00 \sim 13:00$" and "$18:00 \sim 20:00$". So in total there are 4 hours peak-time during a day. Regime switchings occur at the instances marked as red diamond in all the figures.

The control signal for the diesel generator is depicted in Fig. 1. We do see the generator kicks in when the electricity is in the peak price, while keeps in a low production level when electricity price is normal. Since the cost to purchase extra electricity at this peak hours is usually expansive, it is wiser to turn on the generator to meet the demand.

Finally, the dynamics of battery SOC is captured in Fig. 2. It is obvious that the battery keeps being close to fully charged during the off-peak hours, while it discharges to the lower bound of the SOC during the peak hours.

To examine the performance and energy efficiency of the proposed solution, a set of comparison is conducted by the use of MPC. It should be noted that it's not trivial to extend the general MPC solution to the case involving regime switchings. Therefore it's working similarly as the unknown regime case, i.e. MPC would **NOT** directly adjust its strategy based on whether the electricity is in peak price or not, however the electricity price would implicitly affect the control decisions through the cost function.

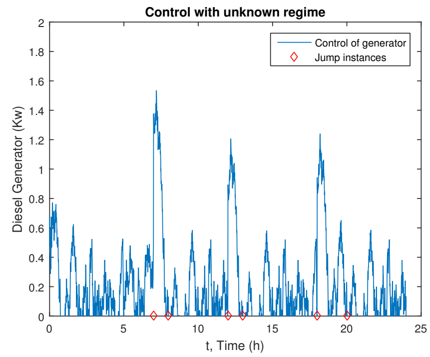

Fig. 1. Control for diesel generator.

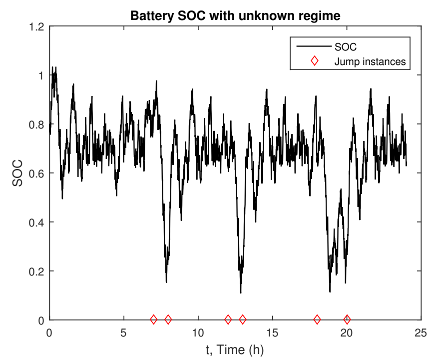

Fig. 2. SOC proposed control.

B. MPC for the Deterministic Problem

x represents the states of the system, $x = [x_1, x_2]^T$; x_1 corresponds to the SOC and $x_2 = P_{bat} + P_{dg}$. u represents the control inputs which need to be designed, $u = [P_{bat}, P_{dg}]^T$. The output of the system is denoted by y, where $y = Cx$. C is set to be identity for simplicity. Therefore, the output vector we pick is $y = [SOC_{ref}, P_L]^T$, which correspond to the desired SOC and predicted load profiles. It should be mentioned that both the two reference signals take the same values as in the previous simulation. Moreover, state matrices A, B are chosen to have compatible dimensions, and carrying certain values corresponding to the same model as we study the previous stochastic problem. Then the state space model for describing the deterministic problem is given as:

$$\dot{x} = Ax + Bu. \quad (23)$$

The considered cost here is to minimize the difference between system output and the reference signal. We denote the cost function as

$$J_{MPC} = \sum_{i=1}^{N} (Y_{ref} - Y_i)^T Q (Y_{ref} - Y_i) + u_i^T R u_i \quad (24)$$

The simulation results using MPC are given in Fig. 3 and Fig. 4.

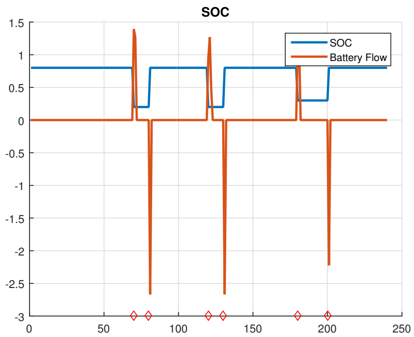

Fig. 3. Battery flow with SOC using MPC.

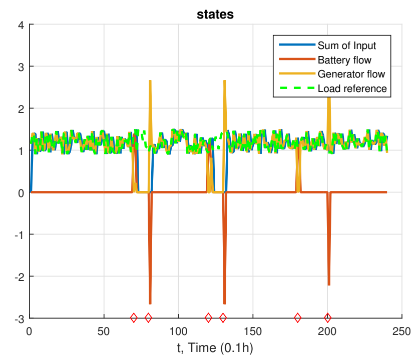

Fig. 4. Control performance using MPC.

C. Comparison of the results

The net demand is generated by normally distributed random values which are similar to the cumulative demand in the previous simulation. As we can see from Fig. 3, the battery flow kicks in during peak electricity regimes to reduce electricity purchase from grid. After passing the peak regimes, it automatically charged the battery to maximum SOC to make it ready for future use. As expected, the sum of all available resources follows exactly the random load demand (marked green in Fig. 4). However, both the magnitudes for battery flow and diesel generator are larger then our proposed control algorithm. If we compute the two norm of diesel flows under both algorithms, we get **8.92** and **18.81** for our method and MPC, respectively. It should be mentioned that this reflects around **53 %** economic saving since total financial cost is linear with two norm (multiplied by unit price of diesel).

V. Conclusion

This paper has been concerned with the optimal stochastic control problem for home energy systems with solar and energy storage devices, when the demand, renewable energy, electricity purchased from grid are all subject to Brownian motions. We assumed the regime information follows an HMM, and then estimated the state by change of measure based on *Girsanov's theorem*. Mean-variance has been used to improve the efficiency of the power grid. We have developed a novel framework to deal with the stochastic processes involved in the home energy management systems.

We are working on the problem involving more detailed and practical models of power grid components and compare our algorithm with stochastic MPC algorithm.

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