TRAVELER-CENTRIC MOBILITY SYSTEMS - ANALYSIS AND PERSPECTIVES USING GAME-THEORETIC FRAMEWORKS

by

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A dissertation submitted to the Faculty of the University of Delaware in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Mechanical Engineering

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Dedicated to my parents

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ABSTRACT

The rapid advancements in transportation technologies have significantly impacted urban social life, necessitating a reevaluation of the relationship between automobility and society. This dissertation is motivated by the need to understand human behavior and the effects of selfish decision-making in mobility systems, as technological advancements make it increasingly convenient for travelers to rely on cars. We are also motivated in providing a sustained scholarly focus on understanding the deeply economic, behavioral game-theoretic relationships and interactions among travelers, passengers, drivers, and the mobility system itself, while addressing key open questions in the domain. Our overarching goal is to ensure accessibility and efficiency in the mobility systems of the future by developing socially-efficient strategies that handle travel demand while focusing on the human preferences. By bridging theories from behavioral economics, microeconomics, game theory, control, and transportation engineering, we provide a comprehensive understanding of traveler-centric mobility systems.

This dissertation investigates social dilemmas in mobility decision-making, strategic traveler routing, mobility market design, and behavioral interactions in multimodal transportation networks. We explore the implications of selfish behavior on transportation systems and provide solutions that address congestion and resource allocation challenges. Our contributions span the development of distinct methodologies to understand the evolving social-mobility dilemma, the integration of game theory and microeconomics with transportation engineering for strategic traveler routing, and the incorporation of prospect theory in examining travel behavior in mobility systems.

In Chapter 2, we investigate the socioeconomic interactions of humans with self-driving cars and public transit. The focus is on the impact of these interactions

on system efficiency, using a game-theoretic perspective to analyze emerging mobility systems (EMS) within a multimodal transportation network. Chapter 3 explores the game-theoretic approach to resource allocation for efficient multimodal mobility systems. The research probes the potential of such systems to enhance accessibility, manage travel demand versus capacity, and improve overall traveler welfare. The design and stability of a shared mobility market are also examined from a game-theoretic perspective. In Chapter 4, the work is expanded by applying prospect theory and establishing the "mobility game" for multimodal transportation systems. This approach aids in understanding the effect of different traveler behaviors on decision-making in EMS and the impact on optimal system-wide performance.

The impact of our research on society and the state of the art is significant, as it provides a deeper understanding of human behavior in mobility systems and offers valuable insights into the implications of selfish decision-making on the future of transportation. By bridging theories from different disciplines, our work lays the foundation for the development of innovative solutions that ensure accessibility, efficiency, and sustainability in the mobility systems of the future. Furthermore, our research advances the state of the art by providing a comprehensive framework that unifies diverse theoretical perspectives, ultimately informing the design of smart, efficient, and accessible mobility systems.

In conclusion, this dissertation advances the state of the art in mobility systems research by providing a comprehensive and interdisciplinary understanding of human behavior and selfish decision-making. Our work contributes to the development of smart strategies for handling travel demand, promoting accessibility and efficiency in future transportation networks. By bridging diverse theoretical perspectives, we pave the way for continued innovation in the design and management of sustainable and efficient mobility systems that cater to the preferences of individual travelers.

Chapter 1

INTRODUCTION AND LITERATURE REVIEW

1.1 Motivation

Commuters in big cities have continuously experienced the frustration of congestion and traffic jams [239]. Travel delays, accidents, and road altercations have consistently impacted the economy, society, and the natural environment by increasing the number of vehicles on city roads [52]. In addition, one of the pressing challenges of our time is the increasing demand for energy, which requires us to make fundamental transformations in how our societies use and access transportation [258]. Thanks to the technological evolution of mobility (e.g., electrification of vehicles, smart vehicles and smart mobility, improved vehicle sensor technology [53]) that is currently afoot, it is highly expected that we will be able to eliminate congestion entirely, and significantly increase mobility efficiency in terms of fuel consumption and travel time [208]. In addition, several studies have shown the benefits of *emerging mobility systems* (EMS) (e.g., ride-hailing, on-demand mobility services, shared vehicles, self-driving cars) in reducing energy and alleviating traffic congestion in a number of different transportation scenarios [178, 255, 172, 137, 232]. But, what are EMS exactly?

One of the most novel and defining characteristics of an EMS is its socioeconomic complexity. Mobility is an indispensable prerequisite for social, cultural, and economic development as well as social participation. Thanks to the unprecedented improvements in mobility, we expect a significant alteration in human behavior and, most importantly, on tendency-to-travel. This may lead to unintended consequences, i.e., rebound effects, in the sense of additional energy use and greenhouse gas emissions, as well as leading to decreases in the density of urban areas and negatively impacting congestion. In addition, future mobility systems will enable human-vehicle interactions between people of any age and abilities, thus allowing enhanced and universal accessibility. One key reason why connectivity (e.g., Internet of Things) and automation in mobility may lead to rebound effects is because of the high levels of comfort and convenience - factors that urge drivers, passengers, and travelers to change their commute and travel tendencies, and thus use their vehicles quite more frequently and more unexpectedly. As urban social life has been greatly associated with the technological impact of the car, this compels us to reassess the relationship between automobility and social life [218, 27]. To add to our argument, evident from similar technological revolutions, for example, the impact of elevators on building design and social class hierarchies [22], human social perspective and view can have a tremendous effect on how technological innovations are utilized and implemented. For all these reasons, it is vital to study the impact of EMS in a sociotechnical context focusing on the social implications and attempt to provide optimal solutions for efficient EMS with widespread societal benefits.

The cyber-physical nature (e.g., data and shared information) of EMS is associated with significant control challenges and gives rise to a new level of complexity in modeling and control [77]. Research efforts over the last twenty years have tended to focus on the technological dimension. Zardini et al [254] provide a comprehensive review of the methods and tools that model and solve problems related to smart mobility-on-demand systems. While there has been considerable research in smart mobility, the area that remains less studied is a comprehensive theoretical examination of its broader social implications. The impact of selfish social behavior in routing networks of regular and autonomous vehicles has been studied in [153, 130, 30]. Other efforts have addressed "how people learn and make routing decisions" with behavioral dynamics [124, 123]. A game-theoretic framework using sequential games was proposed to study the socioeconomic interactions as well as the different tradeoffs that emerge between the mobility stakeholders of a mobility "ecosystem" [253]. It seems though that the problem of how automation in mobility will affect the tendency to travel and decision making has not been adequately approached yet. In a recent study [94], it was shown that when daily commuters were offered a convenient and affordable taxi service for their travels, a change of behavior was noticed; the commuters adjusted their travel behavior and activities and used the taxi service considerably more often leading to a 83% overall increase in vehicle miles traveled. Along with other similar studies [9, 26] this shows that EMS will affect people's tendency to travel and incentivize them to use cars more frequently, which potentially can also lead to a shift away from public transit.

As the reality of *connected and automated vehicles* (CAVs) is coming fast to realization [140], and similarly, with other past technologies, CAVs promise to be an incoming disruptive innovation with vast technological, commercial, and regulatory dimensions. Recently, there has been a significant amount of work on the technological or social impact of CAVs (mostly focusing on congestion, emissions, energy consumption, and safety). CAVs will transform the transportation system of today and revolutionize mobility. On the other hand, one expected social consequence of CAVs is to reshape urban mobility in the sense of altered tendency-to-travel, and thus, highly increase demand in the transportation system. To elaborate on this point, evident from similar technological revolutions (e.g., elevators), human social tendencies and society's perspective have changed the way a technology is used and applied [22]. Thus, we can most certainly expect that the deployment of CAVs in society will have unexpected outcomes, in the form of rebound effects (e.g., increased overall vehicle miles traveled, decreased use of public transportation, higher demand for road usage, etc.) Although there have been numerous studies that provide qualitative analysis for the social impact [261, 227], a formal analysis of the human decision-making regarding the expected social-mobility dilemma of the future travelers is missing from the state of the art.

In addition, shared mobility can provide access to transportation on a custom basis without vehicle ownership [20]. Over the last few years, on-demand ride-sharing services available via our smartphone have proved to be an innovative and adaptive mobility strategy for a broad range of travelers, passengers, and drivers [224]. Besides the apparent benefits to travelers (e.g., short-term and as-needed mobility access [164]), shared mobility services have been shown to have a significant environmental and societal impact. For example, reduced vehicle use, ownership, and vehicle miles traveled [212]. However, it is the authors' belief that shared mobility can also provide a solution to the social impact of connected and automated vehicles (CAVs), which promise to be an incoming disruptive innovation with vast technological, commercial, and regulatory dimensions [140]. Although it is clear that CAVs will transform the urban transportation networks and revolutionize mobility [133, 172, 4, 137], we expect CAVs to have social consequences. For example, CAVs may reshape urban mobility in the sense of altered tendency-to-travel and highly increase traffic demand [128].

The question for improved and efficient EMS is not only on modeling and understanding the human interactions. Wide accessibility to transportation in EMS can be impacted by the socioeconomic background of the travelers, i.e., whether a traveler can afford it. For example, travelers with a low-income background may be unable to use any or all transportation travel options available in a city. A key characteristic of our approach, which allows us to differentiate from other studies on fairness and efficiency [222, 105], is that we adopt the Mobility-as-a-Service (MaaS) concept, which is a system of multi-modal mobility that handles user-centric information and provides travel services (e.g., navigation, location, booking, payment) to a number of travelers. So, for the purposes of our work, travelers are expected to report their preferences to a central authority. The goal is to guarantee mobility as a seamless service across all modes of transportation accessible to all in a socially-efficient and fair way.

In this dissertation, we are motivated to investigate traveler-centric mobility systems (CAVS, shared mobility, smart mobility), provide a formal theoretical understanding of the relationships between travelers, passengers, and drivers and self-driving cars and public transit. At the same time, we investigate and explore how using a game-theoretic framework can enable us to a better understanding of how to offer efficient and accessible solutions for tomorrow's mobility systems.

1.1.1 Why Game Theory in Mobility?

One may ask why use game theory to analyze such a problem. EMS (e.g., CAVs, shared mobility, electric vehicles) will be characterized by their socio-economic complexity: (i) improved productivity and energy efficiency, (ii) widespread accessibility, and (iii) drastic urban redesign and evolved urban culture. In other words, the interplay of economic implications and social tendencies of the travelers can be naturally modeled and analyzed using notions from social choice theory and game theory. One of the main arguments in our work is that the social interaction of humans and CAVs or shared and self-driving cars or even public transit can be modeled as a "social dilemma" or a market. Informally, a social dilemma is any situation where there is a subtle yet unwanted discrepancy between individual and collective interest. We can then find the socially-acceptable equilibrium by ensuring to take into account the most important factors that influence the travelers' decision-making. It is for this reason why we in this dissertation we argue that game-inspired markets offer a complementary analysis of decision-making in EMS.

We offer an example here: Consider a normal-form game of n players. We can acquire a significantly improved way to realistically model social dilemmas that occur in real-life, and most importantly, we can obtain a multiplayer structure that reflects Garrett Hardin's "Tragedy of the Commons" [95]. From its conception, the Tragedy of the Commons has been an important problem in economics and other fields as it describes a plethora of phenomena in which independent members of a society selfishly attempt to maximize their benefit of utilizing at least one common resource which is scarce. Thus, the individuals' selfishness leads to the collective degradation of society's well-being. Noteworthy, even though the decision-makers are selfish and their decisions aim to maximize personal gain, they end up depleting the resource with unavoidable repercussions and losses [58]. In our context, the common resource is the road infrastructure shared by all the travelers, and the utilization is whether to travel with a CAV or not. Intuitively, one can expect that if all travelers make the selfish decision to use a CAV for commuting, then congestion is unavoidable.

The next section presents the state of the art related to mobility systems, game theory, mechanism design.

1.2 Literature Review

In the first decades of the 20th century, economist Arthur Cecil Pigou argued that "if a system's decision-makers take autonomous decisions, then the resulting collective outcome most probably will be inefficient." This key observation is evident in many different studies and analyses of transportation and mobility problems. Based on this, we categorize into four main realms of research the state of the art that serve as the foundations for this dissertation and its contributions.

1.2.1 Emerging Mobility Systems & Game Theory

There is a solid body of research now available for optimizing the efficiency of emerging mobility systems with CAVs. Over the last decade, several research efforts reported in the literature [255, 135, 257, 256] have aimed at addressing questions regarding the CAVs' impact on transportation efficiency. For example, can we consider the problem of optimizing fuel economy and emissions by coordinating a mobility system consisting of CAVs? What would be the appropriate conceptual approaches for modeling and optimizing emerging mobility systems? Recent technological developments can answer the above questions, indicating that CAVs will most likely help us eliminate congestion, significantly decrease fuel consumption, and minimize road accidents. Analytical frameworks have been proposed to quantify and evaluate the impacts of CAVs from the technological perspective [192, 193]. Furthermore, coordination of CAVs at different traffic scenarios (e.g., intersections, vehicle-following) have been extensively evaluated in the literature [194, 133, 251, 19, 106]. Moreover, the impact of CAVs has been identified as one that will enable traffic administrators to monitor transportation network conditions efficiently and effectively, thus improving the operating decisions that are required daily [208, 258].

In recent years, it has been recognized in the literature that further research is required to identify and understand the potential impacts of EMS. Shared mobility has been investigated and studied extensively the last decade. Factors that motivate active research in shared mobility systems are the significant energy savings, the limited importance of parking, and thus, opportunities for urban redesign with more space, and the increased demand for mobility access in developing countries [213]. Even though the promises of shared mobility have been realized with the implementation of various ride-sharing, or car-sharing, programs and initiatives, there are still open questions on how to design a shared mobility system that is socially-acceptable and profitable. Standard techniques of optimization and dynamics pricing have been used to control shared vehicle traffic and the non-strategic behavior of travelers and passengers [179, 248]. These methods focus primarily on formulating and solving a dynamic or stochastic optimization problem with respect to variables that include preferred and expected departure, arrival, and in-vehicle travel. One can control the solution by designing pricing schemes that the travelers, or passengers, react in a predictable manner (travelers are assumed to be price-takers).

There have been different approaches reported in the literature to study shared mobility using ideas from game theory. In particular, game theory has been used to model and analyze non-cooperative, or cooperative, interactions of travelers who seek to accommodate their desired origin-destination commutes through ride-sharing. The authors in [28] modeled a shared mobility system connected with a social network in which travelers could communicate and arrange one-time rides. Their focus was to minimize travel cost. Assignment games have been used to match sets of travelers with sets of capacitated routes in a transportation network [187]. In contrast to game-theoretic techniques, other efforts used a Vickrey-Clarke-Groves-inspired mechanism to design a first-mile, ride-sharing mobility system matching selfish travelers to vehicles [23]. The proposed mechanism was shown to be incentive compatible, individually rational, and price non-negative. In most cases, however, traveler, or passenger, behavior has not been well-understood. This is so, especially, in relevance to the impact that human travelers, passengers, and drivers might have on the traffic and energy efficiency of a mobility system. A very recent study on "social dilemmas" attempted to remedy this lack of understanding on the social impact of shared mobility [185]. The authors provided both a theoretical and an experimental study of how the strategic decision-making of travelers can impact the shared mobility's welfare, and thus, efficiency. A thorough review on ride-sharing can be found in [82] and the references therein.

Although the social effect of selfish-mobility behavior in routing networks of regular and autonomous vehicles has been studied [152, 155, 154], it seems that the problem of how CAVs will affect human tendency-to-travel and mobility frequency has not been adequately approached yet. One of the main questions we will attempt to answer in this proposal is the following: "What is the social impact of the emerging mobility systems with CAVs?" This question is quite important as it is widely accepted that CAVs will revolutionize urban mobility and the way people travel. For example, thanks to CAVs, travelers may use them to make empty trips, i.e., no travelers, to avoid parking, and thus add extra congestion in the network [59]. In addition, CAVs could potentially affect drivers' behavior and negatively impact traffic performance in general [8]. The question of the actual impact of CAVs on travel, energy, and carbon demand has attracted considerable attention [246]. Depending on different environmental indicators, the authors in [245] provided a practical microeconomic environmental rebound effect model. Quite recently, there has been research on the effects of a considerate penetration of shared CAVs in a major metropolitan area [144]. In an emerging mobility system with CAVs, shared mobility (car-sharing, ride-sharing, and on-demand ride service) enables users to access transportation as needed without vehicle ownership [214, 215, 203]. Various research efforts have focused on quantifying the impact of car sharing on vehicle ownership, vehicle miles traveled, greenhouse gas (GHG) emissions, and modal shift [79, 43]. Some studies [142, 141, 143, 17] have shown a decrease in vehicle ownership and GHG emissions for car-sharing services with significant implications on public transit. The feasibility and potential environmental impacts of shared CAVs have been investigated thoroughly in the literature [216, 44, 81, 190, 25, 64, 158, 159, 78, 132]. There have been also studies focusing on cost-benefit analysis of a mobility system with shared CAVs [38, 72, 33, 163], while some other studies have investigated its impact on vehicle ownership by using surveys or comparable analysis with conventional car-sharing systems [234, 237, 80, 93, 157].

1.2.2 Resource Allocation, Mechanism Design, Auctions

The theory of mechanism design was developed as an objective-first approach to efficiently align the individuals' and system's interests in problems of asymmetric information, where the individual agents have private preferences [145, 171]. It can be viewed as the art of designing the rules of a game to achieve a specific desired outcome. A well-established and broadly-used mechanism that has been successful in widely different applications (e.g., auctions, public projects, and cost-minimization problems) is the Vickrey-Clarke-Groves (VCG) mechanism [243, 50, 90]. The VCG mechanism ensures the existence and implementation of a dominant strategy equilibrium, which is an efficient solution and allows selfish agents to make a decision (alternatively choose a strategy) that is best no matter what other agents may decide. Agents are also incentivized to truthfully report their private preferences and have no reason (e.g., chance of receiving negative utility) not to participate in the mechanism. However, the VCG mechanism is known to be an extravagant mechanism, i.e., it can generate big surpluses. Because the VCG mechanisms have certain limitations, there have been attempts to use different approaches to solve the mechanism design problem. For example, by adopting the Nash equilibrium (NE) as the solution concept of the mechanism, a surrogate optimization method can be used where the network manager asks the agents to report a bundle of messages that approximate their private information [108, 5, 73, 96]. Mechanism design has been used extensively in communication networks in the form of decentralized resource allocation problems [112, 107], and also in transportation [240, 262].

Overall, mechanism design has broad applications spanning surprisingly many different fields, including microeconomics, social choice theory, and control engineering. Applications in engineering include communication networks [189], social networks [56], transportation routing [24], online advertising [111], smart grid [206], multi-agent systems [219], and in general resource allocation problems [102], who was the first to formulate a resource allocation problem within the framework of mechanism design. Mechanism design theory then emerged to mathematically model, analyze, and solve informationally decentralized problems involving systems of multiple rational and intelligent agents [167]. In particular, mechanism design is concerned with methodologies that implement system-wide optimal solutions to a myriad of problems - problems in which the strategically interacting agents can hide their true preferences for better individual benefits, thus hurting the overall efficiency of the system. Mechanism design has broad applications spanning different fields, including microeconomics, social choice theory, and control engineering. Applications in engineering include communication networks [189, 112], power markets [221], social networks [56], transportation routing [23, 240, 241, 262], online advertising [111], smart grid [206], multi-agent systems [219], and resource allocation problems [89, 98].

The application of mechanism design is not new in transportation and mobility problems [104, 231, 240, 184, 174]. For example, it has been used to provide solutions to individual route selection under different congestion traffic scenarios (e.g., first-mile ridesharing, selfish routing, tradable driving permits). In particular, auction-based mechanisms treat traffic congestion as an economic problem of supply and demand, focusing on travel time allocation or routing. So, on the one hand, auctions have been proposed to design pricing schemes with tolls in a network of roads leading to a spark of studies in auctioning techniques. On the other hand, this approach has important limitations: (i) the implementability of auction-based tolling on highways is not straightforward due to the dynamic and fast-changing nature of transportation systems; (ii) it is also uncertain how the public (e.g., drivers, passengers, travelers) will respond concerning toll roads in an auction setting. Therefore, understanding the travelers' interests (willingness-to-pay, value of time) and the impacts on different sociodemographic groups become imperative for a socially-efficient design of an emerging mobility system. For these reasons, it is essential to design an emerging mobility system whose focal point is the social aspect and societal impact of CAVs. In conjunction, it is the authors' belief that the emerging mobility systems - CAVs, shared mobility, electric vehicles - will be characterized by their socioeconomic complexity: (1) improved productivity and energy efficiency, (2) widespread accessibility, and (3) drastic urban redesign and evolved urban culture. This characteristic can naturally be modeled and analyzed using game theory/mechanism design and behavioral economics alongside control and optimization techniques. One of the main arguments in this dissertation is that the social interactions of human travelers with CAVs, and other modes of transportation can be modeled as an economically-inspired mobility market, where either monetary incentives (tolls) or well-designed game rules are used to induce the desired socially-efficient outcome.

In a transportation context, travelers can be viewed as agents in a routing game created by the interaction among travelers and the road authority. Mechanism design for routing games have been studied extensively in computer and communication networks [198, 200, 139, 114, 75]. Advances in algorithmic mechanism design [170, 171] provide a promising approach to incentivize rational agents to cooperate with the system in order to reach desirable outcomes. This approach motivates agents to disclose their private information. As usually done in mechanism design

[165, 86, 150, 149, 88, 147, 146, 148], the aim is to construct a mechanism that induces voluntary participation (VP) and incentive compatibility (IC) properties. The former means agents do not suffer any loss when they use the system, and the latter means revealing truthful information is in their best interest. In algorithmic mechanism design, besides the VP and IC properties, designers also concern about computational complexity when computing an allocation rule and a payment rule for intended outcomes [129]. The key technical difficulties lie in the combinatorial nature of the allocation rule and the interweaving relationship of allocation rules and payment rules. On the whole, though, we are more interested in the application of mechanism design in designing a market or an auction. Auctions are processes for allocating goods among bidders, so the challenge of auction design can only be understood by studying the demands of the participants [161]. When each bidder wants to buy only one item from a set of nonidentical items, the mechanism needs to solve the assignment problem: "who gets which item?" Auction design has been the focus of significant theoretical discussion from John Nash's seminal work on bargaining problems [169] to works on multi-object auctions and matching market problems [62, 83]. Recently, advanced optimization techniques have been used to design multi-parameter auctions, either focusing on budget constraints or maximizing revenue [14, 201]. A comprehensive outlook of the theory can be found in [125] and the references therein.

Partly related to our work are matching models which describe systems or markets in which there are agents of disjoint groups and have preferences regarding the "goods" of the opposite agent they associate with. Two-sided matching with transfers have been modeled as assignment problems where one entity (e.g., a firm) needs to pay salaries to individuals (e.g., workers) [217, 115]. Tasks-matching problems under a wide range of constraints have been reported in [119]. A wider literature on matching under constraints can be found in [11]. Notable examples are mechanisms for assigning students to schools [1], interns to hospitals [250], pickup and delivery [233], electric vehicles [252]. It is easy to see that matching markets are quite practical as they offer insights into the more general economic and behavioral real-life situations. These examples are all centralized approaches of determining who gets assigned to whom at what cost and benefit. One of the very first studies was the marriage model which was analyzed by [83] and existence of stable matchings between men and women was established. The authors in [217] extended it by incorporating monetary transactions between the agents to the marriage model and formulated it to the well-known "assignment game." They showed that there exists a set of stable assignments, called the *core* (no agent wants to deviate from their match) and it is identical to the solutions of a dual linear program. However, no explicit mechanism was offered on how to achieve a stable assignment in the core. Thanks to the natural usefulness of matching markets, various extensions of the assignment game have been developed focusing either on different behavioral settings or information structures [62, 223, 7]. Assignment games and matching markets have also been used and studied extensively in auction theory [165, 14]. For a complete and thorough overview of assignment games and matching models, see [197].

1.2.3 Routing and Congestion Games

One of the standard approaches to alleviate congestion in a transportation system has been the management of demand size due to the shortage of space availability and scarce economic resources by imposing an appropriate *congestion pricing scheme* [180, 76]. Such an approach focuses primarily on intelligent and scalable traffic routing, in which the objective is to guide and coordinate decision makers in choosing routes [177, 37, 259] that leads to optimizing the routing decisions in a transportation network [121, 220]. Game theory has been one of the standard tools that can help us investigate the impact of selfish routing on efficiency and congestion [138, 101]. By adopting a game-theoretic approach, advanced systems have been proposed to assign decision makers concrete routes or minimize travel time and study a Nash equilibrium

(NE) under different congestion pricing mechanisms [34, 42, 205, 45, 46, 47, 48, 56, 49].

An important and key theoretical approach in alleviating congestion is *routing/congestion games* [195, 54, 93, 130, 228], which are a generalization of the standard resource-sharing game of an arbitrary number of resources in a network with a finite number of travelers. For example, each traveler may contribute a certain amount of traffic from a source to a destination and affect the overall congestion on a route, thus increasing the travel time for all other travelers. Another important class of games is *potential games*, first introduced in [162], which represent an important branch of game theory. In a potential game the incentive of all players to change their strategy can be expressed using a single global function called the "potential" function. The potential function depends on the action sets of all decision makers and captures the changes of utility as the actions vary. Potential games have been used extensively in wide-ranging applications; for example, tax schemes of public goods [120], economics of shallow lakes [134], electricity markets [85]. Routing/congestion and potential games have played an instrumental role in understanding competition over shared resources. Both classes of games has been studied in multiple disciplines to model transportation and communication networks [175, 199, 211, 76], common-pool resource games in economics [176], and resource dilemma problems in psychology [186, 36].

In cases where a resource fails (e.g., if a road is viewed as a resource then too many people using it may lead to a traffic jam) or there is uncertainty over the resource's quantity/quality, the decision makers cannot collectively reach an efficient equilibrium. Resources with negative congestion externalities have been widely considered in congestion games [97, 16]. In this regard, our work deviates from the literature as we consider the overutilization of multiple different resources on each route in the transportation network as well as considering additional indirect costs to the travelers' (e.g., waiting cost at a transport hub). In our modeling framework, we consider *negative congestion externalities* by supposing that if the number of co-travelers that utilize the same route or mode of transportation increases, then a traveler's utility decreases

[6, 260].

1.2.4 Behavioral Model: Prospect Theory

So far, most of the existing game-theoretical literature in transportation and routing/congestion games assumes that the decision-makers behavior follows the *rational-choice theory*, i.e., each decision-maker is a risk-neutral (e.g., being indifferent to risk and only concerned with the expected outcome), selfish, utility maximizer [219]. This seems to turn most transportation models quite unrealistic, as unexpected travel delays can lead to uncertainty in a traveler's utility. More irrational decision making over uncertainties and risks in utility can play a significant role, and its study can help us understand how large-scale systems perform inefficiently. There is strong evidence with empirical experiments that show how a decision-maker's choices and preferences systematically may deviate from the choice and preferences of a decision-maker under the rational choice theory [110, 39]. This is because real-life decision making is seldomly truly rational, and biases affect how we make decisions. For example, decision makers compare the outcomes of their choices with a known expected amount of utility (called reference) and decide based on that reference whether their utility is a gain or loss. *Prospect theory* has laid down the theoretical foundations to study such biases and the subjective perception of risk in utility of decision makers (see the seminal papers [110, 238]). Prospect theory has been recognized as a closer-to-reality behavioral model for the decision making of humans and has been used in a wide range of applications and fields [39, 15], including recent studies in engineering [69, 168, 70]. There has also been considerable work at the intersection of transportation studies and prospect theory [61, 127, 71].

1.3 Research Gaps and Contributions

As urban social life has been considerably associated with the technological impact of the car, we are compelled to reassess the relationship between automobility and social life. Based on this, this dissertation focuses on paving the way for a sustained scholarly focus on understanding the societal effects and dynamics between human travelers and EMS (shared and self-driving cars and public transit). We do this by investigating several key open questions on the behavioral game-theoretic deeply economic relationships and interactions of travelers, passengers, drivers and the mobility system itself.

We offer a game-theoretic perspective and analysis for EMS by looking at the socioeconomic interactions of human travelers with self-driving cars and public transit in Chapter 2 following our work in:

(i) Chremos, I.V., & Malikopoulos, A.A. (2023). Socioeconomic impact of emerging mobility markets and implementation strategies. In AI-enabled Technologies for Autonomous and Connected Vehicles (pp. 481-510). Cham: Springer International Publishing.

(ii) Chremos, I.V., Beaver, L.E., & Malikopoulos, A.A. (2020, September). A game-theoretic analysis of the social impact of connected and automated vehicles. In 2020 IEEE 23rd International Conference on Intelligent Transportation Systems (pp. 2214-2219).

We ask: *Can we develop an efficient multimodal mobility system that can enhance accessibility while controlling the ratio of travel demand over capacity and and improve indirectly the welfare of all travelers, passengers, and drivers?* Studying this question in a game-theoretic setting in order to better understand how EMS might affect human tendency-to-travel and decision-making allowed us to address a key open question that has not been adequately approached yet. Understanding this "social" aspect of CAVs is critical in our effort to design efficient mobility systems. So, to address this question, our first step is to understand the behavioral interactions of travelers with different modes of transportation along with the implications to system efficiency. Thus, we study the game-theoretic interactions of travelers seeking to travel in a transportation network comprised of roads used by different modes of transportation (e.g., cars, buses, light rail, and bikes). A key characteristic of our approach is that we adopt the Mobility-as-a-Service (MaaS) concept, i.e., a multimodal mobility system that handles centrally the travelers' information and provides travel services (e.g., navigation, location, booking, payment). Our goal is to provide a game-theoretic framework that captures the most significant factors of a traveler's decision making in a transportation network under two different behavioral models. We also expanded our approach by addressing shared mobility.

For this question, we offer multiple different game-theoretic perspectives and analyses in Chapter 3 following our work in

(i) Chremos, I.V., & Malikopoulos, A.A. (2022). Mobility equity and economic sustainability using game theory. arXiv preprint, arXiv-2203.11421. In 2023 American Control Conference (to appear).

(ii) Chremos, I.V., & Malikopoulos, A.A. (2022). An analytical study of a two-sided mobility game. In 2022 American Control Conference (pp. 1254-1259).

We expand our work and incorporate prospect theory and establish formally the "mobility game" for multimodal transportation systems in Chapter 4 following our work in

(i) Chremos, I.V., Bang, H., Dave, A., Le, V.A., & Malikopoulos, A.A. (2023). Modeling travel behavior in mobility systems with an atomic routing game and prospect theory. arXiv preprint arXiv:2303.17790.

(ii) Chremos, I.V., & Malikopoulos, A.A. (2023). A traveler-centric mobility game: Efficiency and stability under rationality and prospect theory. PLoS ONE 18(5): e0285322.

Recall that, in this dissertation, we interested in the question *Can we develop an efficient multimodal mobility system that can enhance accessibility while controlling the* *ratio of travel demand over capacity and and improve indirectly the welfare of all travelers, passengers, and drivers?* So, at the same time, we offer multiple solutions from different perspectives at answering this question. We aim to provide a first-attempt answer and its solution and we argue that a sociotechnical approach focusing on the social dimension of a mobility problem can help us design the next-generation mobility systems. To achieve this, we consider a mobility system with decentralized information (alternatively called "asymmetric information") and multiple selfish and intelligent decision-makers (e.g., travelers), who, in turn, may misreport their true travel preferences for better individual benefits. Hence, based on their background and unique behavioral tendencies, travelers make decisions that generally do not lead to system-wide optimal performance. We tackle this *discrepancy between individual and collective interests* [45] by reverse-engineering the mobility system from its optimal solution (e.g., efficiency, congestion-free) to what should each traveler do via the implementation of monetary incentives. This method in economics is known as "mechanism design," in which by treating systems as economic institutions, we can control and coordinate the selfish agents' "economic activity" (e.g., which mode of transportation to use).

For this question, we explore a game-theoretic approach presented in Chapter 3 in our work:

(i) Chremos, I.V., & Malikopoulos, A.A. (2020). Social resource allocation in a mobility system with connected and automated vehicles: A mechanism design problem. In 2020 59th IEEE Conference on Decision and Control (pp. 2642-2647).

Another key fundamental question that we ask ourselves is whether the deployment of CAVs in society will give rise to unexpected outcomes. For example, will the overall vehicle miles traveled increase to the point where we observe a decrease in traveler usage of public transit? Shared mobility can be a cost-effective and flexible mode of transportation alongside CAVs and provide mobility access to city travelers without increasing congestion, pollution, accidents, and energy consumption. We design a *shared mobility market*, which is consisted of a finite number of travelers and vehicles, and it is managed by a social planner. Our goal is to measure the "benefit" received of both the travelers and the vehicles' operators, define the social welfare as a function of these benefits, and form a maximization problem with integer solutions subject to physically-related constraints. From a game-theoretic perspective, our proposed shared mobility market can be interpreted as an "assignment game," in which indivisible goods are exchanged between two parties for money [217].

For this work, we present our contributions in Chapter 3, following:

(i) Chremos, I.V., & Malikopoulos, A.A. (2021). Design and stability analysis of a shared mobility market. In 2021 European Control Conference (pp. 375-380).

Another important contribution of our work is the tutorial/review on the state of the art of mechanism design in control engineering following:

(i) Chremos, I.V., & Malikopoulos, A.A. (2023). Mechanism design theory in control engineering: A tutorial and overview of applications in communication, power grid, transportation, and security systems. arXiv preprint arXiv:2212.00756.

This dissertation primarily includes the main contributions of my research; however, there are several other publications listed as follows which were the outcomes of my research and collaborations with other colleagues

(i) Malikopoulos, A.A., Beaver, L.E., & Chremos, I.V. (2021). Optimal time trajectory and coordination for connected and automated vehicles. Automatica. 125. 109469.

(ii) Dave, A., Chremos, I.V., & Malikopoulos, A.A. (2022). Social media and misleading information in a democracy: A mechanism design approach. IEEE Transactions on Automatic Control. 67(5). 2633-2639.

1.4 Dissertation Outline

With the literature review completed and the major research gaps identified, we present our work in these areas in Chapters 2 - 4 in the form of five peer-reviewed conference papers and one journal paper submitted for a peer-review, and a chapter. The overarching goal for this dissertation is presented along with our research aims. Finally, we summarize the main contributions of this dissertation and provide some potential future directions in Chapter 5.

1.4.1 List of Abbreviations

In this subsection, we offer a table that provides a concise list of abbreviations used throughout the dissertation, along with their full forms for ease of reference.

Table 1.1: A summary of all abbreviations.

Abbreviation	Full Form
EMS	Emerging Mobility Systems
CAV	Connected and Automated Vehicles
NE	Nash Equilibrium
VCG	Vickrey-Clarke-Groves
MaaS	Mobility-as-a-Service
VP	Voluntary Participation
IC	Incentive Compatibility
PD	Prisoner's Dilemma
ERC	Equity, Reciprocity, and Competition
SCF	Social Choice Function
KKT	Karush–Kuhn–Tucker
OD	Origin-Destination
POA	Price of Anarchy
POS	Price of Stability
IDM	Intelligent Driver Model
IDS3C	IDS Lab's Scaled Smart City
BFGS	Broyden-Fletcher-Goldfarb-Shanno

1.4.2 List of Notation

In addition, we summarize the notation of key variables of our models that will be used throughout the dissertation. Note that all notation will be formally introduced and defined in each chapter as well.

Table 1.2: A summary of the dissertation's notation.

Symbol	Description
$\mathcal{I}$	Set of travelers
$\mathcal{J}$	Set of mobility services
$\mathcal{J}_h$	Set of mobility services of type h
$\mathcal{H}$	Set of different types of services
ε_j	Physical traveler capacity for service $j \in \mathcal{J}$
$\bar{\varepsilon}_j$	Maximum traveler capacity of service $j \in \mathcal{J}$
$\mathcal{G}$	Network with set of edges $\mathcal{E}$ and set of nodes $\mathcal{V}$
v_i	Node in network $\mathcal{G}$ that represents a transport hub
$\mathcal{P}^{(o,d)}$	Set of routes that connect the origin o to destination d
π_i	Mobility payment of traveler $i \in \mathcal{I}$
ρ_i	Route chosen by traveler $i \in \mathcal{I}$
$\mathcal{S}_{v_i}$	Set of co-travelers at transport hub v_i for traveler $i \in \mathcal{I}$
η_i	Socioeconomic characteristic of traveler $i \in \mathcal{I}$
$\mathcal{A}_i$	Set of actions for traveler $i \in \mathcal{I}$
$\mathcal{A}$	Cartesian product of all action sets
J_e	Total number of services of all types on road $e \in \mathcal{E}$
c_e	Travel time latency function

Chapter 2

MOBILITY AS A SOCIAL DILEMMA AND AN EMERGING MOBILITY MARKET

This chapter has two parts. In the first part, we address the much-anticipated deployment of connected and automated vehicles (CAVs) in society by modeling and analyzing the social-mobility dilemma in a game-theoretic approach. We formulate this dilemma as a normal-form game of players making a binary decision: whether to travel with a CAV (CAV travel) or not (non-CAV travel) and by constructing an intuitive payoff function inspired by the socially beneficial outcomes of a mobility system consisting of CAVs. We show that the game is equivalent to the Prisoner's dilemma, which implies that the rational collective decision is the opposite of the socially optimum. We present two different solutions to tackle this phenomenon: one with a preference structure and the other with institutional arrangements. In the first approach, we implement a social mechanism that incentivizes players to non-CAV travel and derive a lower bound on the players that ensures an equilibrium of non-CAV travel. In the second approach, we investigate the possibility of players bargaining to create an institution that enforces non-CAV travel and show that as the number of players increases, the incentive ratio of non-CAV travel over CAV travel tends to zero. We conclude by showcasing the last result with a numerical study.

The contributions of this chapter are: (i) we provide a game-theoretic analysis of the conflict of interest and model the social-mobility dilemma as a social dilemma, and (ii) we apply two different in mindset mechanisms and approaches that attempt to prevent negative outcomes, e.g., similar to the Tragedy of the Commons. Several research efforts reported in the literature have focused on studying social behavior regarding semi-autonomous driving and the selfish social decision-making of choosing a route to commute in a transportation network [156]. A key difference between our work and the frameworks already reported in the literature is that we focus on modeling the human decision-making of which mode of transportation to be used rather than modeling selfish routing. Our analysis will complement these efforts by providing a framework that attempts to integrate the human social behavior in a mobility system consisting of CAVs. Moreover, our work in this chapter expands the much-needed discussion on understanding the social impact and implications of CAVs by providing insights on how human behavior might react to an emerging mobility system. More specifically, our most important contribution is to rigorously show that without a well-thought intervention via regulations or incentives, a society of selfish travelers will make the wrong collective decisions, and thus, we will end up with a catastrophically sub-optimal performance of the emerging mobility system.

In the second part, our aim is to develop a holistic and rigorous framework to capture the societal impact of connectivity and automation in emerging mobility systems and provide solutions that prevent any potential rebound effects (e.g., increased vehicle-miles-traveled, increased travel demand, empty trips). To achieve this aim, as a first attempt, we study an emerging mobility system consisting of a finite group of travelers who seek to travel in a "smart city," where a central authority (alternatively called social planner) seeks to ensure the efficient distribution and operation of the different modes of transportation offered by the city. We call these different modes of transportation "mobility services." A few examples of mobility services are CAVs, shared vehicles, and public transit (e.g., train, bus, light rail, subway). The travelers request to use at most one service to satisfy their mobility needs, i.e., to reach their destination, via a smartphone app easily accessible to all travelers. The social planner (e.g., a central computer) compiles all travelers' origin-destination requests and other information (e.g., preferred travel time, value of time, and maximum willingness-to-pay) in order to provide a travel recommendation to each traveler. The social planner's goal is to ensure that the aggregate travel recommendations are *socially-efficient*. Informally, by socially-efficient, we mean that the endmost collective travel recommendation must achieve two objectives: (i) respect and satisfy the travelers' preferences regarding mobility, and (ii) ensure the alleviation of congestion in the system. Since our focus is to provide socially-efficient solutions, we consider a city that supports connected and automated mobility technologies on its roads and public transit infrastructure. Subsequently, the social planner is fully aware of the system's capabilities and network's capacity. In other words, the social planner is fully capable of computing the maximum capacity of each mobility service and the associated costs aimed at providing travel recommendations to all travelers. Our objective is to design a mobility market of an emerging mobility system and provide a socially-efficient solution consisting of well-designed and appropriate monetary incentives (e.g., tolls, fares, fees) for a social planner to guarantee the realization of the desired outcome, i.e., maximize the social welfare of all travelers. At the same time, our solution will ensure to provide such incentives to travelers so that the usage of any mobility service will not lead to congestion in the mobility system. In other words, we design a mobility market that efficiently assigns each traveler to the "right" mode of transportation.

Our contributions are the following: (i) we design a socially-efficient mobility market that assigns mobility services to a finite group of travelers by taking into consideration their travel preferences. We achieve that by implementing a special case of the VCG mechanism after modifying it accordingly for a mobility problem. (ii) We show that the proposed mobility market is incentive compatible and individually rational, two properties that ensure all selfish travelers are truthful in their communication with the social planner and voluntarily participate in the mobility market. (iii) We also show that the proposed market is economically sustainable, i.e., it generates revenue from each traveler and ensures that the operating costs of each mobility service are covered. It is through the appropriate design of monetary incentives that we successfully incentivize all travelers to truthfully report their travel preferences and voluntarily participate in the market. Thus, we are guaranteed a socially-efficient mobility solution. The proposed mobility market also provides an incentive to central authorities to implement it, since as we show, the market ensures that there are minimum acceptable payments to cover the operating costs of the mobility services.

The first part of the chapter proceeds as follows. In Subsection 2.1.1, we provide an overview of Game Theory notions. In Subsection 2.1.2, we present our formulation of the social decision-making regarding the CAVs as a normal-form game and show that it is equivalent to a PD game. In Subsection 2.1.3, we introduce and study a preference structure, and in Subsection 2.1.4, we apply a framework of institutions and provide a numerical study of the results. The second part of the chapter is structured as follows. In Subsection 2.2.1, we review the main concepts of mechanism design and briefly discuss the VCG mechanism. In Subsection 2.2.2, we present the mathematical formulation of the emerging mobility market, which forms the basis for the rest of the chapter. In Subsection 2.2.3, we present the imposed optimization problem. In Subsection 2.2.5, we present the methodology used to design the monetary incentives for each traveler. In Subsection 2.2.6, we study the properties of the mobility market, and finally, we draw conclusions.

2.1 A Mobility System as a Social Dilemma

2.1.1 Mathematical Preliminaries

In this section, we present a brief overview of important notions from non-cooperative Game Theory. First, we assume that the players of the game are *rational*, in the sense that each player's objective is to maximize the expected value of her own payoff. In addition, we assume that the players are *intelligent*, i.e., each player has full knowledge of the game and has the ability to make any inferences about the game that we, the designers, can make. In order to develop a rigorous framework that analyzes the social dilemma as a game, we need to formally define a few important notions of Game Theory that will prove instrumental in our analysis.

Definition 2.1.1. *A finite normal-form game is a tuple* $\mathcal{G} = \langle \mathcal{I}, \mathcal{S}, (u_i)_{i\in\mathcal{I}} \rangle$*, where* $\mathcal{I} = \{1, 2, \ldots, n\}$ *is a finite set of* n *players with* $n \geq 2$*;* $\mathcal{S} = S_1 \times \cdots \times S_n$*, where* S_i *is a finite set of actions available to player* $i \in \mathcal{I}$ *with* $s = (s_1, \ldots, s_n) \in \mathcal{S}$ *being the action profile;* $u = (u_1, \ldots, u_n)$*, where* $u_i : \mathcal{S} \to \mathbb{R}$*, is a real-valued utility function for player* $i \in \mathcal{I}$*.*

Definition 2.1.2. *Let* S_i *be the strategy profile of player* i*,* $s_i, s_i' \in S_i$ *be two strategies of player* i*, and* S_{-i} *be the set of all strategy profiles of the remaining players. Then,* s_i *strictly dominates* s_i' *if, for all* $s_{-i} \in S_{-i}$*, we have* $u_i(s_i, s_{-i}) > u_i(s_i', s_{-i})$*. Also, a strategy is strictly dominant if it (strictly) dominates any other strategy.*

Definition 2.1.3. *A player* i*'s best response to the strategy profile*

$$s_{-i} = (s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_n) \tag{2.1}$$

is the strategy $s_i^* \in S_i$ *such that* $u_i(s_i^*, s_{-i}) \geq u_i(s_i, s_{-i})$ *for all* $s_i \in S_i$*. A strategy profile* s *is a Nash equilibrium (NE) if, for each player* i*,* s_i *is a best response to* s_{-i}*.*

Next, for completeness, we define the notion of Pareto domination. First, an "outcome" of a game is any strategy profile $s \in \mathcal{S}$. Intuitively, an outcome that Pareto dominates some other outcome improves the utility of at least one player without reducing the utility of any other.

Definition 2.1.4. *Let* $\mathcal{G}$ *and* $s', s \in \mathcal{S}$*. Then a strategy profile* s' *Pareto dominates strategy* s *if,* $u_i(s') \geq u_i(s)$*, for all* i*, and there exists some* $j \in \mathcal{I}$ *for which* $u_j(s') > u_j(s)$*.*

Pareto domination is a useful notion to describe the social dilemma in a game. However, Pareto-dominated outcomes are often not played in Game Theory; a NE will always be preferred by rational players. For further discussion of the Game Theory notions presented above, see [219].

Next, we provide our formulation and show that it is equivalent to the PD game.

2.1.2 Game-Theoretical Formulation

We consider a society of $n \in \mathbb{N}$, $n > 2$, travelers who seek to commute on a city's transportation network. We consider the road infrastructure as the common, yet limited, resource that is open-access and shared with all travelers. Each traveler has the option to utilize the roads by traveling in a CAV, which in turn contributes to the capacity of the roads. We expect each traveler to utilize the roads selfishly.

Assumption 2.1.5. *We assume full CAV-penetration, and so each traveler may choose either to travel in a CAV or use another mode of transportation, e.g., train, light rail, bicycling, or walking, thereby not contributing to congestion.*

In a game-theoretic context, each traveler represents a rational player who has two possible actions, namely either NC for not traveling in a CAV (non-CAV travel) or C for traveling in a CAV (CAV travel). From now on, we shall use the terms "player" and "traveler," interchangeably. All players receive a benefit $c \in \mathbb{R}_{>0}$ for deciding to commute in the society. On the other hand, traveling using CAVs conveys benefits arising from flexibility, privacy, convenience. So, if a player chooses to travel in a CAV, then they receive a benefit of $c + d$, where $d \in \mathbb{R}_{>0}$ with $d \cdot (n - 2) > 2$ (this ensures that d provides a significant incentive for CAV travel yet the lower bound decreases as n increases). However, traveling in a CAV is naturally the selfish choice as it exploits the society's resources. Hence, for each player that decides to travel in a CAV, a cost of $e \in \mathbb{R}_{>0}$ is imposed to the society as a whole and is paid out equally by all players, i.e., we define $\phi = e/n$ as the damage done to society. Without losing any theoretical insight, let us define $e = d + 1$ and assume that the original benefit c is strictly greater than e.

Remark 2.1.6. *In our formulation, we want to capture the potential consequence of the players' decision to travel in a CAV. For this reason, contributing to the capacity of the roads (creating congestion, pollution) is represented by the cost and overall by the damage done to society.*

We can write the final form of player's i payoff for traveling in a CAV as $(c+d)-(n-k)\phi$, and accordingly, player's i payoff for not traveling in a CAV as $c-(n-k-1)\phi$, where k is the number of players who choose not to travel in a CAV other than player i. Thus, the payoff function is

$$f_i(s_i, k) = \begin{cases} c - (n-k-1)\phi, & \text{if } s_i = NC, \\ c + d - (n-k)\phi, & \text{if } s_i = C. \end{cases} \tag{2.2}$$

For player i the benefit of traveling in a CAV is denoted by $f_i(C, k)$ and the benefit of not traveling in a CAV by $f_i(NC, k)$, where k is the number of players who decide to non-CAV travel other than player i. Note that (2.2) depends not only on player i's own action but also on k.

At this point, we can formally formulate our game denoted by $\mathcal{G}$. We have the finite set of players $\mathcal{I} = \{1, \dots, n\}$ with $n > 2$; for each player i the action set is $s_i \in \{NC, C\}$, and $f_i(s_i, k)$ with $k = 0, 1, \dots, n-1$ is the payoff function of player i. Thus, our game can be represented by the following tuple:

$$\mathcal{G} = \langle \mathcal{I}, (S_i = \{NC, C\})_{i \in \mathcal{I}}, (f_i(s_i, k))_{i \in \mathcal{I}} \rangle . \tag{2.3}$$

Next, we fully characterize game $\mathcal{G}$.

Lemma 2.1.7. *The payoff difference $\alpha = f_i(C, k) - f_i(NC, k)$ is positive and constant for all values $k \in [0, n-1]$ and for all players $i \in \mathcal{I}$. Furthermore, $f_i(NC, k)$ and $f_i(C, k)$ are strictly increasing in k.*

Proof. We have $f_i(C, k) = c + d - (n-k)\phi$ and $f_i(NC, k) = c - (n-k-1)\phi$ and so the difference is simply $f_i(C, k) - f_i(NC, k) = c + d - (n-k)\phi - [c - (n-k-1)\phi] = d - \phi$. Hence, α is clearly positive by definition of c and d and also constant for all values $k = [0, n-1]$. Furthermore, for $k > k'$, we have

$$f_i(NC, k) = c - (n-k-1)\phi, \quad \text{and} \tag{2.4}$$

$$f_i(NC, k') = c - (n-k'-1)\phi. \tag{2.5}$$

Subtracting (2.5) from (2.4) gives $f_i(NC,k) - f_i(NC,k') = (k-k')\phi > 0$, and so $f_i(NC,k) > f_i(NC,k')$ for all k. In similar lines, we can show that the benefit of CAV travel, $f_i(C,k)$, is strictly increasing in k. Therefore, we conclude that $f_i(NC,k)$ and $f_i(C,k)$ are strictly increasing in k. □

From now on, the payoff difference is denoted by α. We observe that the payoff difference, interpreted as the non-CAV travel cost, increases as n increases. Interestingly enough, the payoff difference is independent of how many players choose not to travel in a CAV. In game-theoretic terms, we can interpret this as the strategy CAV travel dominating strategy non-CAV travel with a degree that is constant and independent of the other players who choose to CAV travel.

Lemma 2.1.8. *The payoff function* (2.2) *is non-negative for all* $k \in [0, n-1]$*, i.e.,* $f_i \geq 0$ *for all* $i \in \mathcal{I}$*. Furthermore, mutual non-CAV travel is preferred to mutual CAV travel, i.e.,* $f_i(NC, n-1) > f_i(C, 0)$ *is a Pareto relation.*

Proof. We have $f_i(NC,0) = c-(n-1)\phi = c-(d+1)+\phi$ and $f_i(C,0) = c+d-n\cdot\phi = c-1$. As $f_i(NC,k)$ and $f_i(C,k)$ are increasing in k, the result follows. Also, we have $f_i(C,0) = c-1$ and $f_i(NC,n-1) = c-(n-(n-1)-1)\phi = c$ leading to $f_i(NC,n-1) > f_i(C,0)$ for all $i \in \mathcal{I}$. □

Lemma 2.1.8 establishes the fact that game $\mathcal{G}$ induces a Pareto relation, which implies that the equilibrium of mutual CAV travel is Pareto inferior to the alternative outcome, i.e., all players choose to non-CAV travel. This is significant since Pareto relations are directly associated with social dilemmas.

Theorem 2.1.9. *Game* $\mathcal{G}$ *defined in* (2.3) *is equivalent to the PD game as both games share an equivalent incentive structure.*

Proof. By Lemma 2.1.7, we have $f_i(NC,k) < f_i(C,k)$ for all $k \in [0, n-1]$ which implies that the dominant strategy by rational players in the game is CAV travel no matter

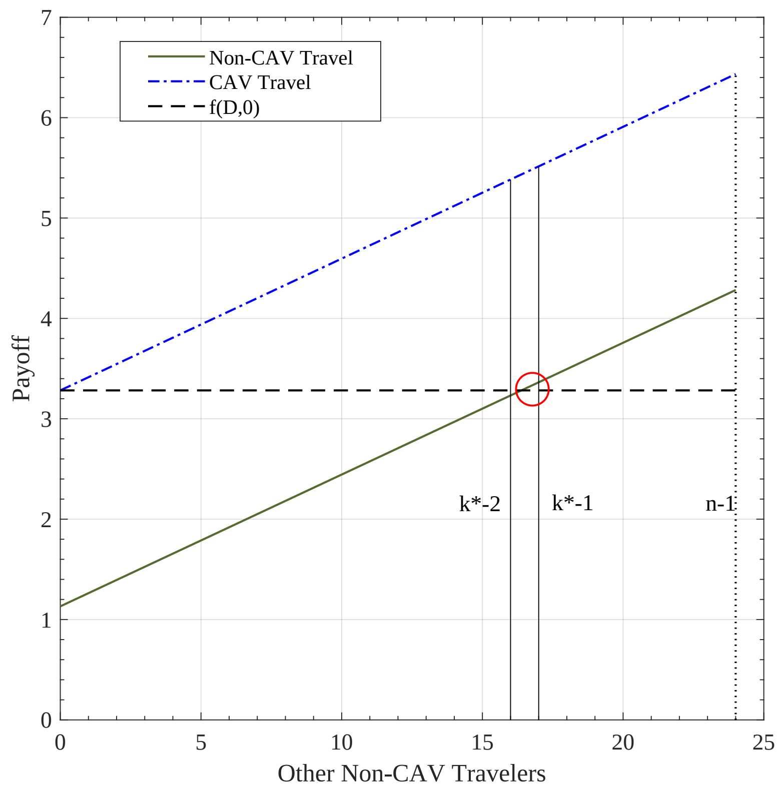

Figure 2.1: A visualization of the payoff function (2.2) evaluated using the values $d = 2.2827$, $c = 4.2827$, and $n = 25$. We notice that, by focusing on the red circle, with a certain number of non-CAV travelers the overall utility of non-CAV travel is greater than the utility of CAV travel. This is the true meaning of a social dilemma in a CAV transportation context.

how many players decide to non-CAV travel. By Lemma 2.1.8, the social dilemma induced structure is equivalent to that of the Prisoner's dilemma. $\square$

Corollary 2.1.10. *The game defined in* (2.3) *and the PD game provide equivalent incentives to the players, and thus, they result in equivalent outcomes.*

Next, we show that by construction of the payoff function (2.2), non-CAV travel is more attractive from both the societal and the player's perspective.

Proposition 2.1.11. *Consider the game $\mathcal{G}$ defined in* (2.3)*. Note that the benefit of CAV travel is given by $f_i(C, k)$ and the cost of non-CAV travel given by α (i.e., the* *payoff difference). Then the strategy non-CAV travel is socially desirable:*

$$nf_i(C, k+1) - (k+1)\alpha > nf_i(C, k) - k\alpha, \quad \forall i \in \mathcal{I}, \tag{2.6}$$

and also individually desirable:

$$f_i(NC, k+1) > f_i(NC, k), \quad \forall i \in \mathcal{I}. \tag{2.7}$$

Proof. Both (2.6) and (2.7) can be verified by substitution of the corresponding functions in (2.2). □

Before we continue, let us introduce the notation $\lfloor x \rfloor$, which denotes the greatest integer that is less than x.

Proposition 2.1.12. *Consider game $\mathcal{G}$ defined in (2.3). There exists a unique integer $2 \le k^* \le n$ given by $k^* = \left\lfloor \frac{nd}{d+1} \right\rfloor + 1$ such that*

$$f_i(NC, k^* - 2) < f_i(C, 0) < f_i(NC, k^* - 1), \tag{2.8}$$

where k^* is the minimum number of non-CAV travelers.

Proof. By substitution, we get the following equations:

$$\begin{aligned} f_i(NC, k^* - 2) &= c - (n - (k^* - 2) - 1)\phi \\ &= c - (n - k^* + 1)\phi, \end{aligned} \tag{2.9}$$

$$f_i(C, 0) = c - 1, \tag{2.10}$$

$$\begin{aligned} f_i(NC, k^* - 1) &= c - (n - (k^* - 1) - 1)\phi \\ &= c - (n - k^*)\phi. \end{aligned} \tag{2.11}$$

We want to find a unique k^* such that (2.8) holds. So, we have

$$c - (n - k^* + 1)\phi < c - 1 < c - (n - k^*)\phi \tag{2.12}$$

which leads to

$$\frac{nd}{d+1} < k^* < \frac{nd}{d+1} + 1. \tag{2.13}$$

As k^* is an integer, the last inequality (2.13) is true if and only if $k^* = \left\lfloor \frac{nd}{d+1} \right\rfloor + 1$ and $\frac{nd}{d+1}$ is not an integer number. □

Proposition 2.1.12 intuitively implies that we need at least k^* non-CAV travelers so that the benefit a player receives when they decide non-CAV travel will be greater than the dominant strategy $f(C, 0)$ (see in Fig. 2.1 the red circle).

Next, we seek a way to characterize an outcome of the game in terms of preference. Now, in most cases, identifying the "best" outcome is not possible, but there are certain situations that might be better from a societal standpoint.

Proposition 2.1.13. *The strategy of universal CAV travel, $f(C, 0)$, is Pareto dominated by outcomes with $k' \geq k^* - 1$.*

Proof. We want to show that the outcomes with $k' \geq k^* - 1$ Pareto dominate the dominant strategy of universal CAV travel. We only have to check two cases, namely $k' \geq k^* - 1$ and $k' < k^* - 1$. For $k' = k^* - 1$, we have

$$f_i(NC, k') = c - \left(n - \left\lfloor \frac{nd}{d+1} \right\rfloor - 1\right)\phi. \tag{2.14}$$

Let $\left\lfloor \frac{nd}{d+1} \right\rfloor = \frac{nd}{d+1} - \varepsilon$, where $\varepsilon > 0$, so that

$$\begin{aligned} f_i(NC, k') &= c - \left(n - \frac{nd}{d+1} - \varepsilon - 1\right)\left(\frac{d+1}{n}\right) \\ &= c - 1 + (\varepsilon + 1)\phi. \end{aligned} \tag{2.15}$$

Subtracting $f_i(C, 0)$ from $f_i(NC, k')$ gives $(\varepsilon + 1)\phi > 0$. Furthermore, for $k' > k^* - 1$, note that f_i is a strictly increasing function in k, thus $f_i(NC, k') > f_i(NC, k^* - 1)$ which implies $f_i(NC, k') > f_i(C, 0)$. Thus, for all players i, $f_i(NC, k') > f_i(C, 0)$, where $k' \geq k^* - 1$. On the other hand, if $k' < k^* - 1$, then $f_i(NC, k') < f_i(C, 0)$ for all players i by the first inequality relation in Proposition 2.1.12. Hence, all outcomes which satisfy $k' \geq k^* - 1$ Pareto dominate the dominant strategy of universal CAV travel. □

We note that by construction, the payoff function (2.2) mutual non-CAV travel is the social optimum but, as a consequence of Proposition 2.1.13, the decision to non-CAV travel is worthwhile to a player only if there are k^* or more non-CAV travelers. Otherwise, everyone is no worse off at the dominant strategy of universal CAV travel. This gives rise to the notion of the state of *minimally effective non-CAV travel*.

Definition 2.1.14. *The state of minimally effective non-CAV travel is the minimum number of non-CAV travelers, k^*, such that an outcome Pareto dominates the universal CAV travel equilibrium.*

Clearly, the state of minimally effective non-CAV travel is given by Propositions 2.1.12 and 2.1.13. This is an important notion that can help in the derivation of the optimal utilization of CAVs in the emerging transportation systems.

Next, we discuss two solution approaches applied in our game $\mathcal{G}$. Our goal is to derive conditions that ensure a coalition of non-CAV travel, which are at least as large as the minimum state of non-CAV travel.

2.1.3 Nash Equilibria and the Population Threshold

Usually, in Game Theory, we assume that players are only interested in their own payoff. One of our goals is to study, in a more realistic setting, the players' social behavior, and so we impose to our game $\mathcal{G}$ a "preference structure."

A preference structure allows us to model a particularly interesting scenario: the rational players are interested not only on their own payoff but also on the relative payoff share they receive, i.e., how their standing compares to that of others [160]. The authors in [31] designed the "equity, reciprocity, and competition (ERC)" model which is a simple model capable of handling a large population of players with an "adjusted utility" function constructed on the premise that players are motivated by both their pecuniary payoff and their relative payoff standing. Notice that we changed our terminology of payoff function to adjusted utility function here. We do this to differentiate the difference between the absolute payoffs that players get from (2.2) and the adjusted payoffs players will get in a preference structure. One of the reasons we use the ERC model is because it has been successful in explaining the behavior of selfish players in social experiments than other standard modeling techniques.

Now, we observe that players rarely play against the same other players, and so it is reasonable enough to analyze each game as one-shot. To further justify this, we only have to argue that it is highly unlikely to "meet" other travelers in a major metropolitan city. Let the absolute payoff of player i be given by f_i from (2.2). Each player i seeks to maximize the expected utility of her motivation function $v_i = v_i(f_i, \sigma_i)$, where

$$\sigma_i = \sigma_i(f_i, \gamma, n) = \begin{cases} f_i/\gamma, & \text{if } \gamma > 0, \\ 1/n, & \text{if } \gamma = 0. \end{cases} \tag{2.16}$$

Equation (2.16) represents player i's relative share of the payoff and $\gamma = \sum_{j=1}^{n} f_j$ is the total pecuniary payout. We can think of the motivation function v_i as the expected benefits that drive the players' behavior. We assume that v_i is twice differentiable.

Next, we allow each player to be characterized by a_i/b_i which is the ratio of weights that are attributed to the pecuniary and relative components of the motivation function. For example, strict relativism is represented by $a_i/b_i = 0$, so $\arg\max_{\sigma_i} v_i(\gamma\sigma_i, \sigma_i) = \pi = 1/2$, where $\pi_i(\gamma)$ is implicitly defined by $v_i(\gamma\pi_i, \pi_i) = v_i(0, 1/n)$ for $\pi_i \leq 1/n$. Strict narrow self-interest is the limiting case $a_i/b_i \to \infty$, so $\arg\max_{\sigma_i} v_i(\gamma\sigma_i, \sigma_i) = 1$ and $s \to 0$ [31]. Based on the above, the adjusted utility function then is given by:

$$u_i(f_i, \sigma_i) = a_i q(f_i) + b_i r(\sigma_i), \tag{2.17}$$

where $q(\cdot)$ is strictly increasing, strictly concave, and differentiable; $r(\cdot)$ is differentiable, concave, and has its maximum at $\sigma_i = 1/n$. Let us discuss a simple example from [31].

Example 1. We can explicitly define both q and r as:

$$q(f_i) = f_i \quad \text{and} \quad r(\sigma_i) = -\frac{1}{2}\left(\sigma_i - \frac{1}{n}\right)^2, \tag{2.18}$$

where function $q(\cdot)$ expresses the standard preferences for the payoff functions (2.2); function $r(\cdot)$ describes in a precise way the collective importance of equal division of the payoffs (this is also called the "comparative effect.") Consequently, the further the allocation moves from player i receiving an equal share, the higher the loss from the comparative effect.

Our analysis in this subsection follows [160], but we apply it to our game $\mathcal{G}$ defined in (2.3) along with the preference structure. Our goal is to study what influences strategic agents to non-CAV travel in our game $\mathcal{G}$.

We start our analysis by looking at the necessary and sufficient conditions for player i to non-CAV travel, i.e.,

$$u_i(f_i(NC, k+1)) \geq u_i(f_i(NC, k)). \tag{2.19}$$

Equivalently, we have from [160] that $a_i/b_i \leq \delta(k)$, where

$$\delta(k) = \frac{r\left(\frac{f_i(NC,k+1)}{nf_i(C,k+1)-(k+1)\alpha}\right) - r\left(\frac{f_i(C,k)}{nf_i(C,k)-k\alpha}\right)}{q\big(f_i(C,k)\big) - q\big(f_i(NC,k+1)\big)}. \tag{2.20}$$

From (2.20), we can deduce that player i will non-CAV travel if, and only if, there is overcompensation for the loss in absolute gain by moving closer to the average gain [160]. Hence, we can state the general conditions of a NE:

$$a_i/b_i \leq \delta(k-1), \quad \text{for } k \text{ players non-CAV travel}, \tag{2.21}$$

$$a_i/b_i \geq \delta(k), \quad \text{for } n-k \text{ players CAV travel}. \tag{2.22}$$

We now have a better understanding of how the number of other non-CAV travelers, and its value can make non-CAV travel a rational strategy.

Lemma 2.1.15. *For a given distribution of ERC-types, $\delta(k-1) > 0$ is necessary but not sufficient to get a coalition size of k where $n-k$ players free-ride. For a given payoff structure with $\delta(k-1) > 0$, there exist ERC-types such that k is an equilibrium coalition size.*

Proof. If $\delta(k-1) < 0$, it is impossible for a coalition to form in the game of size k. On the other hand, if $a_i/b_i > \delta(k-1)$ then condition (2.21) cannot hold for any player. However, conditions (2.21) and (2.22) imply that if $\delta(k-1) > 0$, then there are types $(a_i/b_i)_{i \in \mathcal{I}}$ such that k players non-CAV travel and $n-k$ players free-ride. □

Proposition 2.1.16. *By construction of the game $\mathcal{G}$ together with the ERC preference structure, there always exists a Nash equilibrium of universal CAV travel.*

Proposition 2.1.16 follows directly from Lemma 2.1.15. We are though interested in finding a threshold of players that decide to non-CAV travel. The next proposition will help us do that.

Proposition 2.1.17. *The necessary condition for an equilibrium of non-CAV travel $\delta(k-1) > 0$ is equivalent to*

$$n\left[(k-1)f_i(C,k) - kf_i(C,k-1)\right] + \left[nf_i(C,k-1) - (k-1)\alpha\right]\left[2k-n\right] > 0. \quad (2.23)$$

Proof. In order to obtain $\delta(k^*-1) > 0$, it is necessary that by choosing the strategy CAV travel, a player further deviates from the equal share $1/n$ than by choosing strategy non-CAV travel, i.e.,

$$\frac{f_i(C,k-1)}{nf_i(C,k-1) - (k-1)\alpha} - \frac{1}{n} > \frac{1}{n} - \frac{f_i(NC,k)}{nf_i(C,k) - k\alpha}. \quad (2.24)$$

Rearranging and by eliminating the denominators, we get

$$nf_i(C,k-1)(nf_i(C,k) - k\alpha) + n(f_i(C,k) - \alpha)(nf_i(C,k-1) - (k-1)\alpha) \\ - 2(nf_i(C,k-1) - (k-1)\alpha)(nf_i(C,k) - k\alpha) > 0,$$

where we have used $\alpha = f_i(C,k) - f_i(NC,k)$. Substituting the payoff functions from (2.2) and further simplification yield

$$n\left[(k-1)f_i(C,k) - kf_i(C,k-1)\right] + n(2k-n)f_i(C,k-1) \\ - (k-1)\alpha[2k-n] > 0. \quad (2.25)$$

Simplifying (2.25) gives

$$n\left[(k-1)f_i(C,k) - kf_i(C,k-1)\right] + \left[nf_i(C,k-1) - (k-1)\alpha\right][2k-n] > 0. \quad (2.26)$$

Therefore, the result follows immediately. □

We are now ready to prove the main result of the section.

Theorem 2.1.18. *For any given vector of types, a rational player chooses to non-CAV travel when at least half of the players non-CAV travel.*

Proof. We only have to check on what conditions relation (2.23) is positive. By construction the payoff functions are non-negative, and thus $nf_i(C,k-1) - (k-1)\alpha > 0$, i.e.,

$$nf_i(C,k-1) - (k-1)\alpha = n(c-1) + (k-1)(1+\phi) \quad (2.27)$$

which is clearly positive for all values of n, c, and k. Hence, the second component of (2.23) is positive for $2k - n > 0$. Next, we look at the first component of (2.23). By substituting the payoff function from (2.2), we get

$$(k-1)f_i(C,k) - kf_i(C,k-1) = 1 - c, \quad (2.28)$$

which is negative for all values of c. We observe though that the second component is much bigger and dominates the first component as long as $2k - n > 0$. Hence, relation (2.23) is positive and we have $\delta(k-1) > 0$ for $2k > n$. Therefore, for any given vector of types, if a player cooperates at the equilibrium, then at least half of the players cooperate. □

The interpretation of Theorem 2.1.18 is that for any coalition to exist with size $k \geq 2$, a minimum of $n/2$ players must join. We showed that given the specific payoff structure of our game $\mathcal{G}$ and along with the ERC preference structure, a coalition of players choosing strategy non-CAV travel could be formed provided that it is rather large. Thus, even if we impose a social mechanism that enforces strategy non-CAV travel in a society of travelers and satisfying (2.23), a coalition of at least size $n/2$ must be formed to create an equilibrium of non-CAV travel. Therefore, the social mechanism will require significant influence over the players' behaviors in order to create a state of effective non-CAV travel. On the other hand, this result is promising as it shows that a social solution can potentially prevent self-centered and destructive behavior towards society.

2.1.4 Creating an Institutional Arrangement

In this section, we take advantage of the equivalency of our game $\mathcal{G}$ to the Prisoner's dilemma in order to use the non-cooperative game model of institutional arrangements framework of [173]. We prove in Theorem 3 that the ratio of non-CAV travel and CAV travel in a deregulated society as the number of players increases, tends to zero. In other words, as the society becomes larger and larger, the incentive to cooperatively agree not to travel in a CAV tends to zero.

Players are free to create a social institution that binds them by selecting their actions. In other words, players agree to have an institutional arrangement with the purpose of enforcing an agreement of non-CAV travel. The first stage is the creation of a social institution, and this is done through participation negotiations, and thus the first stage is called "participation decision stage." All players have to decide whether they will participate in negotiations for collective decision making, or not, without any knowledge of each others' decisions. The outcome of the game at this first stage is either that some group of players is formed or not. All players decide to participate in negotiations or not based on their expectations about what will happen in the rest of the game. The possibility of non-CAV travel is significantly affected by the number of players. That means that the outcome of the institutional arrangements depends on the players' decisions in the first stage [173].

Remark 2.1.19. *In contrast to Cooperative Game Theory, there is no external binding enforcement, and players are free to make their decisions (whether it is beneficial* *to them only). Thus, we treat the institutional arrangements framework as a non-cooperative game.*

The goal here is to investigate the question: *does the number of travelers affect the possibility of non-CAV travel?* The next proposition addresses the basic cases.

Proposition 2.1.20. *([173]) Let $d_i = 1$ denote a player i's decision to participate in bargaining for installing an enforcement agency; otherwise $d_i = 0$. When $k^* = n$, the participation decision stage has a unique solution $d^* = (1, \dots, 1)$.*

It is interesting enough that in the special case $n = 2$, both players agree to create an enforcement agency and also to non-CAV travel in the institutional arrangements.

Definition 2.1.21. *The incentive ratio of non-CAV travel and CAV travel can be defined as a positive number given by:*

$$\beta = \frac{f_i(NC, k^* - 1) - f_i(C, 0)}{f_i(C, k^* - 1) - f_i(NC, k^* - 1)}. \tag{2.29}$$

In words, β represents the ratio of players' incentive to form the minimum group for non-CAV travel, i.e., the group of k^* non-CAV travelers, over their incentive to deviate unilaterally from the minimum group for non-CAV travel. Given our game $\mathcal{G}$ defined in (2.3), we have

$$\beta = \frac{k^*\phi - d}{d - \phi} = \frac{k^*(d + 1) - nd}{d(n - 1) - 1}. \tag{2.30}$$

Proposition 2.1.22. *([173]) The uncooperative solution of the institutional arrangements for our game $\mathcal{G}$ prescribes the following player behavior:*

1. *If $n = \lfloor \frac{nd}{d+1} \rfloor + 1 = k^*$, then all players participate in bargaining and they agree to non-CAV travel.*

2. *If $n \geq \lfloor \frac{nd}{d+1} \rfloor + 2$, then every player participates in bargaining with probability $t(n)$ satisfying:*

$$\beta = \sum_{k^* \leq k \leq n-1} \frac{(n - k^*) \cdot \ldots \cdot (n - k)}{k^* \cdot \ldots \cdot k} \left(\frac{t}{1 - t} \right)^{k - k^* + 1},$$

where k^* and β are given by Proposition 2.1.12 and (2.30), respectively.

We are ready now to prove our main result of this section, which has to do with the limiting behavior of β.

Theorem 2.1.23. *As the number of players increases, the incentive ratio of non-CAV travel and CAV travel vanishes, i.e., β tends to zero as n tends to infinity.*

Proof. Substitute $k^* = \left\lfloor \frac{nd}{d+1} \right\rfloor + 1$ into β to get

$$\beta = \frac{\left(\left\lfloor \frac{nd}{d+1} \right\rfloor + 1\right)(d+1) - nd}{d(n-1) - 1}. \tag{2.31}$$

By Proposition 2.1.12, $\frac{nd}{d+1}$ is not an integer, thus we can write $\left\lfloor \frac{nd}{d+1} \right\rfloor = \frac{nd}{d+1} - \varepsilon$, where $\varepsilon > 0$. Now taking the limit of β as n goes to infinity gives

$$\lim_{n\to\infty} \beta = \lim_{n\to\infty} \frac{\left(\left\lfloor \frac{nd}{d+1} \right\rfloor + 1\right)(d+1) - nd}{d(n-1) - 1}, \tag{2.32}$$

or equivalently

$$\lim_{n\to\infty} \beta = \lim_{n\to\infty} \frac{(\frac{nd}{d+1} - \varepsilon + 1)(d+1) - nd}{d(n-1) - 1} \tag{2.33}$$

$$= \lim_{n\to\infty} \frac{nd + (-\varepsilon + 1)(d+1) - nd}{d(n-1) - 1} \tag{2.34}$$

$$= \lim_{n\to\infty} \frac{(-\varepsilon + 1)(d+1)}{d(n-1) - 1}. \tag{2.35}$$

We divide both numerator and denominator by $1/n$ and using the standard limit $\lim_{x\to\infty} \frac{1}{x} = 0$ gives the result, i.e.,

$$\frac{\lim_{n\to\infty} \frac{(-\varepsilon+1)(d+1)}{n}}{d - \lim_{n\to\infty} \frac{d}{n} - \lim_{n\to\infty} \frac{1}{n}} = 0. \tag{2.36}$$

Thus, we conclude that $\lim_{n\to\infty} \beta = 0$. □

To complement our understanding, we performed a numerical study of the limiting behavior of $t(n)$, given in Table 2.1. In the table, we have included the additional probabilities: $p_A(n)$ shows the probability of some group of size k^* or greater reaching

n	k^*	β	$t(n)$	$p_A(n)$	$p_I(n)$	$p_F(n)$
3	3	0.930	1.000	1.000	1.000	0.000
4	3	0.166	0.333	0.111	0.086	0.025
5	4	0.253	0.503	0.192	0.160	0.032
6	5	0.302	0.602	0.236	0.204	0.031
7	5	0.066	0.139	0.001	0.001	0.000
8	6	0.129	0.269	0.006	0.005	0.001
9	7	0.175	0.363	0.014	0.011	0.003
10	7	0.037	0.078	0.000	0.000	0.000
11	8	0.083	0.174	0.000	0.000	0.000
12	9	0.120	0.252	0.000	0.000	0.000
13	9	0.023	0.048	0.000	0.000	0.000
14	10	0.059	0.124	0.000	0.000	0.000
15	11	0.089	0.188	0.000	0.000	0.000
20	14	0.034	0.072	0.000	0.000	0.000
25	17	0.002	0.004	0.000	0.000	0.000
30	21	0.033	0.070	0.000	0.000	0.000
35	24	0.011	0.023	0.000	0.000	0.000
40	28	0.033	0.069	0.000	0.000	0.000
45	31	0.016	0.033	0.000	0.000	0.000
50	34	0.002	0.004	0.000	0.000	0.000

Table 2.1: Numerical study for game $\mathcal{G}$ with the institutional arrangements where $d \approx 2$.

an agreement, $p_I(n)$ the probability of each player being an insider of some group with at least k^* non-CAV travelers, and $p_F(n)$ is the probability of each player being a free rider, i.e., existing outside of a group of at least k^* non-CAV travelers.

From Theorem 2.1.23, the incentive ratio goes to zero as the number of players increases. In addition, from the numerical study summarized in Table 2.1 and Figure 2.2, the likelihood of bargaining for an institution, $t(n)$ and probability of being an insider, $p_A(n)$ approach zero as n gets large. This implies that for large societies, the impact of self-realized non-CAV travel is non-existent, and the universal CAV travel strategy dominates. For small societies with $k^* = n$, it is a certainty that players agree to bargain and create an institution for CAV travel (which is not ideal).

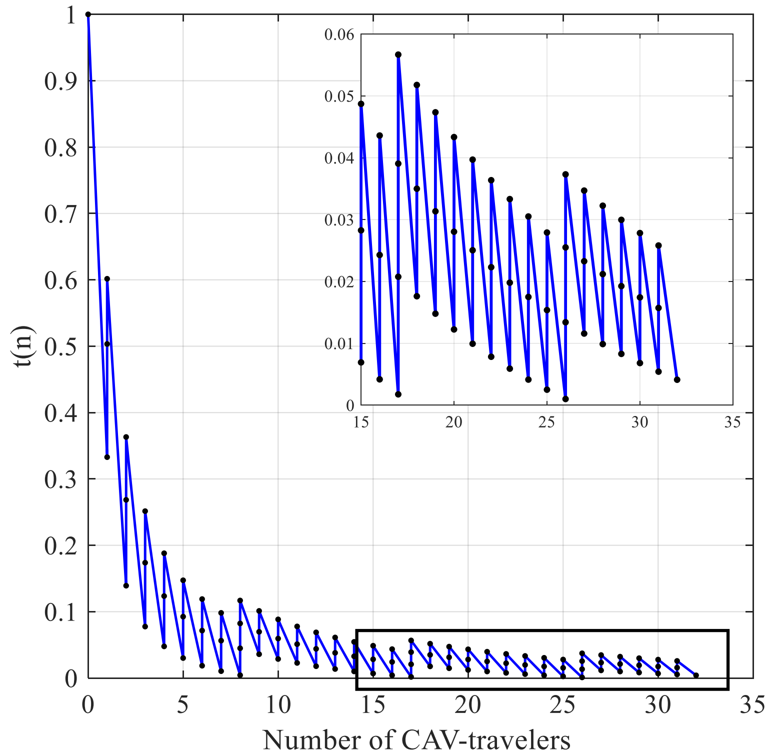

Figure 2.2: Plot of $t(n)$ as a function of the number of CAV travelers. The blue line shows the sequence of $t(n)$ as n increases from 0 to 100.

2.2 A VCG-inspired Mechanism for Efficient Mobility

Emerging mobility systems such as connected and automated vehicles (CAVs) provide the most intriguing opportunity for more accessible, safe, and efficient transportation. CAVs are expected to significantly improve safety by eliminating the human factor and ensure transportation efficiency by allowing users to monitor transportation network conditions and make better operating decisions. However, CAVs could alter the users' tendency-to-travel, leading to a higher traffic demand than expected, thus causing rebound effects (e.g., increased vehicle-miles-traveled). In this chapter, we focus on tackling social factors that could drive an emerging mobility system to unsustainable congestion levels. We propose a mobility market that models the economic in-nature interactions of the travelers in a smart city network with roads and public transit infrastructure. Using techniques from mechanism design, we introduce appropriate monetary incentives (e.g., tolls, fares, fees) and show how a mobility system consisting of selfish travelers that seek to travel either with a CAV or use public transit can be socially efficient. Furthermore, the proposed mobility market ensures that travelers always report their true travel preferences and always benefit from participating in the market; lastly, we also show that the market generates enough revenue to potentially cover its operating costs.

2.2.1 Theoretical Preliminaries

In this section, we provide the theoretical preliminary material related to this chapter's proposed modeling framework, and we formally introduce all important concepts needed to prove our principal results.

Most generic control systems can be viewed as a specification of how decisions (e.g., how to utilize a number of resources) are determined as a function of the information that is known by the agents in the system. What interests us in most cases is *efficiency*, i.e., realizing the best possible allocation of resources with the best use of information to achieve an outcome where collectively agents are satisfied, and there is no overutilization of the system's resources [148]. One key challenge in ensuring efficiency in a control system is the fact that different agents may have conflicting interests and act selfishly. In other words, systems that incorporate human decision-making, if remained uninfluenced, are not guaranteed to exhibit optimal performance. This is well-known to be the case in control theory, and economics [166, 34]. There are various different theories and approaches that attempt to guarantee efficiency in such systems and can provide solutions of varying degrees of success. One such theory is mechanism design, in which we are concerned with how to implement system-wide optimal solutions to problems involving multiple selfish agents, each with private information about their preferences [170, 65]. Within the context of mobility, agents are the travelers, and their private information can be either tolerance to traffic delays, value of time, preferred travel time, or any disposition to a specific mode of transportation.

Our goal in mechanism design is to design appropriate incentives in order to align the interests of agents with the interests of the system [102]. For example, in mobility, given that each traveler/driver/passenger "competes" with everyone else to reach their destination first, we want to ensure that given this inherent conflict of interest, we can still guarantee uncongested roads, no traffic accidents, and no travel time delays. Mechanism design can help us design the rules of systems where information is decentralized (different agents know different things), and agents do not necessarily have an immediate incentive to cooperate [32]. In particular, mechanism design helps us design rules that align all agents' decision-making by providing the right incentives to achieve a well-defined objective for the system (e.g., aggregate optimal performance, system-level efficiency). Thus, mechanism design entails solving an optimization problem with sometimes unverifiable and always incomplete information structure [150]. We call such a problem *an incentive design and preference elicitation problem*.

We start by supposing that there is a system consisted of a finite group of agents, each competing with each other for a limited, fixed allocation of resources. Each agent evaluates different allocations based on some private information that is known only to them. We consider a *social planner*, playing the role of a centralized entity, whose task is to align the selfish and conflicting interests of the agents with the overall system's objective (e.g., an efficient allocation of resources or the maximization of social welfare). As it can be seen in Fig. 2.3, there are four components: (1) There is a group of decision-makers, (2) who make a decision based on their personal information, and (3) that decision is reported as a message to the social planner who is tasked to design the rules of which (4) it can be determined what each agent gets. What follows next is a mathematically formal presentation of the social planner's task.

Consider a set of selfish agents $\mathcal{I}$, $|\mathcal{I}| = n \in \mathbb{N}$ with preferences over different outcomes in a set $\mathcal{O}$. Each agent Each agent $i \in \mathcal{I}$ is assumed to possess private information, denoted by $\theta_i \in \Theta_i$. Since an agent i's θ_i can characterize and influence their decision-making in a significant way, we call θ_i the *type* of agent i. We write

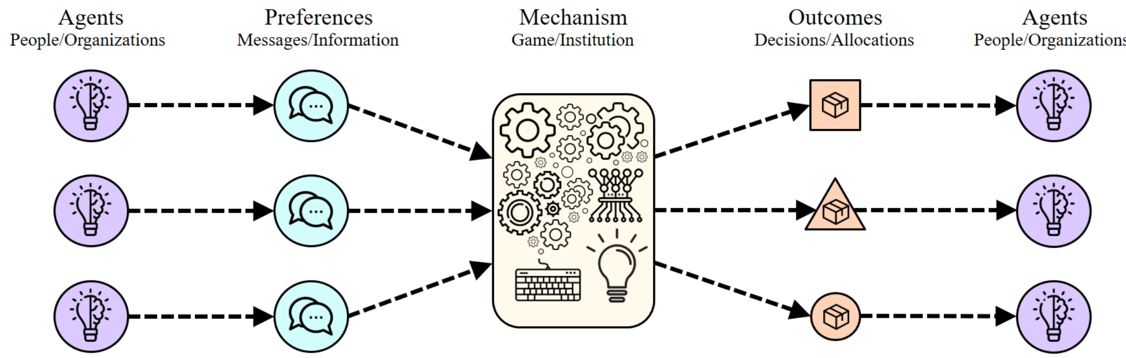

Figure 2.3: A visualization of how an arbitrary control system (agents, preferences, allocations) can be viewed under a mechanism design framework. Agents hold private information, of which they send reports to the social planner who is responsible for designing a mechanism. How efficient the mechanism is can depend on whether the agents' messages are truthful or not.

$(\theta_i)_{i\in\mathcal{I}} = \theta \in \Theta = \prod_{i\in\mathcal{I}} \Theta_i$ to represent the type profile of all agents. Next, an agent i's preferences over different outcomes can be represented by a utility function $u_i : \mathcal{O} \times \Theta_i \to \mathbb{R}$. Although the exact form of u_i can vary depending on the application of the problem [230, 229, 29, 113], what is common in the literature [219, 32, 171] is a quasilinear function of the form

$$u_i(o, \theta_i) = v_i(o, \theta_i) - p_i, \tag{2.37}$$

where $v_i : \mathcal{O} \times \Theta_i \to \mathbb{R}_{\geq 0}$ represents an arbitrary valuation function, and $p_i \mapsto \mathbb{R}$ is a monotonically increasing function. If outcome $o \in \mathcal{O}$ represents an allocation of a resource, then p_i can be thought of as a transfer of agent i's wealth or a cost imposed to agent i for that particular allocation o. Intuitively, a quasilinear function defined as in (2.37) ensures that the marginal value of v_i does not depend on how large p_i becomes, and vice-versa. Furthermore, (2.37) assumes u_i is linear with respect to p_i. We can now naturally define the *social welfare* as the collective summation of all agents' valuations, i.e.,

$$w(o, \theta) = \sum_{i\in\mathcal{I}} v_i(o, \theta_i). \tag{2.38}$$

If our system objective is to maximize w, then immediately we observe that there is an important obstacle, i.e., any agent i may misreport their type θ_i in the hopes to increase their own utility. So, the question is now: How can we incentivize agents to truthfully report their type? The answer is through the appropriate design of p_i. Next, we outline the building blocks that can help us design p_i. Formally, we can define a *mechanism* as the tuple $\langle f, p \rangle$ composed of a *social choice function* (SCF) $f : \Theta \to \mathcal{O}$ and a vector of *payment functions* $p = (p_i)_{i \in \mathcal{I}}$, with $p_i : \Theta \to \mathbb{R}$. In words, a mechanism $\langle f, p \rangle$ defines the rules of which we can implement a system objective by mapping the agents' types to an outcome while using the payments to ensure the optimality or efficiency of that outcome (see Fig. 2.4 for an illustration of the mechanism design framework). We can now state the social planner's problem as follows

$$\max_{o \in \mathcal{O}} w(o, \theta) \tag{2.39}$$

$$\text{subject to: } \hat{\theta}_i = \theta_i, \quad \forall i \in \mathcal{I}, \tag{2.40}$$

$$\sum_{i \in \mathcal{I}} v_i(o, \theta_i) \geq \sum_{i \in \mathcal{I}} v_i(o', \theta_i), \quad \forall o' \in \mathcal{O}, \tag{2.41}$$

$$\sum_{i \in \mathcal{I}} p_i(s(\theta)) \geq 0, \quad \forall \theta \in \Theta, \tag{2.42}$$

$$v_i(f(s(\theta))) - p_i(s(\theta)) \geq 0, \quad \forall i \in \mathcal{I}, \forall \theta \in \Theta, \tag{2.43}$$

where $\hat{\theta}_i$ denotes the reported type of agent i, $s(\cdot)$ is the equilibrium strategy profile (e.g., Nash equilibrium). Constraints (2.40) ensure the truthfulness in the agents' reported types, (2.41) impose an efficiency condition, (2.42) make certain that no external payments are required, and (2.43) incentivize all agents to voluntarily participate in the mechanism. If we could know for certain the true types of all agents, then we would be able solve the optimization problem (2.39) - (2.43) using standard optimization techniques. However, as this is unreasonable to expect from selfish decision-makers, the social planner needs to elicit $\theta = (\theta_i)_{i \in \mathcal{I}}$ by designing the appropriate $p = (p_i)_{i \in \mathcal{I}}$. We discuss in the next subsection one such mechanism that elicits the private information of agents truthfully.

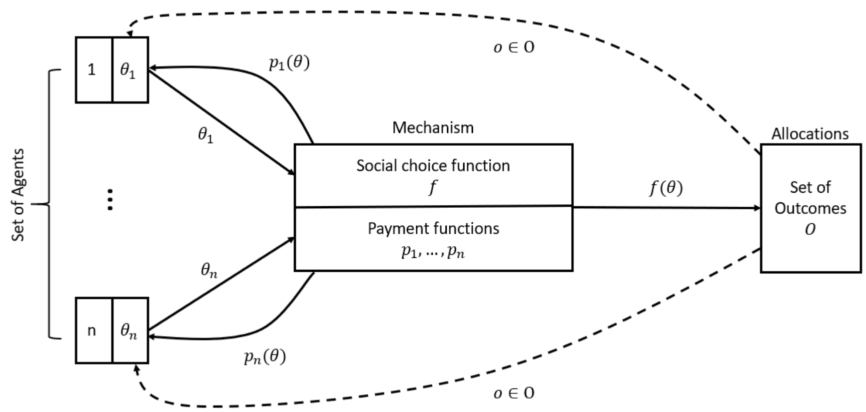

Figure 2.4: A theoretical representation of the mechanism design framework.

2.2.2 The Vickrey-Clarke-Groves Mechanism

In the previous subsection, we reviewed the main concepts of mechanism design and formulated the incentive design and preference elicitation problem. In words, we asked "How can we design the payments $p = (p_i)_{i\in\mathcal{I}}$ so that every agent makes the decision that agrees with what *we* have chosen as the system's objective (e.g., efficiency)? To answer this question, in this subsection, we review the Vickrey-Clarke-Groves (VCG) mechanism [243, 50, 90], one of the most successful mechanisms as it incentivizes agents to be truthful and guarantees efficiency.

As we discussed earlier, a mechanism is a tuple $\langle f, p\rangle$. In a VCG mechanism, the SCF f is defined as an allocation rule (who gets what) based on the optimization problem (2.39) - (2.43), i.e.,

$$f(\hat{\theta}) = \arg\max_{o\in\mathcal{O}} W(o, \hat{\theta}_i). \tag{2.44}$$

where $\hat{\theta} = (\hat{\theta}_i)_{i\in\mathcal{I}}$. In words, assuming that the agents disclose their true information, (2.44) provides to the social planner who attempts to maximize the social welfare a formal way to compute the allocations of each agent. At the same time, the VCG mechanism charges each agent for their allocation as follows

$$p_i(\hat{\theta}) = \sum_{j\neq i} v_j(f(\hat{\theta}_{-i})) - \sum_{j\neq i} v_j(f(\hat{\theta})), \tag{2.45}$$

where $\hat{\theta}_{-i}$ denotes the type profile of all agents except agent i. Note that the payments defined in (2.45) do not depend on an agent i's own declaration $\hat{\theta}_i$. Let us assume for a moment that all agents declare their types truthfully. Then, the first sum in (2.45) computes the value of the social welfare with agent i not participating in the mechanism. The second sum in (2.45) computes the value of the social welfare of all other agents $j \neq i$ with agent i participating in the mechanism. Thus, agent i when they report $\hat{\theta}_i$ are made to pay the *marginal effect* of their decision (in our case that is agent i's reported type $\hat{\theta}_i$). In other words, this particular design of the payments in (2.45) internalizes an agent i's social externality, i.e., agent i's impact on every other agents' welfare.

The VCG mechanism represented by the SCF f defined by (2.44) and the payment functions p defined by (2.45) satisfies the following properties:

1. For any agent, truth-telling is a strategy that dominates any other strategy that is available for that agent. We say then that truth-telling is a *dominant strategy*. Note that such strategies are "always optimal" no matter what the other agents decide.

2. The VCG mechanism successfully aligns the agents' individual interests with the system's objective. In our case, that objective was to maximize the social welfare of all agents. We call this property, *economic efficiency*.

3. For any agent, the VCG mechanism incentivizes them to voluntarily participate in the mechanism as no agent loses by participation (in terms of utility).

4. The VCG mechanism ensures no positive transfers are made from the social planner to the agents. Thus, the mechanism does not incur a loss. We call this *weakly budget balanced*.

The VCG mechanism essentially ensures the realization of a *socially-efficient outcome*, i.e., satisfying properties 1 - 3, in a system of selfish agents, where each possesses private information. It is noteworthy to note how powerful the VCG mechanism is as it induces a dominant strategy equilibrium maximizing the social welfare while also making sure no agent is hurt by participating.

We conclude with the following remark: although the main motivation of mechanism design is the microeconomic study of institutions and relies heavily on game-theoretic techniques, it can prove a powerful theory providing a systematic methodology in the design of systems of asymmetric information, consisted of strategic decision-makers, and whose performance must attain a specified system objective. The rest of the chapter shall present how we can use this theory to design a socially-efficient mobility system consisting of travelers who compete with each other for the utilization of a limited number of mobility services.

2.2.3 The Emerging Mobility Market

We consider an emerging mobility system consisting of a transportation city network managed by a social planner and a finite group of travelers who seek to travel in the network. Informally, this network represents the high-level mobility connections of multiple and different city neighborhoods. In other words, we move away from the concept of "personally-owned" modes of transportation and focus our modeling towards mobility provided as a service. This means that a social planner (e.g., a central computer) offers travelers a unified gateway of public and private transportation providers capable of providing mobility solutions to manage and realize their trip. For example, travelers can plan their journey via a smartphone app by specifying their preferences (e.g., cost, time, and convenience) and their desired destination. The social planner then is tasked to offer a travel recommendation to each traveler, i.e., which mode of transportation to take. In addition, we consider that multiple and different travel options can be offered to each traveler focusing on urban modes of transportation (e.g., CAVs, bus, train). We call these options "mobility services" or "services" for short. Within this framework, we propose a mobility market for a socially-efficient implementation of connectivity and automation in an emerging mobility system. The goal of the mobility market is twofold: (i) ensure that all travelers voluntarily participate and truthfully report their travel preferences, and (ii) be economically sustainable by generating revenue from each traveler and setting a minimum acceptable mobility payment for each traveler to potentially cover the operating costs.

The proposed mobility market is managed by a social planner who aims to allocate $m \in \mathbb{N}$ mobility services to $n \in \mathbb{N}$ travelers, where $n \geq m \neq 0$. We denote the set of travelers by $\mathcal{I}$, $|\mathcal{I}| = n$ and the set of mobility services by $\mathcal{J}$, $|\mathcal{J}| = m$. For example, each service $j \in \mathcal{J}$ can either represent a shared CAV, a train, or a bus. Both sets $\mathcal{I}$ and $\mathcal{J}$ are nonempty, disjoint, and finite. The set of all mobility services $\mathcal{J}$ can be partitioned to a finite number of disjoint subsets, each representing a specific "type" of a mobility service, i.e., $\mathcal{J} = \bigcup_{h=1}^{H} \mathcal{J}_h$, where $H \in \mathbb{N}$ is the number of subsets of $\mathcal{J}$. For example, $\mathcal{J} = \mathcal{J}_1 \cup \mathcal{J}_2$, where $|\mathcal{J}_1|$ represents the number of all available CAVs, and $|\mathcal{J}_2|$ represents the number of all available busses. Next, travelers seek to travel using these mobility services in a transportation network represented by an undirected multigraph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where each node in $\mathcal{V}$ represents a different city area or neighborhood, and each link $e \in \mathcal{E}$ represents a sequence of city roads or a public transit connection. For our purposes, we think of $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ as a representation of a smart city network with a road and public transit infrastructure. In $\mathcal{G}$, a traveler $i \in \mathcal{I}$ seeks to travel from their current location $o_i \in \mathcal{V}$ to their desired destination $d_i \in \mathcal{V}$. So, on one hand, each traveler $i \in \mathcal{I}$ is associated with a origin-destination pair (o_i, d_i). On the other hand, each type of mobility services (e.g., one type is shared CAVs, another is trains) is associated with a unique link that connects any two nodes. At the same time, we do not limit the number of different mobility services that connect any origin o_i to any destination d_i of any traveler $i \in \mathcal{I}$. We suppose that any traveler $i \in \mathcal{I}$ has at least two travel options for their origin-destination pair (o_i, d_i). Furthermore, each traveler $i \in \mathcal{I}$ can travel in $\mathcal{G}$ with any mobility service $j \in \mathcal{J}$ that satisfies their origin-destination pair (o_i, d_i) and each service $j \in \mathcal{J}$ can be used by multiple travelers.

Remark 2.2.1. *Network $\mathcal{G}$ represents the upper-level connections of different city neighborhoods. By connections, we mean either roads or public transit routes. Instead of modeling each node to represent travelers' exact location, we consider dividing a city into zones. By grouping travelers' exact locations into such zones, we can use network $\mathcal{G}$ to model the mobility connections between the different city zones.*

Next, we partition the set of travelers $\mathcal{I}$ into different smaller subsets characterized by a common origin-destination pair.

Definition 2.2.2. *The set of travelers with the exact same origin-destination pair is $\mathcal{I}_k = \{i \in \mathcal{I} \mid (o_i, d_i) = (o_k, d_k)\}$, $k = 1, 2, \ldots, K$, where $K \in \mathbb{N}$ is the number of subclasses over the complete set of travelers, i.e., $\mathcal{I} = \bigcup_{k=1}^{K} \mathcal{I}_k$.*

The justification of Definition 2.2.2 is that in an emerging mobility system, we can acquire verifiable location data of travelers either by using a global positioning system or estimating the average number of travelers using public transit [51, 235].

Mathematically, the allocation of the finite number of mobility services to travelers can be described by a vector of binary variables.

Definition 2.2.3. *The* traveler-service assignment *is a vector $\mathbf{a} = (a_{ij})_{i\in\mathcal{I}, j\in\mathcal{J}}$, where a_{ij} is a binary variable of the form:*

$$a_{ij} = \begin{cases} 1, & \text{if } i \in \mathcal{I} \text{ is assigned to } j \in \mathcal{J}, \\ 0, & \text{otherwise.} \end{cases} \tag{2.46}$$

Note that we have $(a_{ij})_{i\in\mathcal{I}, j\in\mathcal{J}} = (a_{11}, \ldots, a_{ij}, \ldots, a_{nm})$. By partitioning the set of travelers in $K \in \mathbb{N}$ subclasses, the traveler-service assignment of subclass $\mathcal{I}_k$ is given by $\mathbf{a}_k = (a_{ij})_{i\in\mathcal{I}_k, j\in\mathcal{J}}$.

Naturally, we need to impose a physical limit on the use of each mobility service $j \in \mathcal{J}$ in network $\mathcal{G}$ as well as a connection capacity of a mobility service for each link in the network. Note that each link in $\mathcal{G}$ represents a road or a public transit connection, which means that multiple mobility services of one type use that one link. For example, one link can be a bus lane with stops between two different city areas; another can be a train route between two stations.

Definition 2.2.4. *The* usage capacity *of any mobility service $j \in \mathcal{J}$ is given by $\varepsilon_j \in \mathbb{N}$. The* link capacity *in network $\mathcal{G}$ is given by $\gamma_e \in \mathbb{R}_{\geq 0}$.*

For example, ε_j can represent the maximum number of travelers (or passengers) in a shared vehicle or the maximum number of travelers in a train vehicle (seated and standing). Similarly, γ_e can represent a critical traffic density of mobility services, which means that any additional input of vehicles or trains can lead to a reduced traffic flow and eventually to traffic congestion. For example, we can use the GreenShields model to define explicitly the critical traffic density [84].

As in any mobility problem that involves travelers, we need to consider the travelers' preferences (e.g., preferred travel time, value of time, willingness-to-pay for service). Hence, we formally define the notion of "personal travel requirements" by introducing three important parameters (our selection of those three parameters is justified by recent transportation studies [181, 99].)

Definition 2.2.5. *For any traveler $i \in \mathcal{I}_k$, $k = 1, \ldots, K$, let $\alpha_i \in (0, 1)$ be the* value of time*, $\theta_i \in \mathbb{R}_{\geq 0}$ the* preferred travel time*, and $\bar{v}_i \in \mathbb{R}_{\geq 0}$ the* maximum willingness-to-pay*. Then, the* personal travel requirements *of traveler i is a tuple of the form $\pi_i = (\alpha_i, \theta_i, \bar{v}_i)$.*

We offer the intuition behind each parameter: traveler i's value of time α_i transforms the traveler's time urgency in monetary units as it can model, for example, the acceptable amount of compensation for lost time. Similarly, a traveler i's preferred travel time θ_i is a non-negative real value representing how fast traveler i wishes to reach their destination. The last term in π_i captures how much traveler i appraises a direct and completely convenient mobility service. For example, $\bar{v}_i$ can measure the maximum willingness-to-pay of traveler i traveling with the fastest and most convenient service (e.g., taking a taxicab with no co-travelers) to their destination.

For each traveler $i \in \mathcal{I}_k$, the tuple π_i is considered private information, known only to traveler i. Hence, as the social planner does not know $(\pi_i)_{i\in\mathcal{I}}$, each traveler i must report their π_i. This is one of the key challenges in the proposed mobility market: *How can we incentivize the travelers to be truthful and elicit the private information needed to provide a socially-efficient solution to the emerging mobility market?* The answer to this question will be given in Section 2.2.5.

Next, we introduce an "inconvenience" metric for any traveler $i \in \mathcal{I}_k$ using any mobility service $j \in \mathcal{J}$. Quantitatively, the inconvenience metric can represent the extra monetary value of travel disutility from any costs, travel delays, or violation of personal travel requirements caused by the use of a mobility service.

Definition 2.2.6. *The* mobility inconvenience metric *for traveler* $i \in \mathcal{I}_k$*,* $k = 1, \dots, K$*, assigned to service* $j \in \mathcal{J}$ *is a function denoted by* $\phi_i\left(\alpha_i, \theta_i, \tilde{\theta}_i(\mathbf{a}_k)\right) \mapsto \mathbb{R}_{\geq 0}$*, where* $\tilde{\theta}_i(\mathbf{a}_k) \in \mathbb{R}_{\geq 0}$ *is the experienced travel time. We assume that* ϕ_i *is continuous, increasing, and convex.*

Note that the mobility inconvenience metric ϕ_i increases when $\tilde{\theta}_i(\mathbf{a}_k)$ increases. From a modeling perspective, traveling with time delays or during peak times can cause significant inconveniences to any traveler $i \in \mathcal{I}_k$. Although, an exact form of ϕ is beyond the scope of this chapter, our definition of ϕ is consistent with general inconvenience functions in the literature [68, 55].

Next, a traveler i's satisfaction is captured by a valuation function v_i, which can reflect the traveler's *willingness-to-pay* for their travel, i.e.,

$$v_i(\mathbf{a}_k) = \bar{v}_i - \phi_i\left(\alpha_i, \theta_i, \tilde{\theta}_i(\mathbf{a}_k)\right), \tag{2.47}$$

where $\bar{v}_i \in \mathbb{R}_{\geq 0}$ is the value gained by traveler $i \in \mathcal{I}_k$ when their origin-destination pair (o_i, d_i) is satisfied using service $j \in \mathcal{J}$ without any travel delays, i.e., $\theta_i = \tilde{\theta}_i(\mathbf{a}_k)$.

Naturally, for any traveler i and any service j, we have $v_i(\mathbf{a}_k) \in [0, \bar{v}_i]$, where $v_i(\mathbf{a}_k) = 0$ means that traveler i is unwilling to use service j. Below we summarize the two extreme cases and their interpretation:

$$v_i(\mathbf{a}_k) = \begin{cases} \bar{v}_i, & \text{if } \phi_i = 0, \\ 0, & \text{if } \phi_i = \bar{v}_i. \end{cases} \tag{2.48}$$

When $\phi_i = 0$, we say that traveler i travels to their destination in the fastest and most convenient mobility service offered by the mobility market (e.g., a taxicab with no co-travelers). On the other hand, when $\phi_i = \bar{v}_i$, we say that traveler i's personal travel requirements are not satisfied, and the traveler is most inconvenienced with regards to mobility.

Although our analysis can treat the valuation function v_i in its most general form, given by (2.47), we explicitly define the second component of (2.47) in our mathematical exposition. Thus, the explicit form for the inconvenience mobility metric ϕ_i is

$$\phi_i\left(\alpha_i, \theta_i, \tilde{\theta}_i(\mathbf{a}_k)\right) = \alpha_i \cdot (\tilde{\theta}_i(\mathbf{a}_k) - \theta_i), \tag{2.49}$$

Basically, (2.49) gives the monetary value of the difference between the travel times (experienced vs preferred), and can be interpreted as the travel time tolerance that the traveler can accept (in monetary units).

In our modeling framework, the total utility $u_i(\mathbf{a}_k)$ of traveler $i \in \mathcal{I}_k$, $k = 1, \ldots, K$, is given by

$$u_i(\mathbf{a}_k) = v_i(\mathbf{a}_k) - p_i(\mathbf{a}_k), \tag{2.50}$$

where $v_i(\mathbf{a}_k)$ is the willingness-to-pay and $p_i(\mathbf{a}_k) \in \mathbb{R}_{\geq 0}$ is the mobility payment that traveler i is required to make to use service $j \in \mathcal{J}$ (e.g., pay road tolls or buy a public transit fare). Hence, (2.50) establishes a "quasi-linear" relationship between a traveler's satisfaction and payment, both measured in monetary units [145].

In contrast to the traveler's satisfaction, we also introduce an "operating cost" to capture the needed investment that public and private mobility providers and operators make to ensure the proper function of their mobility services.

Definition 2.2.7. *The operating cost of service $j \in \mathcal{J}$ can be computed by*

$$c_j(\mathbf{a}_k) = \sum_{i \in \mathcal{I}_k} c_{ij}(a_{ij}), \tag{2.51}$$

where $c_{ij}(a_{ij}) \in \mathbb{R}_{\geq 0}$ is traveler i's corresponding share of the operating cost of vehicle $j \in \mathcal{J}$. In the case of $a_{ij} = 0$, we have $c_{ij} = 0$.

Intuitively, the operating cost c_{ij} captures traveler i's fair share of the costs of service $j \in \mathcal{J}$. These costs can be associated with fuel/energy consumption, drivers' labor reimbursement, maintenance, and environmental impact.

Definition 2.2.8. *Given the traveler-service assignment $\mathbf{a}_k = (a_{ij})_{i \in \mathcal{I}_k, j \in \mathcal{J}}$, the travelers' payments are given by the vector $\mathbf{p}_k = (p_i(a_{ij}))_{i \in \mathcal{I}_k, j \in \mathcal{J}}$. Then, for a subclass $\mathcal{I}_k$, $k = 1, \ldots, K$, the proposed mobility market can be fully described by the tuple*

$$\left\langle \mathcal{I}_k, \mathcal{J}, (\pi_i)_{i \in \mathcal{I}_k}, (u_i)_{i \in \mathcal{I}_k}, \mathbf{a}_k, \mathbf{p}_k \right\rangle, \tag{2.52}$$

where $(\pi_i)_{i \in \mathcal{I}_k}$ is considered private information (unknown to the social planner), and the experienced travel time $\tilde{\theta}_i$ and operation costs c_j of all mobility services are considered known to the social planner.

Note that in Definition 2.2.8, we have also defined the informational structure of the proposed market. The operation costs $(c_j)_{j \in \mathcal{J}}$ are considered public information as well as the minimum acceptable mobility payments $(\sigma_i)_{i \in \mathcal{I}}$. In general, though, any VCG-based mechanism requires agents to report their entire valuation function [207]. In our case, we can take advantage of more advanced and sophisticated data gathering techniques so that we may infer the form and shape of a traveler's valuation (and utility) function using, for example, historical and empirical data [40, 2]. Hence, the functional form of v_i can be considered known, but the realization of $v_i(\cdot)$ is agent

i's private information. It is important to note that the evaluation of any traveler i's valuation function can be learned using the three-parameter tuple π_i, which provides the personal travel requirements of any traveler $i \in \mathcal{I}_k$. In addition, we expect any social planner of a generic transportation system to have the ability (e.g., using regression analysis [21]) to approximate the experienced travel time of any mobility service and its operating costs. Hence, the only private information that we are required to elicit from the travelers is their personal travel requirements $(\pi_i)_{i\in\mathcal{I}_k}$, $k = 1, \ldots, K$. At the same time, receiving communication in the form of messages from all travelers regarding the $(\pi_i)_{i\in\mathcal{I}_k}$, $k = 1, \ldots, K$ can be an unrealistic burden. That is why, in our framework, any traveler $i \in \mathcal{I}$ is expected to report the evaluation of their valuation function v_i, which depends on their π_i. Essentially, we parameterize the private information of travelers into a one-dimensional number. In future research, we plan to address a multi-dimensional mechanism to ensure there is no loss of information of the traveler's preferences.

On a different note, a natural question to ask here is whether there is any guarantee that the travelers' mobility payments will meet the providers' operating costs. As we saw in Section 2.2.1, the VCG mechanism can only charge travelers their social cost or impact into the mobility system. Thus, this might lead to very low mobility payments for a significant number of travelers, leading to deficits to cover operating costs for the providers. Since, in reality, we cannot expect any providers to serve travelers indefinitely when their costs have not been met, we introduce a "pricing base" for the mobility payments. Essentially, these bases can be chosen by the providers to ensure that no payment by any traveler is below a set value (e.g., minimum acceptable payment), which can be determined approximately by the traveler's location and destination, supply and demand, and operator's reimbursement fee [92].

Definition 2.2.9. *The* minimum acceptable mobility payment *of any service $j \in \mathcal{J}$ is given by $\sigma_i(\mathbf{a}_k) \in \mathbb{R}_{\geq 0}$, for any traveler $i \in \mathcal{I}_k$, $k = 1, \ldots, K$. If for an arbitrary traveler i, we have $p_i(\mathbf{a}_k) \geq \sigma_i(\mathbf{a}_k)$, then we say that the mobility market, defined in*

(2.52), *is* economically sustainable.

The minimum acceptable mobility payments $\sigma = (\sigma_i)_{i\in\mathcal{I}}$ are considered public information set by the providers and may be different for each traveler $i \in \mathcal{I}_k$, $k = 1, \dots, K$.

In the modeling framework described above, we impose the following assumption:

Assumption 2.2.10. *For all subclasses $\mathcal{I}_k$, $k = 1, \dots, K$, $K \in \mathbb{N}$, any traveler $i \in \mathcal{I}_k$ is modeled as a selfish decision-maker with private information $\pi_i = (\alpha_i, \theta_i, \bar{v}_i)$. Traveler i's objective is to maximize their total utility $u_i(\mathbf{a}_k) = v_i(\mathbf{a}_k) - p_i(\mathbf{a}_k)$ in a non-cooperative game-theoretic setting.*

Assumption 2.2.10 essentially says that each traveler is selfish in the sense that they are only interested in their own well-being. In economics, such behavior is called "strategic" since agents attempt to misreport their private information to the social planner if that means higher individual benefits.

Assumption 2.2.11. *The aggregate usage capacities of all mobility services can adequately serve all travel requests of travelers. Mathematically, we have $\sum_{j\in\mathcal{J}} \varepsilon_j = n = |\mathcal{I}|$.*

Intuitively, Assumption 2.2.11 ensures that no traveler will remain unassigned. We can justify this assumption as follows: our focus is on efficiently allocating the different mobility services to travelers in a mobility market, a multimodal mobility system that incorporates public transit services with high traveler capacity capabilities. A relaxation of this assumption must consider scenarios where the existing mobility services cannot meet the travelers' demand, thus transforming our problem into a "mobility and equity" problem (giving priority to a subset of travelers in a fair way).

2.2.4 The Optimization Problem Statement of the Emerging Mobility Market

In the proposed mobility market, travelers request (via a smartphone app), in advance, a travel recommendation from the social planner that satisfies their origin-destination. Given the travelers' origin-destination pairs, the social planner partitions all travelers to different subclasses, as described in Definition 2.2.2. Thus, travelers from the same neighborhood have the same origin. Similarly, travelers going to the same neighborhood have the same destination. The social planner's task is to elicit the travelers' preferences, attempt to satisfy all travel requests, and provide recommendations to the travelers (e.g., which mobility service to use) by considering the social optimum subject to the city network's physical constraints. Hence, we are interested in minimizing the travel inconvenience of all travelers and the operating costs.

Remark 2.2.12. *Without loss of generality and to simplify the mathematical analysis in our exposition, we consider that both the mobility inconvenience metrics* $(\phi_i)_{i\in\mathcal{I}_k}$*,* $k = 1, \ldots, K$*, the minimum mobility payments* $(\sigma_i)_{i\in\mathcal{I}_k}$*,* $k = 1, \ldots, K$*, and the operating costs* $(c_j)_{j\in\mathcal{J}}$ *are normalized. This ensures that* ϕ_i*,* σ_i*, and* c_j *do not dominate each other in Problem 2.2.13 next, while all three are measured in the same monetary units.*

Problem 2.2.13. *For each subclass* $\mathcal{I}_k$*,* $k = 1, \ldots, K$*, the optimization problem is*

$$\min_{\mathbf{a}_k} \sum_{i\in\mathcal{I}_k} \left[\phi_i\left(\alpha_i, \theta_i, \tilde{\theta}_i(\mathbf{a}_k)\right) + \sigma_i(\mathbf{a}_k)\right] + \sum_{j\in\mathcal{J}} c_j(\mathbf{a}_k), \tag{2.53}$$

subject to:

$$\sum_{j\in\mathcal{J}} a_{ij} \leq 1, \quad \forall i \in \mathcal{I}_k, \tag{2.54}$$

$$\sum_{i\in\mathcal{I}_k} a_{ij} \leq \varepsilon_j, \quad \forall j \in \mathcal{J}, \tag{2.55}$$

$$\sum_{j\in\mathcal{J}_h} \sum_{i\in\mathcal{I}_k} a_{ij} \leq \gamma_e, \quad \forall h \in \{1, 2, \ldots, H\}, \quad \forall e \in \mathcal{E}, \tag{2.56}$$

where (2.54) assures that each traveler $i \in \mathcal{I}_k$ *will be assigned at most one mobility service, and (2.55) stipulates that service* j*'s maximum usage capacity* ε_j *must not* *be exceeded. Lastly,* (2.56) *ensures that there will be no congestion on the links that represent roads or public transit connections. Note also that even though in Problem 2.2.13 we focus only on the* k*th partition of the set of travelers* $\mathcal{I}$*, we do not need to do the same for the mobility services. In other words, since each type of mobility services is associated with a unique link that connects any two nodes, any services that do not satisfy* (o_k, d_k) *will not be considered in the optimization.*

Problem 2.2.13 is similar to the many-to-one assignment problem, and standard algorithmic approaches (e.g., Jonker-Volgenant algorithm [109]) exist to find its global optimal solution or, in worst-case scenarios, a second-best optimal approximation of a solution. We can also reformulate Problem 2.2.13 to a linear program by relaxing to a non-negativity constraint the binary optimization variable a_{ij} for all $i \in \mathcal{I}$ and $j \in \mathcal{J}$. We can then guarantee that an optimal solution of zeros and ones exists by noting that the constraint matrix formed by (2.54) - (2.56) satisfies the total unimodularity property [210]. Note, though, that these approaches assume complete information of all parameters and variables in the model. Such an assumption is unreasonable to expect from strategic decision-makers, so, in our framework, travelers are not expected to report their private information truthfully. This turns our problems to a *preference elicitation problem*. Our task in Section 2.2.5 is to provide a theoretical approach that elicits the necessary private information of all travelers using monetary incentives in the form of mobility payments (e.g., tolls, fares, fees).

2.2.5 Methodology for the Design of Mobility Incentives

We can reformulate Problem 2.2.13 as a standard social welfare maximization problem. First, recall that $\phi_i\left(\alpha_i, \theta_i, \tilde{\theta}_i(\mathbf{a}_k)\right) = \bar{v}_i - v_i(\mathbf{a}_k)$, so the objective function (2.53) becomes

$$\max_{\mathbf{a}_k} \sum_{i \in \mathcal{I}_k} [v_i(\mathbf{a}_k) - \sigma_i(\mathbf{a}_k)] - \sum_{j \in \mathcal{J}} c_j(\mathbf{a}_k). \tag{2.57}$$

This reformulation will prove useful as the design of the monetary payments relies on the social welfare impact (or mobility externality) caused by one traveler to the rest of the travelers in the proposed mobility market.

Problem 2.2.14. *We rewrite Problem 2.2.13 as follows:*

$$\max_{\mathbf{a}_k} \sum_{i \in \mathcal{I}_k} [v_i(\mathbf{a}_k) - \sigma_i(\mathbf{a}_k)] - \sum_{j \in \mathcal{J}} c_j(\mathbf{a}_k), \tag{2.58}$$

subject to:

$$\sum_{j \in \mathcal{J}} a_{ij} \leq 1, \quad \forall i \in \mathcal{I}_k, \tag{2.59}$$

$$\sum_{i \in \mathcal{I}_k} a_{ij} \leq \varepsilon_j, \quad \forall j \in \mathcal{J}, \tag{2.60}$$

$$\sum_{j \in \mathcal{J}_h} \sum_{i \in \mathcal{I}_k} a_{ij} \leq \gamma_e, \quad \forall h \in \{1, 2, \dots, H\}, \quad \forall e \in \mathcal{E}, \tag{2.61}$$

where $\mathbf{a}_k = (a_{ij})_{i \in \mathcal{I}_k, j \in \mathcal{J}}$ *denotes the solution of Problem 2.2.14.*

In order for the solution of Problem 2.2.14 to be socially-efficient, we would need a control input in utility function (2.50) to incentivize all travelers to report their personal travel requirements truthfully. In our case, this control input is the payments $\mathbf{p}_k$, $k = 1, \dots, K$, which can be designed to be the difference between the *maximum social welfare with traveler* $\ell \in \mathcal{I}_k$ *not participating* and the *maximum social welfare of other travelers with traveler* ℓ *participating*. Thus, to capture the first term, we revise Problem 2.2.14 by adding constraint (2.66) to help us capture the "mobility externality" of traveler ℓ rejecting any travel recommendations from the social planner. For example, traveler ℓ may use a taxicab with no other co-travelers. Thus, Problem 2.2.14 takes the following form.

Problem 2.2.15. *For each traveler* $i \in \mathcal{I}_k$, $k = 1, \dots, K$, *we fix traveler* $\ell \in \mathcal{I}_k$ *and* *solve the following optimization problem:*

$$\max_{\mathbf{b}_k} \sum_{i\in\mathcal{I}_k} [v_i(\mathbf{b}_k) - \sigma_i(\mathbf{b}_k)] - \sum_{j\in\mathcal{J}} c_j(\mathbf{b}_k), \tag{2.62}$$

subject to:

$$\sum_{j\in\mathcal{J}} b_{ij} \le 1, \quad \forall i \in \mathcal{I}_k, \tag{2.63}$$

$$\sum_{i\in\mathcal{I}_k} b_{ij} \le \varepsilon_j, \quad \forall j \in \mathcal{J}, \tag{2.64}$$

$$\sum_{j\in\mathcal{J}_h} \sum_{i\in\mathcal{I}_k} b_{ij} \le \gamma_e, \quad \forall h \in \{1, 2, \dots, H\}, \quad \forall e \in \mathcal{E}, \tag{2.65}$$

$$b_{\ell j} = 0, \quad \forall j \in \mathcal{J}, \tag{2.66}$$

where $\mathbf{b}_k = (b_{ij})_{i\in\mathcal{I}_k, j\in\mathcal{J}}$ *defined similarly as in* (2.46) *denotes the solution of Problem 2.2.15, and* (2.66) *states that traveler* $\ell \in \mathcal{I}_k$ *is not considered in the optimization problem.*

Remark 2.2.16. *In what follows, to simplify the mathematical exposition, we introduce the following notation:*

$$w_2(\mathbf{a}_k) = \sum_{i\in\mathcal{I}_k} [v_i(\mathbf{a}_k) - \sigma_i(\mathbf{a}_k)] - \sum_{j\in\mathcal{J}} c_j(\mathbf{a}_k), \tag{2.67}$$

$$w_3(\mathbf{b}_k) = \sum_{i\in\mathcal{I}_k} [v_i(\mathbf{b}_k) - \sigma_i(\mathbf{b}_k)] - \sum_{j\in\mathcal{J}} c_j(\mathbf{b}_k), \tag{2.68}$$

where w_2 *and* w_3 *denote the objective functions of Problems 2.2.14 and 2.2.15, respectively.*

We can now propose the exact form of the mobility payment p_ℓ for an arbitrary traveler $\ell \in \mathcal{I}_k$, $k = 1, \dots, K$, of the proposed mobility market. For any subclass $\mathcal{I}_k$, $k = 1, \dots, K$, traveler $\ell \in \mathcal{I}_k$ makes the following payment:

$$p_\ell(\mathbf{a}_k, \mathbf{b}_k) = w_3(\mathbf{b}_k) - [w_2(\mathbf{a}_k) - v_\ell(\mathbf{a}_k)]. \tag{2.69}$$

Since $w_3(\mathbf{b}_k)$ yields the maximum social welfare from the traveler-service assignment $\mathbf{b}_k$ when traveler $\ell \in \mathcal{I}_k$ does not participate in the mobility market, it can be viewed by traveler $\ell \in \mathcal{I}_k$ in (2.69) as a constant, regardless of what traveler ℓ reports to the social planner about their own personal travel requirements π_ℓ. The term $[w_2(\mathbf{a}_k) - v_\ell(\mathbf{a}_k)]$ in (2.69) represents the maximum social welfare of all travelers other than traveler $\ell \in \mathcal{I}_k$, when traveler $\ell \in \mathcal{I}_k$ partakes in the mobility market. As a consequence, p_ℓ can be interpreted as the externality caused by traveler $\ell \in \mathcal{I}_k$ to all other travelers. In addition, the computation of the mobility payments (2.69) requires solving Problem 2.2.15 repeatedly for each traveler. As shown in Algorithm 1, first we derive the optimal solution of Problem 2.2.14, and then we use the optimal solution of Problem 2.2.15 to compute the monetary payment of each traveler $\ell \in \mathcal{I}_k$.

Algorithm 1: Solution of Problem 2.2.14 with Problems 2.2.15

Data: $\mathcal{I}_k, \mathcal{J}, (\pi_i)_{i\in\mathcal{I}_k}, (u_i)_{i\in\mathcal{I}_k}$
Result: $\mathbf{a}_k^*$ and $\mathbf{p}_k$
Solve for the optimal solution $\mathbf{a}_k^*$ of Problem 2.2.14;
for $\ell \in \mathcal{I}_k$ **do**
  Solve for the optimal solution $\mathbf{b}_k^*$ of Problem 2.2.15;
  Next, compute

$$p_\ell(\mathbf{a}_k^*, \mathbf{b}_k^*) = w_3(\mathbf{b}_k^*) - [w_2(\mathbf{a}_k^*) - v_\ell(\mathbf{a}_k^*)] \,.$$

end

Before we move on to the next section, we note that informally we talked about a traveler not participating in the mobility market in solving Problem 2.2.15. This idea helps us design the mobility payments in (2.69) by identifying the mobility externalities in the welfare of all travelers. Thus, we introduce the notion of "mobility exclusion," which will help us capture the socioeconomic impact of any traveler on the rest of the mobility market.

Definition 2.2.17. *For any subclass $\mathcal{I}_k$, $k = 1, \ldots, K$, given a traveler-service assignment $\mathbf{a}_k$ of Problem 2.2.14, a traveler $\ell \in \mathcal{I}_k$ is said to be* mobility excluded *if they are not assigned to any mobility service in the traveler-service assignment $\mathbf{b}_k$ of Problem 2.2.15.*

Problem 2.2.15 is used to compute the mobility payments for each traveler in the mobility market by identifying the mobility externality caused by the decision-making of the traveler to the rest of the market. In addition, however, we are also interested in identifying the traveler's impact on (i) operating costs and (ii) overall welfare. We shall see in the next section how we can achieve this.

2.2.6 Properties of the Mobility Market

Our first result is an immediate and straightforward consequence of Definition 2.2.17. Recall that the operating cost $c_{ij}(a_{ij})$ captures traveler i's fair share of the mobility service j's costs that they use under the traveler-service assignment $\mathbf{a}_k$.

Corollary 2.2.18. *Let $\mathbf{b}_k^\ell$ be a feasible traveler-service assignment of Problem 2.2.15. Given that traveler $\ell \in \mathcal{I}_k$ is mobility excluded, the operating cost that is associated with the traveler-service assignment $\mathbf{b}_k^\ell$ is smaller than or equal than the operating cost associated with the optimal assignment $\mathbf{a}_k^*$ of Problem 2.2.14, i.e., we have*

$$\sum_{i \in \mathcal{I}_k} c_{ij}(a_{ij}^*) \geq \sum_{i \in \mathcal{I}_k \setminus \{\ell\}} c_{ij}(b_{ij}^\ell). \tag{2.70}$$

Similarly, using Definition 2.2.17, we show that the sum of valuations (or welfare) of all travelers other than the traveler, who is mobility excluded specifically in Problem 2.2.15, is greater or equal than the sum of valuations evaluated at the traveler-service assignment of Problem 2.2.14.

Lemma 2.2.19. *Let $\mathbf{b}_k^\ell$ be a feasible traveler-service assignment of Problem 2.2.15, in which traveler $\ell \in \mathcal{I}_k$ is mobility excluded. Then, we have*

$$\sum_{i \in \mathcal{I}_k \setminus \{\ell\}} v_i(\mathbf{a}_k) \leq \sum_{i \in \mathcal{I}_k} v_i(\mathbf{b}_k^\ell). \tag{2.71}$$

Proof. Given that traveler $\ell \in \mathcal{I}_k$ is mobility excluded in the traveler-service assignment $\mathbf{b}_k^\ell$ of Problem 2.2.15, we know that there is one less traveler required to be served by any mobility service in the market. Naturally, this affects the experienced travel times of any other traveler $i \in \mathcal{I}_k$, i.e., we have either a decreased or constant $\tilde{\theta}_i(\mathbf{b}_k^\ell)$. So, mathematically this means that with traveler-service assignment $\mathbf{a}_k$ of Problem 2.2.14, we have

$$\tilde{\theta}_i(\mathbf{b}_k^\ell) \leq \tilde{\theta}_i(\mathbf{a}_k), \tag{2.72}$$

where $\tilde{\theta}_i(\mathbf{b}_k^\ell)$ is the experienced travel time of traveler $i \in \mathcal{I}_k$ evaluated at $\mathbf{b}_k^\ell$ and $\tilde{\theta}_i(\mathbf{a}_k)$ is the experienced travel time of traveler i evaluated at $\mathbf{a}_k$. Intuitively, (2.72) means there is one less traveler leading to better travel times for other travelers (better here means less). Hence, since the explicit form of traveler i's valuation is given by

$$v_i(\mathbf{a}_k) = \bar{v}_i - \phi_i\left(\alpha_i, \theta_i, \tilde{\theta}_i(\mathbf{a}_k)\right) = \bar{v}_i - \alpha_i \cdot (\tilde{\theta}_i(\mathbf{a}_k) - \theta_i), \tag{2.73}$$

if we compare the two valuations $v_i(\mathbf{a}_k)$ and $v_i(\mathbf{b}_k^\ell)$, we get $v_i(\mathbf{a}_k) \leq v_i(\mathbf{b}_k^\ell)$. This completes the proof. $\square$

Next, we show that for any traveler, their valuation will always be greater or equal than the minimum mobility payment. This will be instrumental in our attempt to show individual rationality later on.

Lemma 2.2.20. *Let $\mathbf{a}_k^*$ denote the optimal solution of Problem 2.2.14. Then the minimum mobility payment σ_ℓ in the objective function (2.58) of Problem 2.2.14 ensures that, for any $\ell \in \mathcal{I}_k$, $k = 1, \ldots, K$, $v_\ell(\mathbf{a}_k^*) \geq \sigma_\ell(\mathbf{a}_k^*)$.*

Proof. Let $\mathbf{a}_k^*$ denote the optimal solution of Problem 2.2.14 and $\mathbf{b}_k^{\ell*}$ the corresponding solution of Problem 2.2.15. Hence, traveler ℓ has been assigned a mobility service in the optimal traveler-service assignment $\mathbf{a}_k^*$, but they are mobility excluded in $\mathbf{b}_k^{\ell*}$. Thus, we have

$$\begin{aligned} w_3(\mathbf{b}_k^{\ell*}) &= \sum_{i \in \mathcal{I}_k} \left[v_i(\mathbf{b}_k^{\ell*}) - \sigma_i(\mathbf{b}_k^{\ell*})\right] - \sum_{j \in \mathcal{J}} c_j(\mathbf{b}_k^{\ell*}) \\ &\geq \sum_{i \in \mathcal{I}_k \setminus \{\ell\}} v_i(\mathbf{a}_k^*) - \sum_{i \in \mathcal{I}_k} \sigma_i(\mathbf{b}_k^{\ell*}) - \sum_{j \in \mathcal{J}} c_j(\mathbf{a}_k^*), \end{aligned} \tag{2.74}$$

where (2.74) follows from Corollary 2.2.18 and Lemma 2.2.19. Next, we look at the welfare of an arbitrary traveler $i \in \mathcal{I}_k$ under $\mathbf{a}_k^*$, i.e.,

$$\begin{aligned} w_2(\mathbf{a}_k^*) &= \sum_{i\in\mathcal{I}_k} [v_i(\mathbf{a}_k^*) - \sigma_i(\mathbf{a}_k^*)] - \sum_{j\in\mathcal{J}} c_j(\mathbf{a}_k^*) \\ &= \sum_{i\in\mathcal{I}_k} v_i(\mathbf{a}_k^*) - \sum_{i\in\mathcal{I}_k} \sigma_i(\mathbf{a}_k^*) - \sum_{j\in\mathcal{J}} c_j(\mathbf{a}_k^*), \end{aligned} \tag{2.75}$$

where it also follows that $w_2(\mathbf{a}_k^*) \geq w_3(\mathbf{b}_k^{\ell *})$ from the fact that $\mathbf{b}_k^{\ell *}$ is not an optimal solution of Problem 2.2.14. Thus, if we compare (2.74) and (2.75), we get

$$\sum_{i\in\mathcal{I}_k} v_i(\mathbf{a}_k^*) - \sum_{i\in\mathcal{I}_k} \sigma_i(\mathbf{a}_k^*) - \sum_{j\in\mathcal{J}} c_j(\mathbf{a}_k^*) \geq \sum_{i\in\mathcal{I}_k\setminus\{\ell\}} v_i(\mathbf{a}_k^*) - \sum_{i\in\mathcal{I}_k} \sigma_i(\mathbf{b}_k^{\ell *}) - \sum_{j\in\mathcal{J}} c_j(\mathbf{a}_k^*). \tag{2.76}$$

So, by simplifying and rearranging (2.76), we have

$$\begin{aligned} \sum_{i\in\mathcal{I}_k} v_i(\mathbf{a}_k^*) - \sum_{i\in\mathcal{I}_k\setminus\{\ell\}} v_i(\mathbf{a}_k^*) &\geq \sum_{i\in\mathcal{I}_k} \sigma_i(\mathbf{a}_k^*) - \sum_{i\in\mathcal{I}_k} \sigma_i(\mathbf{b}_k^{\ell *}), \\ &= \sigma_\ell(\mathbf{a}_k^*) - \sigma_\ell(\mathbf{b}_k^{\ell *}) = \sigma_\ell(\mathbf{a}_k^*), \end{aligned} \tag{2.77}$$

since $\sigma_\ell(\mathbf{b}_k^{\ell *}) = 0$ as traveler ℓ is not assigned any mobility service under the traveler-service assignment $\mathbf{b}_k^{\ell *}$. Therefore, (2.77) simplifies to $v_\ell(\mathbf{a}_k^*) \geq \sigma_\ell(\mathbf{a}_k^*)$. □

Our first main result is incentive compatibility, which means that all travelers are incentivized to report their private information truthfully. Formally, for an arbitrary traveler $i \in \mathcal{I}_k$, $k = 1, \ldots, K$, given that u_i' is the utility gained with misreported π_i and u_i is the "actual" utility, showing that $u_i' \leq u_i$ guarantees truthfulness.

Theorem 2.2.21. *The mobility market defined in (2.52) provides the appropriate monetary incentives to each traveler $i \in \mathcal{I}_k$, $k = 1, \ldots, K$ to report their personal travel requirements $\pi_i = (\alpha_i, \theta_i, \bar{v}_i)$ truthfully regardless of what other travelers report.*

Proof. It is sufficient to show incentive compatibility only for an arbitrary mobility market for some arbitrary $k \in \{1, \ldots, K\}$. Suppose some traveler $\ell \in \mathcal{I}_k$ misreports their personal travel requirements denoted by $\pi_\ell = (\alpha'_\ell, \theta'_\ell, \bar{v}'_\ell)$ to the social planner. Thus, we have

$$v'_\ell(\mathbf{a}_k) = \bar{v}'_\ell - \phi_\ell\left(\alpha'_\ell, \theta'_\ell, \tilde{\theta}_\ell(\mathbf{a}_k)\right). \tag{2.78}$$

The objective function of Problem 2.2.14 becomes

$$w'_2(\mathbf{a}_k) = \sum_{i\in\mathcal{I}_k\setminus\{\ell\}} [v_i(\mathbf{a}_k) - \sigma_i(\mathbf{a}_k)] - \sum_{j\in\mathcal{J}} c_j(\mathbf{a}_k) + v'_\ell(\mathbf{a}_k), \tag{2.79}$$

where the feasible solution of (2.79) is subject to the same constraints as in Problem 2.2.14. We denote the optimal solution of the optimization problem that traveler ℓ has misreported their personal travel requirements π_ℓ with (2.79) as the objective function by $\tilde{\mathbf{a}}_k^*$. Then, for traveler $\ell \in \mathcal{I}_k$ their mobility payment can be computed as follows:

$$p'_\ell(\tilde{\mathbf{a}}_k^*, \tilde{\mathbf{b}}_k^*) = w_3(\tilde{\mathbf{b}}_k^*) - \left[w_2^\ell(\tilde{\mathbf{a}}_k^*) - v'_\ell(\tilde{\mathbf{a}}_k^*)\right] = w_3(\mathbf{b}_k^*) - \left[w_2^\ell(\tilde{\mathbf{a}}_k^*) - v'_\ell(\tilde{\mathbf{a}}_k^*)\right], \tag{2.80}$$

where $\tilde{\mathbf{b}}_k^*$ denotes the optimal solution of Problem 2.2.15 with traveler $\ell \in \mathcal{I}_k$ misreporting. However, $w_3(\tilde{\mathbf{b}}_k^*) = w_3(\mathbf{b}_k^*)$ since, in Problem 2.2.15, it does not matter what traveler $\ell \in \mathcal{I}_k$ reports. Thus, the total utility of traveler $\ell \in \mathcal{I}_k$ is

$$u_\ell(\tilde{\mathbf{a}}_k^*) = v_\ell(\tilde{\mathbf{a}}_k^*) - p'_\ell(\tilde{\mathbf{a}}_k^*, \mathbf{b}_k^*), \tag{2.81}$$

where for traveler $\ell \in \mathcal{I}_k$ the term $v_\ell(\tilde{\mathbf{a}}_k^*)$ is the actual satisfaction gained by misreporting their private information. Substituting (2.80) into (2.81) yields

$$u_\ell(\tilde{\mathbf{a}}_k^*) = v_\ell(\tilde{\mathbf{a}}_k^*) - \left[w_3(\mathbf{b}_k^*) - \left(w_2^\ell(\tilde{\mathbf{a}}_k^*) - v'_\ell(\tilde{\mathbf{a}}_k^*)\right)\right], \tag{2.82}$$

which after a few simplifications gives

$$u_\ell(\tilde{\mathbf{a}}_k^*) = v_\ell(\tilde{\mathbf{a}}_k^*) - w_3(\mathbf{b}_k^*) - \left[\left(\sum_{i\in\mathcal{I}_k\setminus\{\ell\}} [v_i(\tilde{\mathbf{a}}_k^*) - \sigma_i(\tilde{\mathbf{a}}_k^*)] - \sum_{j\in\mathcal{J}} c_j(\tilde{\mathbf{a}}_k^*) + v'_\ell(\tilde{\mathbf{a}}_k^*)\right) - v'_\ell(\tilde{\mathbf{a}}_k^*)\right]. \tag{2.83}$$

Hence, as the term $v'_\ell(\tilde{\mathbf{a}}_k^*)$ appears in opposite signs in (2.83), we have

$$\begin{aligned} u_\ell(\tilde{\mathbf{a}}_k^*) &= \left[\sum_{i\in\mathcal{I}_k} [v_i(\tilde{\mathbf{a}}_k^*) - \sigma_i(\tilde{\mathbf{a}}_k^*)] - \sum_{j\in\mathcal{J}} c_j(\tilde{\mathbf{a}}_k^*)\right] - w_3(\mathbf{b}_k^*) \\ &= w_2(\tilde{\mathbf{a}}_k^*) - w_3(\mathbf{b}_k^*). \end{aligned} \tag{2.84}$$

Note that $\tilde{\mathbf{a}}_k^*$ is not necessarily the optimal solution of Problem 2.2.14. Thus, we have $w_2(\tilde{\mathbf{a}}_k^*) \leq w_2(\mathbf{a}_k^*)$. So, we observe that

$$u_\ell(\tilde{\mathbf{a}}_k^*) = w_2(\tilde{\mathbf{a}}_k^*) - w_3(\mathbf{b}_k^*) \leq w_2(\mathbf{a}_k^*) - w_3(\mathbf{b}_k^*) = u_\ell(\mathbf{a}_k^*). \tag{2.85}$$

Therefore, from (2.85), it follows immediately that the proposed mobility market is incentive compatible. □

Our next result is individual rationality, which implies that all travelers voluntarily participate in the proposed mobility market. Formally, for any traveler $i \in \mathcal{I}_k$, $k = 1, \ldots, K$, if traveler i's utility u_i is non-negative, i.e., $u_i \geq 0$, then we say traveler i voluntarily participates in the mobility market. This is important as we can guarantee for any traveler i that what they are willing to pay, v_i, will never be less than what they actually pay, p_i.

Theorem 2.2.22. *The mobility market is individually rational. For any subclass $\mathcal{I}_k$, $k = 1, \ldots, K$, and for any traveler $i \in \mathcal{I}_k$, the utility of any traveler is non-negative, i.e., we have for all $i \in \mathcal{I}_k$, $u_i(\mathbf{a}_k) \geq 0$. Equivalently, $v_i(\mathbf{a}_k) \geq p_i(\mathbf{a}_k)$.*

Proof. It is sufficient to show the result only for one instance of a mobility market for some $k = \{1, \ldots, K\}$. There are two cases to consider. First, let us suppose that traveler $\ell \in \mathcal{I}_k$ rejects any travel recommendations from the social planner; denote such an assignment by $\hat{\mathbf{a}}_k$. From (2.69), traveler ℓ would be required to make a monetary payment equal to their maximum willingness-to-pay, i.e., $p_\ell = \bar{v}_\ell$. This implies that $u_\ell(\hat{\mathbf{a}}_k) = 0$. This is justifiable as traveler ℓ seeks to travel and the only alternative travel option to our mobility market is a taxicab service.

For the second case, let us consider the utility of an arbitrary traveler $i \in \mathcal{I}_k$ evaluated at the optimal solution $\mathbf{a}_k^*$ is given by

$$u_i(\mathbf{a}_k^*) = v_i(\mathbf{a}_k^*) - p_i(\mathbf{a}_k^*, \mathbf{b}_k^*). \tag{2.86}$$

Note that by Theorem 2.2.21 all travelers report their true private information at equilibrium. So, substituting (2.69) into (2.86) yields

$$u_i(\mathbf{a}_k^*) = v_i(\mathbf{a}_k^*) - [w_3(\mathbf{b}_k^*) - [w_2(\mathbf{a}_k^*) - v_i(\mathbf{a}_k^*)]] = w_2(\mathbf{a}_k^*) - w_3(\mathbf{b}_k^*). \tag{2.87}$$

Note that for each $k = 1, \ldots, K$, the feasible regions of Problems 2.2.14 and 2.2.15, say $\mathcal{F}_2$ and $\mathcal{F}_3$, respectively, satisfy the relation $\mathcal{F}_3 \subset \mathcal{F}_2$. This is because Problem 2.2.15 has the exact same constraints plus an additional one, i.e., (2.66), thus the maximization of w_3 (which is almost similar to the one in Problem 2.2.14) will always be less or equal than the maximization of w_2. Hence, it follows that $u_i(\mathbf{a}_k^*) = w_2(\mathbf{a}_k^*) - w_3(\mathbf{b}_k^*) \geq 0$. Therefore, the result follows. □

Next, we establish that the proposed mobility market is economically sustainable (see Definition 2.2.9).

Theorem 2.2.23. *The mobility market is economically sustainable, i.e., it is guaranteed to generate revenue from each traveler and always meet the minimum acceptable mobility payments. In other words, for each subclass $\mathcal{I}_k$, $k = 1, \ldots, K$, and for an arbitrary $\ell \in \mathcal{I}_k$, we have*

$$p_\ell(\mathbf{a}_k^*, \mathbf{b}_k^*) = w_3(\mathbf{b}_k^*) - [w_2(\mathbf{a}_k^*) - v_\ell(\mathbf{a}_k^*)] \geq \sigma_\ell(\mathbf{a}_k^*). \tag{2.88}$$

Proof. Let $\mathbf{b}_k^*$ be an optimal solution of Problem 2.2.15 and $\mathbf{b}_k^{\ell\,*}$ be the corresponding feasible solution of Problem 2.2.15 with $\mathbf{a}_k^*$ an optimal solution of Problem 2.2.13. Since $\mathbf{b}_k^{\ell\,*}$ is only a feasible solution, we have

$$w_3(\mathbf{b}_k^*) \geq w_3(\mathbf{b}_k^{\ell\,*}). \tag{2.89}$$

Given the mobility payments (2.69), if we subtract the term $[w_2(\mathbf{a}_k^*) - v_\ell(\mathbf{a}_k^*)]$ from both sides of (2.89), we have

$$p_\ell(\mathbf{a}_k^*, \mathbf{b}_k^*) = w_3(\mathbf{b}_k^*) - [w_2(\mathbf{a}_k^*) - v_\ell(\mathbf{a}_k^*)] \geq w_3(\mathbf{b}_k^{\ell\,*}) - [w_2(\mathbf{a}_k^*) - v_\ell(\mathbf{a}_k^*)]\,. \tag{2.90}$$

The RHS of (2.90) can be expanded as follows:

$$w_3(\mathbf{b}_k^{\ell *}) - [w_2(\mathbf{a}_k^*) - v_\ell(\mathbf{a}_k^*)] = \sum_{i \in \mathcal{I}_k} \left[v_i(\mathbf{b}_k^{\ell *}) - \sigma_i(\mathbf{b}_k^{\ell *}) \right] - \sum_{j \in \mathcal{J}} c_j(\mathbf{b}_k^{\ell *})$$
$$- \left[\left(\sum_{i \in \mathcal{I}_k} [v_i(\mathbf{a}_k^*) - \sigma_i(\mathbf{a}_k^*)] - \sum_{j \in \mathcal{J}} c_j(\mathbf{a}_k^*) \right) - v_\ell(\mathbf{a}_k^*) \right]. \quad (2.91)$$

After a few simplifications and rearranging of (2.91), we have

$$p_\ell(\mathbf{a}_k^*, \mathbf{b}_k^*) \geq \left[\sum_{i \in \mathcal{I}_k} v_i(\mathbf{b}_k^{\ell *}) - \sum_{i \in \mathcal{I}_k \setminus \{\ell\}} v_i(\mathbf{a}_k^*) \right] + \sum_{i \in \mathcal{I}_k} \left[\sigma_i(\mathbf{a}_k^*) - \sigma_i(\mathbf{b}_k^{\ell *}) \right]$$
$$+ \left[\sum_{j \in \mathcal{J}} c_j(\mathbf{a}_k^*) - \sum_{j \in \mathcal{J}} c_j(\mathbf{b}_k^{\ell *}) \right]. \quad (2.92)$$

So, by Corollary 2.2.18, the last term in (2.92) is non-negative. Similarly, by Lemma 2.2.19, the first term in (2.92) is non-negative. So, we get

$$p_\ell(\mathbf{a}_k^*, \mathbf{b}_k^*) \geq \sigma_\ell(\mathbf{a}_k^*) - \sigma_\ell(\mathbf{b}_k^{\ell *}) = \sigma_\ell(\mathbf{a}_k^*), \quad (2.93)$$

since under $\mathbf{b}_k^{\ell *}$ traveler ℓ has not been assigned any mobility service, thus $\sigma_\ell(\mathbf{b}_k^{\ell *}) = 0$, and so the result follows immediately. □

2.3 Summary

In this chapter, we addressed the problem of the social consequences of decision-making of human interaction with connectivity and automation in a game-theoretic setting. We formulated the problem as a multiplayer normal-form game and showed that the incentive structure is equivalent to the PD game. The proposed approach has the benefit of capturing the social dilemma that is expected to arise from the future social-mobility dilemma. We considered two different approaches: one was with a preference structure and one with institutions. We investigated and derived conditions for the unselfish strategy, i.e., non-CAV travel, to appear in the game. In the first case, we came up with conditions for a NE and derived a threshold for non-CAV travel; in the second case, we allowed players to create an institution that can enforce non-CAV travel. We concluded that the incentive ratio of non-CAV travel over CAV travel tends to zero as the number of players increases.

This chapter demonstrates how we can model and study the mobility decision-making of selfish travelers who are faced with the dilemma of "which mode of transportation to use" as an economically-inspired mobility market. First, the proposed market provides a *socially-efficient solution*, i.e., the endmost collective travel recommendation respects and satisfies the travelers' preferences regarding mobility and ensures that, implicitly, there will be an alleviation of congestion in the system. We achieve the latter by introducing appropriate constraints in the optimization problem; thus, our solution efficiently allocates all the available mobility services to the travelers. Furthermore, we showed that the proposed mobility market attains the properties of *incentive compatibility* and *individual rationality*. In other words, all travelers are incentivized to participate in the market while also truthfully reporting their personal travel requirements. Last, we introduced the notion of *minimum acceptable mobility payments* to ensure that the tolls and fares collected by the social planner will meet the mobility services' operating costs. Hence, the proposed market satisfies a status of *economic sustainability*.

Chapter 3

RESOURCE ALLOCATION AND MARKET-BASED MECHANISMS FOR MOBILITY SYSTEMS

This chapter has four parts and focuses on resource allocation and market-based approaches for efficient mobility. First, we focus on the social perspective of the emerging mobility systems with CAVs. It is widely accepted that CAVs will revolutionize urban mobility and the way people commute. An example would be for CAVs to make empty trips, i.e., no travelers, to avoid parking, and thus add extra congestion in the network [60]. In addition, CAVs could potentially affect drivers' behavior and have an impact on traffic performance in general [8]. The question of the actual impact of CAVs on travel, energy, and carbon demand has attracted considerable attention [246]. Depending on different environmental indicators, the authors in [245] provided a practical microeconomic environmental rebound effect model. So far, there has been research on the effects of a considerate penetration of shared CAVs in a major metropolitan area [144]. However, most studies on CAVs have focused on how to coordinate CAVs in different traffic scenarios [191, 258]. And so, we investigate the travel time provisioning in transportation networks with CAVs and strategic travelers. The main contribution of this chapter is the development of an informationally decentralized travel time social allocation mechanism with strategic travelers possessing the following properties: (i) existence of at least one Nash equilibrium (NE), (ii) budget balanced at equilibrium, (iii) individually rational, (iv) strongly implementable at NE, and (v) feasible at or off of equilibrium. Another contribution is that the design of our mechanism's tolls for the travelers' utilization of the network's resources is intuitive enough to provide a good understanding of the practical implementation of the mechanism.

What if the resource to be allocated in an emerging mobility market needs to be shared? To answer this, we provide the design of a shared mobility market for the stable assignment of travelers to shared vehicles. By stable we mean that, considering the decision-making of both travelers and vehicles' operators, no other assignment is preferred. We formulate a binary linear optimization problem and we show that our shared mobility market can produce optimal assignments with a feasible and stable traveler-vehicle profit allocation. For the latter, we also give the necessary and sufficient conditions when stability can be guaranteed.

Next, we offer the development of a game-theoretical framework to study the economic interactions of travelers and providers in a two-sided many-to-one assignment game. By two-sided we mean that we consider the preferences of both the travelers and the providers. By many-to-one we mean that we impose constraints of how many travelers can be assigned to one provider and how many providers to one traveler. Our analysis can be divided into two parts. First, we use linear programming arguments to showcase the existence of an optimal assignment between travelers and providers that is also stable, i.e., no one will seek to deviate from their match. Second, we consider an asymmetric informational structure, where no traveler/provider is expected to provide their private information willingly. We provide a pricing mechanism (Algorithm 2) for this case and show how we can successfully elicit the private information while also ensure efficiency (maximization of social welfare).

One key aspect of resource allocation is fairness and equity in how we allocate the limited resources to the decision-makers. And so, the final part of our chapter focuses on the game-theoretic development of framework to study the socioeconomic interactions of travelers in a multi-modal mobility system. We focus our analysis in the *economic sustainability* and *mobility equity* of the mobility system. We offer a game-theoretic definition of equity based on accessibility, i.e., we ensure our framework satisfies the following properties: truthfulness, voluntary participation, and budget fairness. In particular, we formulate our problem as a linear program to compute the assignments between travelers and mobility services that maximize the worst-case revenue of the system. Under an asymmetric informational structure [135], where a social planner has no knowledge of the individual travelers' valuations of the services, we provide a pricing mechanism and show how we can elicit the private information truthfully (Theorem 3.4.19) while ensuring budget fairness (Theorem 3.4.20). We also show that every traveler voluntary participates in our proposed framework (Theorem 3.4.21).

In Section 3.1.1, we present the mathematical formulation of our proposed mechanism. Then, in Section 3.1.2, we formally show that our proposed mechanism has properties (a) - (e). Finally, we draw some concluding remarks and discuss potential avenues for future research. The remainder of the chapter is structured as follows. In Section 3.2.1, we present the mathematical formulation of our shared mobility market, which forms the basis for the rest of the chapter. In Section 3.2.2, we provide a feasibility and stability analysis of the shared mobility market. In Section 3.3.1, we present the mathematical formulation of the proposed game-theoretic framework. In Section 3.3.2, we derive the theoretical properties of the proposed framework, and finally, we discuss the implementation of the proposed framework and provide a numerical example in 3.3.3. Then in Section 3.4.1, we present the mathematical formulation of the proposed mobility game which forms the basis of our theoretical study for the rest of the chapter. In Section 3.4.3, we derive the theoretical properties of our framework, and finally, in Section 3.4.4, we discuss the implementation of the proposed framework.

3.1 Social Resource Allocation in a Mobility System with Connected and Automated Vehicles: A Mechanism Design Problem

In this chapter, we investigate the social resource allocation in an emerging mobility system consisting of connected and automated vehicles (CAVs) using mechanism design. CAVs provide the most intriguing opportunity for enabling travelers to monitor mobility system conditions efficiently and make better decisions. However, this new reality will influence travelers' tendency-of-travel and might give rise to rebound effects, e.g., increased-vehicle-miles traveled. To tackle this phenomenon, we propose a mechanism design formulation that provides an efficient social resource allocation of travel time for all travelers. Our focus is on the socio-technical aspect of the problem, i.e., by designing appropriate socio-economic incentives, we seek to prevent potential rebound effects. In particular, we propose an economically inspired mechanism to influence the impact of the travelers' decision-making on the well-being of an emerging mobility system.

3.1.1 Mathematical Formulation

We consider a transportation network represented by a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, \ldots, V\}$ corresponds to the index set of vertices and $\mathcal{E} = \{1, \ldots, E\}$ the index set of directed edges. Each edge $e \in \mathcal{E}$ has a fixed capacity, i.e., $c_e \in \mathbb{R}_{>0}$, e.g., a high capacity c_e corresponds to a highway while a low capacity corresponds to an urban road. There are $n \in \mathbb{N}_{\geq 2}$ travelers represented by the set $\mathcal{I} = \{1, 2, \ldots, n\}$. Each traveler i is associated with an origin-destination pair $(o_i, d_i) \in \mathcal{V} \times \mathcal{V}$. The utilization of the roads in $\mathcal{G}$ is done by the use of CAVs, where each CAV corresponds to one traveler. We consider 100% penetration rate of CAVs.

Definition 3.1.1. *A traveler $i \in \mathcal{I}$ seeks to commute from o_i to d_i via a given and fixed route $p_i(o_i, d_i)$ at preferred travel time, denoted by $\theta_i \in \Theta_i = [0, +\infty)$. In game theoretic terms, θ_i is the type of traveler i. We denote the type profile of all travelers by $\theta = (\theta_1, \theta_2, \ldots, \theta_n)$.*

In addition, each edge $e \in \mathcal{E}$ in the network is characterized by $\underline{\theta}^e$ which represents the minimum possible travel time that any traveler can experience if edge $e \in \mathcal{E}$ is an empty (uncongested) road. This allows us to take into account rural or urban roads of different traffic capacities in the transportation network $\mathcal{G}$.

Next, each traveler $i \in \mathcal{I}$ has a cost function v_i which expresses the "commute-satisfaction" that traveler i experiences from commuting in (o_i, d_i) with travel time θ_i.

We expect v_i and θ_i to be traveler i's private information (i.e., unknown to the network manager).

Assumption 3.1.2. *Assume that $v_i : \mathbb{R}_{\geq 0} \to \mathbb{R}$ is continuously differentiable, strictly concave, and strictly decreasing in θ_i with $v_i(0) = 0$.*

Next, we denote by t_i the monetary payment made by traveler i to the network manager. We have $t_i \in \mathbb{R}$, i.e., a positive t_i means that traveler i pays a toll and a negative t_i means that i receives a monetary subsidy. Thus, in our mechanism, traveler i's total utility is given by

$$u_i(\theta_i, t_i) = v_i(\theta_i) - t_i, \tag{3.1}$$

where recall that v_i represents traveler i's satisfaction function and θ_i is traveler i's travel time.

We consider that all travelers are rational and intelligent decision-makers in the system. Each traveler $i \in \mathcal{I}$ has two objectives: (i) to reach their destination, and (ii) to maximize their own utility. A social consequence of the travelers' behavior is that there is an individual disregard of the overall good of the system and it is natural to expect that at least one edge $e \in \mathcal{E}$ will exceed its maximum capacity. If the network manager does not intervene, then congestion is to be expected. So, using appropriate monetary payments, the network manager can incentivize travelers to report truthfully their type θ_i and allocate travel time on each edge $e \in \mathcal{E}$ in such a way that all travelers are satisfied and congestion is prevented. To achieve this, the network manager's objective is to maximize the overall "social welfare" of the network and ensure that the network remains congestion-free. The social welfare function is defined as the $\sum_{i \in \mathcal{I}} v_i(\theta_i)$ and denoted by $\mathcal{W}$. We choose to define the social welfare as the sum of the utilities of all travelers because we follow the utilitarian principles, i.e., we measure the collective benefits gained by the travelers in the transportation network.

Next, note that the travelers' strategic behavior indicates a natural competition over the utilization of the edges.

Definition 3.1.3. *Given $e \in \mathcal{E}$, we define the following sets: (i) the set $\mathcal{S}_e$ of all travelers that edge e is part of their route that connects o_i and d_i, and (ii) the set $\mathcal{R}_i$ of traveler i's edges that consist of their route $p_i(o_i, d_i)$.*

Before we continue, we introduce the notion of *reverse value of time*, say parameter $\alpha_i \in \mathbb{R}_{\geq 1}$, that can vary among each traveler $i \in \mathcal{I}$. The social parameter $\alpha_i \in (\underline{\alpha}, \overline{\alpha})$, where $\underline{\alpha} \geq 1$, can be interpreted as follows. If $\alpha_i \to \overline{\alpha}$, traveler i is willing to tolerate a slightly higher travel time, while if $\alpha_i \to \underline{\alpha}$, traveler i is not willing to tolerate a higher travel time. We assume that each traveler $i \in \mathcal{I}$ can be classified based on socio-economic demographic data (e.g., mobility choices and travel tendencies, civil status and income) [35].

Problem 3.1.4. *The centralized social-welfare maximization problem is presented below:*

$$\max_{\theta_i^e} \sum_{i \in \mathcal{I}} \sum_{e \in \mathcal{R}_i} v_i(\theta_i^e),$$

$$\textit{subject to: } \theta_i^e \geq \underline{\theta}^e, \quad \forall e \in \mathcal{E}, \quad \forall i \in \mathcal{I}, \tag{3.2}$$

$$\sum_{i \in \mathcal{S}_e} \alpha_i \cdot \theta_i^e \leq c_e, \quad \forall e \in \mathcal{E}, \tag{3.3}$$

where θ_i^e is the travel time of traveler i on edge e with $\theta_i = \sum_{e \in \mathcal{R}_i} \theta_i^e$, and $v_i(\theta_i) = \sum_{e \in \mathcal{R}_i} v_i(\theta_i^e)$; inequalities (3.2) ensure that each traveler i's travel time θ_i^e on all edges $e \in \mathcal{E}$ is non-negative but not zero at any case; and inequality (3.3) expresses the network's capacity on each edge $e \in \mathcal{E}$.

By Assumption 3.1.2, it is imperative to impose a network threshold on the feasible values of each traveler i's travel time. We can achieve this in (3.2) by only accepting travelers' travel times that are above $\underline{\theta}^e$. Also, we interpret $\theta_i = 0$ to be the case of traveler i not seeking to commute instead of wishing to commute in zero time.

Problem 3.1.4 would be a standard convex optimization problem if the strategic travelers were expected to report their private information truthfully. As this is unreasonable to expect from strategic decision-makers, the network manager in order to solve Problem 3.1.4 is tasked to elicit the necessary information using monetary incentives.

In our formulation, we use the NE as our solution concept. However, a NE requires complete information. But, we can interpret a NE as the fixed point of an iterative process in an incomplete information setting [188, 91]. This is in accordance with J. Nash's interpretation of a NE, i.e., the complete information NE can be a possible equilibrium of an iterative learning process.

In this section, we present the fundamentals of an indirect and decentralized resource allocation mechanism following the framework presented in [102]. First, we need to specify a set of messages that all travelers have access and are able to use in order to communicate information. Based on this information, travelers make decisions which affect the reaction of the network manager. Once the communication between the network manager and the travelers is complete, we say that the mechanism induces a game; strategic travelers then compete for the network's resources. In this line of reasoning, we define formally below what we mean by indirect mechanism and induced game.

An indirect mechanism can be described as a tuple of two components, namely $\langle M, g \rangle$. We write $M = (M_1, M_2, \ldots, M_n)$, where M_i defines the set of possible messages of traveler i. Thus, the travelers' complete message space is $\mathcal{M} = M_1 \times \cdots \times M_n$. The component g is the outcome function defined by $g : \mathcal{M} \to \mathcal{O}$ which maps each message profile to the output space $\mathcal{O} = \{(\theta_1, \ldots, \theta_n), (t_1, \ldots, t_n) \mid \theta_i \in \mathbb{R}_{\geq 0},\ t_i \in \mathbb{R}\}$, i.e., the set of all possible travel time allocations to the travelers and the monetary payments (e.g., toll, subsidies) made or received by the travelers. The outcome function g determines the outcome, namely $g(\mu)$ for any given message profile $\mu = (m_1, \ldots, m_n) \in \mathcal{M}$. The payment function $t_i : \mathcal{M} \to \mathbb{R}$ determines the monetary payment made or received by a traveler $i \in \mathcal{I}$.

Definition 3.1.5. *A mechanism $\langle M, g\rangle$ together with the utility functions $(u_i)_{i\in\mathcal{I}}$ induce a game $\langle M, g, (u_i)_{i\in\mathcal{I}}\rangle$, where each utility u_i is evaluated at $g(\mu)$ for each traveler $i \in \mathcal{I}$.*

Definition 3.1.6. *Consider a game $\langle M, g, (u_i)_{i\in\mathcal{I}}\rangle$. The solution concept of NE is a message profile μ^* such that $u_i(g(m_i^*, m_{-i}^*)) \geq u_i(g(m_i, m_{-i}^*))$, for all $m_i \in M_i$ and for each $i \in \mathcal{I}$, where $m_{-i} = (m_1, \dots, m_{i-1}, m_{i+1}, \dots, m_n)$.*

Definition 3.1.7. *Let the* utility of no participation *of a traveler $i \in \mathcal{I}$ to be given by $u_i(0, 0) = v_i(0) = 0$. Then, we say that a mechanism is individually rational if $u_i(g(\mu^*)) \geq 0$, for all $i \in \mathcal{I}$, and all NE $\mu^* \in \mathcal{M}$.*

We show now how the network manager can design monetary incentives which achieve the desirable goal, i.e., align everyone's decisions by incentivizing them to send social-welfare supporting messages. But first, we need to establish the informational structure of our mechanism. The network manager has complete knowledge of the network's topology and resources and travelers know only their own utility which they report privately to the network manager. Before we continue, let us define explicitly a traveler's message. For each $i \in \mathcal{I}$, message $m_i \in M_i$ is given by $m_i = (\tilde{\theta}_i, \tau_i)$, where $\tilde{\theta}_i = (\tilde{\theta}_i^e : e \in \mathcal{R}_i)$ is the reported preferred travel time of traveler i, and $\tau_i = (\tau_i^e : e \in \mathcal{R}_i)$ is the price traveler i is willing to pay for $\tilde{\theta}_i$ along their route.

Definition 3.1.8. *The average price of all travelers that compete to utilize edge $e \in \mathcal{E}$ other than traveler i is given by $\tau_{-i}^e = \sum_{j\in\mathcal{S}_e : j\neq i} \frac{\tau_j^e}{|\mathcal{S}_e|-1}$.*

Next, for each traveler i and for each edge $e \in \mathcal{E}$ of their route, we endow a fair share for each edge $e \in \mathcal{E}$, i.e., $c_e/|\mathcal{S}_e|$. This can help us design the monetary payments that each traveler is asked to pay. Using Definition 3.1.8, we propose the following payments, for a particular edge $e \in \mathcal{E}$,

$$t_i^e(\mu) = \tau_{-i}^e \cdot \left(\alpha_i \cdot \tilde{\theta}_i^e - \frac{c_e}{|\mathcal{S}_e|}\right) + (\tau_i^e - \nu_e)^2 + \tau_{-i}^e \cdot (\tau_i^e - \tau_{-i}^e) \cdot \left(c_e - \sum_{i\in\mathcal{S}_e} \alpha_i \cdot \tilde{\theta}_i^e\right)^2. \quad (3.4)$$

The first term in (3.4) is the monetary payments (e.g., toll, subsidies) made or received by traveler i corresponding to their travel time allocation $\tilde{\theta}_i^e$ on edge $e \in \mathcal{E}$. Intuitively, this means that traveler i will pay a toll that is determined by the other travelers' recommendations and only for the excess of the fair share of travelers over a particular edge. Using this formulation, there is no incentive for traveler i to lie in an attempt to reduce their payment to the network. The second term in (3.4) corresponds to a penalty that traveler i will pay if she reports a different price τ_i^e from ν_e, where ν_e represents the Lagrange multiplier corresponding to the capacity constraint defined formally next. The third term in (3.4), collectively incentivizes all travelers to bid the same price per unit of travel time and to utilize the full capacity of each edge $e \in \mathcal{E}$.

Thus, given any message profile μ, the total monetary payment $t_i(\mu)$ for traveler i is

$$t_i(\mu) = \sum_{e \in \mathcal{R}_i} t_i^e(\mu) + \phi_i(\tilde{\theta}_i), \tag{3.5}$$

where ϕ_i is a monetary incentive that encourages traveler i to report a reasonable travel time demand respecting road rules and the network's efficiency goals. In detail, we have

$$\phi_i(\tilde{\theta}_i) = \begin{cases} \gamma, & \exists e \in \mathcal{R}_i, \text{ s.t. } \tilde{\theta}_i^e > \underline{\theta}^e \text{ and } |\mathcal{S}_e| = 1, \\ 0, & \exists e \in \mathcal{R}_i, \text{ s.t. } \tilde{\theta}_i^e > \underline{\theta}^e \text{ and } |\mathcal{S}_e| \geq 2, \\ \delta, & \exists e \in \mathcal{R}_i, \text{ s.t. } \tilde{\theta}_i^e = \underline{\theta}^e \text{ and } |\mathcal{S}_e| \geq 2, \\ 0, & \exists e \in \mathcal{R}_i, \text{ s.t. } \tilde{\theta}_i^e = \underline{\theta}^e \text{ and } |\mathcal{S}_e| = 1, \end{cases} \tag{3.6}$$

where $\gamma, \delta \in \mathbb{R}_{>0}$ represent the imposition of very high penalties. It is necessary to impose such penalties since for the first case in (3.6), traveler i violates the goal of efficiency in the network and for the third case in (3.6), traveler i violates the goal of road safety. In the severe case of $\tilde{\theta}_i^e < \underline{\theta}^e$, we have $\phi_i(\tilde{\theta}_i) = +\infty$.

3.1.2 Properties of the Mechanism

In this section, we present the properties of our proposed mechanism.

Lemma 3.1.9. *Problem 3.1.4 has a unique optimal solution.*

Proof. The objective function of Problem 3.1.4 is a sum of several strictly concave functions. Hence, it is strictly concave. Thus, the necessary KKT conditions are also sufficient for optimality. Since the feasible region is non-empty, convex, and compact, we conclude that Problem 3.1.4 has always a unique optimal solution. □

Lemma 3.1.10. *A solution to Problem 3.1.4 is unique and optimal if, and only if, it satisfies the feasibility conditions of Problem 3.1.4 and there exist Lagrange multipliers* $\lambda = (\lambda_i^e : e \in \mathcal{E})_{i\in\mathcal{I}}$ *and* $\nu = (\nu_e)_{e\in\mathcal{E}}$ *that satisfy the following conditions:*

$$\frac{\partial v_i(\theta_i^{e*})}{\partial \theta_i^e} + \lambda_i^{e*} - \sum_{e\in\mathcal{R}_i} \alpha_i \cdot \nu_e^* = 0, \tag{3.7}$$

$$\lambda_i^{e*} \cdot (\theta_i^{e*} - \underline{\theta}^e) = 0, \quad \forall e \in \mathcal{E}, \quad \forall i \in \mathcal{I}, \tag{3.8}$$

$$\nu_e^* \cdot \left(\sum_{i\in\mathcal{S}_e} \alpha_i \cdot \theta_i^{e*} - c_e \right) = 0, \quad \forall e \in \mathcal{E}, \tag{3.9}$$

$$\lambda_i^{e*}, \nu_e^* \geq 0, \quad \forall e \in \mathcal{E}, \quad \forall i \in \mathcal{I}. \tag{3.10}$$

Proof. First, let us derive the Lagrangian of Problem 3.1.4:

$$\mathcal{L}(\theta, \lambda, \nu) = \sum_{i\in\mathcal{I}} \sum_{e\in\mathcal{R}_i} v_i(\theta_i^e) + \sum_{i\in\mathcal{I}} \sum_{e\in\mathcal{E}} \lambda_i^e \cdot (\theta_i^e - \underline{\theta}^e) - \sum_{e\in\mathcal{E}} \nu_e \cdot \left(\sum_{i\in\mathcal{S}_e} \alpha_i \cdot \theta_i^e - c_e \right). \tag{3.11}$$

From (3.11), it is easy to derive the KKT conditions, i.e.,

$$\frac{\partial v_i(\theta_i^{e*})}{\partial \theta_i^e} + \lambda_i^{e*} - \sum_{e\in\mathcal{R}_i} \alpha_i \cdot \nu_e^* = 0, \tag{3.12}$$

$$\lambda_i^{e*} \cdot (\theta_i^{e*} - \underline{\theta}^e) = 0, \quad \forall e \in \mathcal{E}, \quad \forall i \in \mathcal{I}, \tag{3.13}$$

$$\nu_e^* \cdot \left(\sum_{i\in\mathcal{S}_e} \alpha_i \cdot \theta_i^{e*} - c_e \right) = 0, \quad \forall e \in \mathcal{E}, \tag{3.14}$$

$$\lambda_i^{e*}, \nu_e^* \geq 0, \quad \forall e \in \mathcal{E}, \quad \forall i \in \mathcal{I}. \tag{3.15}$$

Since the KKT conditions are necessary and sufficient to guarantee the optimality of any allocation of travel time that satisfies them, it is enough to find λ_i^{e*} and ν_e^* such that the above conditions are satisfied. □

Theorem 3.1.11 (Feasibility). *For any message profile μ, the corresponding travel time allocation θ is a feasible point of Problem 3.1.4.*

Proof. Consider any traveler i and denote by $\mathcal{C}$ the constraint set of Problem 3.1.4. Then, for a reported preferred travel time $\tilde{\theta}_i$, the travel time θ_i of Problem 3.1.4 generated by the outcome function is equal to (i) $\tilde{\theta}_i$ if $\tilde{\theta}_i \in \mathcal{C}$; or (ii) θ_i^0 if $\tilde{\theta}_i \notin \mathcal{C}$, where $\tilde{\theta}_i = (\tilde{\theta}_i^e : e \in \mathcal{R}_i)$, and θ_i^0 is the point on the boundary of $\mathcal{C}$ (i.e., we ignore the "unreasonable" demand of traveler i and allocate only the portion of the resource that is available). By construction, it follows immediately that if $\tilde{\theta}_i \in \mathcal{C}$, then the allocation θ_i is feasible for any traveler $i \in \mathcal{I}$. In the case of $\tilde{\theta}_i \notin \mathcal{C}$, the allocation is on the boundary of $\mathcal{C}$, hence it is still feasible as the constraint set of Problem 3.1.4 is closed. Thus, the result follows. □

Lemma 3.1.12. *Let μ^* be a NE of the induced game. Then, we have $\tau_i^{e*} = \nu_e^*$, for all $i \in \mathcal{I}$ and each $e \in \mathcal{R}_i$. In addition, it follows that $\tau_{-i}^e = \sum_{j \in \mathcal{S}_e : j \neq i} \frac{\tau_j^e}{|\mathcal{S}_e|-1} = \tau_i^{e*}$.*

Proof. Suppose there is one traveler, say i, that deviates from the NE message profile μ^* and instead reports the message $m_i = (\tilde{\theta}_i^*, \tau_i)$. This deviation to be justifiable has to provide a higher utility to traveler $i \in \mathcal{I}$. But, we have

$$v_i(\tilde{\theta}_i^*) - t_i(m_i^*, m_{-i}^*) \geq v_i(\tilde{\theta}_i^*) - t_i(m_i, m_{-i}^*). \tag{3.16}$$

Next, we substitute (3.4) into (3.16). For ease of notational exposition, let $\xi = \left(c_e - \sum_{i \in \mathcal{S}_e} \alpha_i \cdot \tilde{\theta}_i^{e*}\right)^2$. Thus,

$$\sum_{e \in \mathcal{R}_i} (\tau_i^{e*} - \nu_e^*)^2 + {\tau_{-i}^e}^* \cdot (\tau_i^{e*} - {\tau_{-i}^e}^*) \cdot \xi \leq \sum_{e \in \mathcal{R}_i} (\tau_i^e - \nu_e^*)^2 + {\tau_{-i}^e}^* \cdot (\tau_i^e - {\tau_{-i}^e}^*) \cdot \xi. \tag{3.17}$$

Since traveler i behaves as a utility-maximizer, we need to minimize the right hand side of (3.17). Thus, the best price is $\tau_i^e = {\tau_{-i}^e}^*$, and also the solution of the minimization problem $\min_{(\tau_i^e)} \sum_{e \in \mathcal{R}_i} (\tau_i^e - \nu_e^*)^2$. Therefore, at μ^*, we have $\tau_i^{e*} = \nu_e^*$, for all $e \in \mathcal{E}$ and for all $i \in \mathcal{I}$ and $\tau_{-i}^e = \sum_{j \in \mathcal{S}_e : j \neq i} \frac{\tau_j^e}{|\mathcal{S}_e|-1} = \tau_i^{e*}$ follows immediately. □

Lemma 3.1.13. *Let μ^* be a NE of the induced game. Then, for every traveler $i \in \mathcal{I}$, we have $\phi_i(\tilde{\theta}_i^*) = 0$.*

Proof. We prove this by contradiction. Suppose there exists a NE message $\mu^* = (m_i^* = (\tilde{\theta}_i^*, \tau_i^*))_{i\in\mathcal{I}}$ such that $\phi_i(\tilde{\theta}_i^*) \neq 0$ for traveler $i \in \mathcal{I}$. By (3.6), we only have two cases to consider: let $\tilde{\theta}_i^{e*} > \underline{\theta}^e$ with $|\mathcal{S}_e| = 1$ (the proof for the other case is similar). Suppose traveler i deviates from the NE with message $m_i = ((\tilde{\theta}_i^e = \underline{\theta}^e : e \in \mathcal{R}_i), \tau_i^*)$. By Definition 3.1.6, we have

$$u_i(g(m_i, m_{-i}^*)) \leq u_i(g(m^*)). \tag{3.18}$$

Substitute (3.1), (3.4), and (3.6) into (3.18) and then Lemma 3.1.12 gives

$$[v_i(\tilde{\theta}_i) - v_i(\tilde{\theta}_i{}^*)] - \sum_{e\in\mathcal{S}_e} \alpha_i \cdot \nu_e^* \cdot (\tilde{\theta}_i^e - \tilde{\theta}_i^{e*}) + \phi_i(\tilde{\theta}_i^*) \leq 0, \tag{3.19}$$

where by Assumption 3.1.2, the first difference term of (3.19) is negative; likewise the difference of $(\tilde{\theta}_i^e - \tilde{\theta}_i^{e*})$ is positive. Thus, it follows that, since $\phi_i(\tilde{\theta}_i^*) \gg 0$, traveler i rightfully deviates from the NE $\mu^* = (m_i^* = (\tilde{\theta}_i^*, \tau_i^*))_{i\in\mathcal{I}}$ such that $\phi_i(\tilde{\theta}_i^*) \neq 0$ as (3.19) cannot be true (by construction of (3.6)). Since the case of $\tilde{\theta}_i^e < \underline{\theta}^e$, where $\phi_i(\tilde{\theta}_i) = +\infty$ is straightforward to show, and the proof is complete. □

Theorem 3.1.14 (Budget Balance)**.** *Let the message profile μ^* be a NE of the induced game. The proposed mechanism at μ^* does not require any external or internal monetary payments, i.e., $\sum_{i\in\mathcal{I}} t_i(\mu^*) = 0$ for all μ^*.*

Proof. Summing (3.4) over all travelers yields $\sum_{i\in\mathcal{I}} t_i(\mu^*) = \sum_{i\in\mathcal{I}} \left[\sum_{e\in\mathcal{R}_i} t_i^e(\mu^*)\right] = \sum_{e\in\mathcal{H}} \sum_{i\in\mathcal{S}_e} t_i^e(\mu^*)$, where $\mathcal{H}$ is the set of competitive edges in the network (i.e., any edge utilized by more than two travelers). Hence,

$$\sum_{i\in\mathcal{I}} t_i(\mu^*) = \sum_{e\in\mathcal{H}} \sum_{i\in\mathcal{S}_e} \tau_{-i}^e{}^* \cdot \left(\alpha_i \cdot \tilde{\theta}_i^{e*} - \frac{c_e}{|\mathcal{S}_e|}\right) + (\tau_i^{e*} - \nu_e^*)^2 + \tau_{-i}^e{}^* \cdot (\tau_i^{e*} - \tau_{-i}^e{}^*) \cdot \left(c_e - \sum_{i\in\mathcal{S}_e} \alpha_i \cdot \tilde{\theta}_i^{e*}\right). \tag{3.20}$$

By Lemma 3.1.12, we have for all $e \in \mathcal{H}$, $\sum_{i\in\mathcal{I}} t_i(\mu^*) = \sum_{e\in\mathcal{H}} \nu_e^* \cdot \Big(\sum_{i\in\mathcal{S}_e} \alpha_i \cdot \tilde{\theta}_i^{e*} - c_e\Big)$, which is equal to zero by the KKT conditions in Lemma 3.1.10. □

Theorem 3.1.15 (Individually Rational)**.** *The proposed mechanism is individually rational. In particular, each traveler prefers the outcome of any NE of the induced game to the outcome of no participation.*

Proof. Let the message profile μ^* be an arbitrary NE of the induced game. We need to show that $u_i(\mu^*) \geq u_i(0) = 0$ for each traveler i (see Definition 3.1.7). Consider the message $m_i = (\tilde{\theta}_i, \tau_i)$ with $\tilde{\theta}_i = 0$ and $\tau_i = (\tau_i^e = \nu_e : e \in \mathcal{R}_i)$. That is, traveler i deviates with m_i while the other travelers adhere to the NE μ^*. By Definition 3.1.6, we have the following:

$$\begin{aligned} u_i(g(\mu^*)) &\geq u_i(g(m_i, m_{-i}^*)) \\ &= v_i(0) - \sum_{e\in\mathcal{R}_i} \tau_{-i}^{e\;*} \cdot \left(0 - \frac{c_e}{|\mathcal{S}_e|}\right) \\ &= \sum_{e\in\mathcal{R}_i} \nu_e^* \cdot \left(\frac{c_e}{|\mathcal{S}_e|}\right) \geq 0. \end{aligned} \tag{3.21}$$

Thus, from (3.21), the result follows. □

In our next result, we show that our mechanism is strongly implementable at NE. Strong implementation ensures that the efficient allocation of travel time to the travelers is implemented by all equilibria of the induced game [151].

Theorem 3.1.16 (Strong Implementation)**.** *At an arbitrary NE μ^* of the induced game, the allocation travel time $(\tilde{\theta}_i^*)_{i\in\mathcal{I}}$ is equal to the optimal solution $(\theta_i^*)_{i\in\mathcal{I}}$ of Problem 3.1.4 for each $i \in \mathcal{I}$.*

Proof. Suppose μ^* is a NE of the induced game. Then, by Lemma 3.1.12, it follows that $\tau_i^{e*} = \tau_{-i}^{e\;*} = \nu_e^*$ for each $e \in \mathcal{R}_i$. Next, consider some traveler i that participates in the mechanism and has preferred travel time θ_i. The utility of traveler i for such an allocation is given by

$$u_i(g(m_i, m_{-i}^*)) = v_i(\theta_i) - t_i(m_i, m_{-i}^*), \tag{3.22}$$

where $t_i(m_i, m_{-i}^*) = \sum_{e\in\mathcal{R}_i} \nu_e^* \left(\alpha_i \cdot \theta_i^e - \frac{c_e}{|\mathcal{S}_e|}\right)$. By Definition 3.1.6, it follows that at NE no traveler should have an incentive to deviate. Hence, the maximization of traveler i's utility (3.22) must be attained at the NE travel time allocation, i.e., $\theta_i^* = \tilde{\theta}_i^*$. The Nash-maximization problem is

$$\tilde{\theta}_i^{e*} = \arg\max_{\theta_i^e} \left[\sum_{e\in\mathcal{R}_i} v_i(\theta_i^e) - \sum_{e\in\mathcal{R}_i} \nu_e^* \left(\alpha_i \cdot \theta_i^e - \frac{c_e}{|\mathcal{S}_e|}\right)\right], \tag{3.23}$$

subject to the exact same constraints as in Problem 3.1.4. Now, it is easy to derive the KKT conditions that will give the optimal "Nash solution." By Lemma 3.1.10, the KKT conditions are necessary and sufficient to guarantee the optimality of any travel time allocation $(\theta_i)_{i\in\mathcal{I}}$ that satisfies them. Thus, it is sufficient to show that there exist appropriate Lagrange multipliers λ_i^{e*} and ν_e^* such that (3.7) - (3.9) are satisfied. By setting $\lambda_i^e = 0$ and $\nu_e = \tau_i^e$ for all $e \in \mathcal{E}$, by differentiation of (3.4) with respect to θ_i^e and τ_i^e, we get

$$\frac{\partial v_i(\tilde{\theta}_i^{e*})}{\partial \tilde{\theta}_i^e} = \sum_{e\in\mathcal{R}_i} \alpha_i \cdot \nu_e^*, \quad \forall i \in \mathcal{I}, \tag{3.24}$$

$$\nu_e^* \cdot \left(\sum_{i\in\mathcal{S}_e} \alpha_i \cdot \tilde{\theta}_i^{e*} - c_e\right) = 0, \quad \forall e \in \mathcal{E}. \tag{3.25}$$

It is straightforward to see that (3.24) and (3.25) are identical to (3.7) and (3.9), respectively. Condition (3.8) in both problems holds trivially. Consequently, the solution $\tilde{\theta}^* = (\tilde{\theta}_1^*, \ldots, \tilde{\theta}_n^*)$ of (3.24) and (3.25) along with the specification of the payment functions (3.4) are equivalent to the optimal unique solution of Problem 3.1.4. Thus, at any NE μ^*, we get an identical allocation $g(\mu^*) = (\tilde{\theta}_1^*, \ldots, \tilde{\theta}_n^*, t_1^*, \ldots, t_n^*)$ that is equal to the optimal solution of Problem 3.1.4, and the proof is complete. □

Theorem 3.1.17 (Existence)**.** *Let θ^* be the optimal solution of Problem 3.1.4 and ν_e^* be the corresponding Lagrange multipliers of the KKT conditions. If for each $i \in \mathcal{I}$, $m_i^* = (\tilde{\theta}_i^* = \theta_i^*, \tau_i^*)$, where $\tau_i^* = (\tau_i^{e*} = \nu_e^* : \forall e \in \mathcal{R}_i)$ and $\phi_i(\tilde{\theta}_i^*) = 0$ for all $i \in \mathcal{I}$, then the message $\mu^* = (m_i^*)_{i\in\mathcal{I}}$ is a NE of the induced game.*

Proof. We show that the message profile $\mu^* = (m_i^*)_{i\in\mathcal{I}}$ where $m_i^* = (\tilde{\theta}_i = \theta_i^*, \tau_i^*)$ is a NE. By Lemma 3.1.10, it follows that θ^* along with the appropriate Lagrange multipliers satisfies the KKT conditions of Problem 3.1.4 and is the only feasible allocation. For any traveler i, the utility at message μ^* is $u_i(g(\mu^*)) = v_i(\tilde{\theta}_i^*) - \sum_{e\in\mathcal{R}_i} \nu_e^* \cdot \left(\alpha_i \cdot \tilde{\theta}_i^{e*} - \frac{c_e}{|\mathcal{S}_e|}\right)$. Now, suppose traveler i deviates from μ^* by changing their message while all the other travelers adhere to the message μ^* (though we would still have $\tau_{-i}^{e\,*} = \nu_e^*$). We have

$$\begin{aligned} u_i(g(m_i, m_{-i}^*)) &\leq v_i(\tilde{\theta}_i') - \sum_{e\in\mathcal{R}_i} \nu_e^* \cdot \left(\alpha_i \cdot \tilde{\theta}_i^{e\prime} - \frac{c_e}{|\mathcal{S}_e|}\right) \\ &\leq \max_{\tilde{\theta}_i'} \left[v_i(\theta_i') - \sum_{e\in\mathcal{R}_i} \nu_e^* \cdot \left(\alpha_i \cdot \tilde{\theta}_i^{e\prime} - \frac{c_e}{|\mathcal{S}_e|}\right)\right]. \end{aligned} \tag{3.26}$$

The maximization problem (3.26) is equivalent to (3.23). As the message μ^* clearly satisfies the KKT conditions of (3.23), we have $\theta_i = \tilde{\theta}_i = \tilde{\theta}_i'$, which in turn implies:

$$u_i(g(m_i, m_{-i}^*)) \leq v_i(\tilde{\theta}_i^*) - \sum_{e\in\mathcal{R}_i} \nu_e^* \cdot \left(\alpha_i \cdot \tilde{\theta}_i^{e} - \frac{c_e}{|\mathcal{S}_e|}\right), \tag{3.27}$$

where the right hand side of (3.27) is equal to $u_i(g(\mu^*))$, for all m_i^* and all $i \in \mathcal{I}$. Therefore, message μ^* is a NE. □

3.2 Design and Stability Analysis of a Shared Mobility Market

In recent years, we have witnessed a remarkable surge of usage in shared vehicles in our cities. Shared mobility offers a future of no congestion in busy city roads with increasing populations of travelers, passengers, and drivers. Given the behavioral decision-making of travelers and the shared vehicles' operators, however, the question is "how can we ensure a socially-acceptable assignment between travelers and vehicles?" In other words, how can we design a shared mobility system that assigns each traveler to the "right" vehicle? In this chapter, we design a shared mobility market consisted of travelers and vehicles in a transportation network. We formulate a binary linear program problem and derive the optimal assignment between travelers and vehicles. In addition, we provide the necessary and sufficient conditions for the stable traveler-vehicle profit allocation. Our objective is to (1) maximize the social welfare of all travelers with the optimal assignment, and (2) ensure the feasibility and stability of the traveler-vehicle profit allocation while respecting the decision-making of both the travelers and the vehicles' operators.

3.2.1 Mathematical Formulation

We consider a mobility system managed by a social planner whose objective is to assign $m \in \mathbb{N}$ vehicles to $n \in \mathbb{N}$ travelers, where $n \geq m$. We denote the set of travelers by $\mathcal{I} = \{0\} \cup \{1, 2, \ldots, n\}$ and the set of vehicles by $\mathcal{J} = \{1, 2, \ldots, m\}$. In $\mathcal{I}$, the index 0 has no practical meaning other than helping us to assign any vehicles that have not been assigned to travelers. Travelers seek to travel in a transportation network represented by a directed multi-graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where each vertex in $\mathcal{V}$ represents a different city area, or neighborhood, and each edge $e \in \mathcal{E}$ represents a city road connection. In this network, an arbitrary traveler $i \in \mathcal{I}$ wants to travel from their current location $o_i \in \mathcal{V}$ to their self-chosen destination $d_i \in \mathcal{V}$. So, we say that traveler $i \in \mathcal{I}$ is associated with an origin-destination pair (o_i, d_i). Similarly, each vehicle is associated with a route, i.e., a specific sequence of edges. Hence, the social planner aims to assign any traveler i to a vehicle so that their (o_i, d_i) can be satisfied by the vehicle's route.

Definition 3.2.1. *The* traveler-service assignment *is a vector*

$$\mathbf{a} = (a_{11}, \ldots, a_{ij}, \ldots, a_{nm}) = (a_{ij})_{i \in \mathcal{I}, j \in \mathcal{J}}, \tag{3.28}$$

where a_{ij} is a binary variable of the form:

$$a_{ij} = \begin{cases} 1, & \text{if } i \in \mathcal{I} \text{ is assigned to } j \in \mathcal{J}, \\ 0, & \text{otherwise.} \end{cases} \tag{3.29}$$

A traveler i's satisfaction is represented by a valuation function $v_i(a_{ij}) \in [\underline{v}_i, \bar{v}_i]$ when assigned to vehicle $j \in \mathcal{J}$, where $\underline{v}_i \in \mathbb{R}_{\geq 0}$ represents the lower bound of traveler i's satisfaction, and $\bar{v}_i \in \mathbb{R}_{\geq 0}$ represents the upper bound of traveler i's satisfaction.

Intuitively, a traveler's satisfaction reflects the traveler's value of the service they expect to receive from a vehicle $j \in \mathcal{J}$.

The satisfaction $v_i(\cdot)$ can be defined in terms of several factors (e.g., preferred and experienced number of co-travelers, in-vehicle travel time, or pickup time) that measure how satisfied the traveler can be with vehicle $j \in \mathcal{J}$. For example, a traveler can have a preferred travel time and their satisfaction can measure the monetary value of the difference between preferred and experienced travel time. The disutility caused by vehicle $j \in \mathcal{J}$ to traveler $i \in \mathcal{I}$ is given by $\phi_i(a_{ij}) \in \mathbb{R}_{\geq 0}$. We call $\phi_i(\cdot)$ the *inconvenience cost* as it can measure the travel inconvenience caused to traveler i. Thus, we have

$$v_i(a_{ij}) = \bar{v}_i - \phi_i(a_{ij}). \tag{3.30}$$

where $\bar{v}_i$ is the upper bound of traveler i's satisfaction. Although our analysis will treat $v_i(a_{ij})$ in its most general form (3.30), one can explicitly define $v_i(a_{ij})$ as follows,

$$v_i(a_{ij}) = \begin{cases} \bar{v}_i, & \text{if } \phi_i = 0, \\ \lambda_i \cdot \bar{v}_i, & \text{if } \phi_i = (1 - \lambda_i) \cdot \bar{v}_i, \\ 0, & \text{if } \phi_i = \bar{v}_i, \end{cases} \tag{3.31}$$

where $\lambda_i \in (0, 1)$ is a discount rate.

Next, the total utility of traveler $i \in \mathcal{I}$ is given by

$$u_i(a_{ij}) = v_i(a_{ij}) - t_i(a_{ij}), \tag{3.32}$$

where $t_i \in \mathbb{R}_{>0}$ is the monetary payment that traveler $i \in \mathcal{I}$, e.g., a fare that traveler $i \in \mathcal{I}$ may make for the services of vehicle $j \in \mathcal{J}$. Hence, (3.32) establishes a "quasi-linear" relationship between a traveler's satisfaction and payment, both measured in monetary units.

Definition 3.2.2. *For each vehicle $j \in \mathcal{J}$, the* vehicle maximum capacity $\varepsilon_j \in \mathbb{N}$ *yields how many travelers can receive a ride from vehicle $j \in \mathcal{J}$.*

Definition 3.2.3. *The* social welfare *of the shared mobility market is the collective summation of all travelers' utilities, i.e.,* $W(\mathbf{a}) = \sum_{i \in \mathcal{I}} u_i(a_{ij})$.

As we will see at a later subsection, our objective is to maximize the social welfare.

Definition 3.2.4. *The* operating cost *of vehicle* $j \in \mathcal{J}$ *denoted by* $c_j \in \mathbb{R}_{>0}$ *is shared (not necessarily equally) by each traveler* $i \in \mathcal{I}$ *assigned to vehicle* $j \in \mathcal{J}$ *and can be given by*

$$c_j = \sum_{i \in \mathcal{I} \setminus \{0\}} c_{ij}(a_{ij}), \tag{3.33}$$

where $c_{ij}(a_{ij})$ *is traveler* i*'s share of the operating cost of vehicle* $j \in \mathcal{J}$.

So far, we have described how the shared mobility market works to assign travelers to shared vehicles. Next, we explicitly define the "end" of our market in terms of monetary payments and net profits for both the travelers and the vehicles.

Definition 3.2.5. *At the end of travel, each traveler is asked to make a payment* $t_i(a_{ij})$ *for the service of vehicle* $j \in \mathcal{J}$ *(e.g., a share-mobility fare). The monetary net profit* $\rho_{ij}(a_{ij})$ *of vehicle* $j \in \mathcal{J}$ *from traveler* $i \in \mathcal{I}$ *is given by*

$$\rho_{ij}(a_{ij}) = t_i(a_{ij}) - c_{ij}(a_{ij}). \tag{3.34}$$

On the other hand, the monetary net profit of traveler $i \in \mathcal{I}$ *is*

$$\pi_{ij}(a_{ij}) = v_i(a_{ij}) - t_i(a_{ij}) - \underline{v}_i. \tag{3.35}$$

We call $(\pi_{ij}(a_{ij}), \rho_{ij}(a_{ij}))_{i \in \mathcal{I}, j \in \mathcal{J}}$ *the* traveler-vehicle profit allocation.

Remark 3.2.6. *Naturally, (3.34) gives the net profit of a vehicle* $j \in \mathcal{J}$ *generated by one traveler* $i \in \mathcal{I}$ *as the difference between the monetary payment* t_i *(e.g., fare) made by the traveler, and the traveler's share of the operating cost,* c_{ij}*. In a similar line of arguments, in (3.35) the net profit of traveler* $i \in \mathcal{I}$ *is the difference between what they are willing to pay,* v_i*, what they actually pay,* t_i*, and the minimum accepted value that they expect to get from vehicle* $j \in \mathcal{J}$.

Next, following a similar notion from [223], we define when the traveler-vehicle profit allocation $(\pi_{ij}(a_{ij}), \rho_{ij}(a_{ij}))$ for each traveler $i \in \mathcal{I}$ and each vehicle $j \in \mathcal{J}$ is feasible.

Definition 3.2.7. *Let $\widehat{\mathcal{J}} \subseteq \mathcal{J}$ denote the set of all vehicles that are actually assigned to travelers. We say $(\pi_{ij}(a_{ij}), \rho_{ij}(a_{ij}))_{i\in\mathcal{I}, j\in\mathcal{J}}$ is feasible if (i) for all vehicles $j \in \widehat{\mathcal{J}}$, both the traveler's and vehicle's net profit are nonnegative, i.e., $\pi_{ij}(a_{ij}), \rho_{ij}(a_{ij}) \geq 0$; (ii) the net profit of any traveler $i \in \mathcal{I}$ assigned to any vehicle $j \in \mathcal{J}$ and its net profit is equal to the total utility of traveler $i \in \mathcal{I}$ minus the operating cost of vehicle $j \in \mathcal{J}$, i.e.,*

$$\pi_{ij}(a_{ij}) + \rho_{ij}(a_{ij}) = u_i(a_{ij}) - c_{ij}(a_{ij}); \tag{3.36}$$

(iii) for all unassigned vehicles $j \in \mathcal{J} \setminus \widehat{\mathcal{J}}$, $\rho_{ij}(a_{ij}) = 0$; and (iv) for any traveler $i \in \mathcal{I}$ left unassigned, $\pi_{ij}(a_{ij}) = 0$.

Definition 3.2.8. *A feasible traveler-vehicle profit allocation $(\pi_{ij}(a_{ij}), \rho_{ij}(a_{ij}))_{i\in\mathcal{I}, j\in\mathcal{J}}$ is stable if for all $i \in \mathcal{I}$,*

$$u_i(a_{ij}) - c_{ij}(a_{ij}) \geq u_i(a'_{ij}) - c_{ij}(a'_{ij}), \tag{3.37}$$

for any assignment a'_{ij}.

In other words, Definition 3.2.8 implies that for any traveler i and any vehicle j that are not assigned together, if $u_i(a_{ij}) - c_{ij}(a_{ij}) < u_i(a'_{ij}) - c_{ij}(a'_{ij})$, then neither traveler i or vehicle j would be satisfied with that assignment. If we can eliminate those cases, then the traveler-vehicle profit allocation is socially-acceptable and no traveler, or vehicle, will seek to deviate.

In our modeling framework of a shared mobility market we impose the following assumptions.

Assumption 3.2.9. *All travelers participate in the market since sharing a vehicle is the only commute option.*

We impose Assumption 3.2.9 in our modeling framework since the focus is on identifying the best assignment between travelers and shared vehicles. By including alternative commute options, we would just add complexity in our analysis without any compelling reason. However, in future work, we plan to relax this assumption, and allow travelers to have multiple commute options using different modes of transportation to reach their destination in the network.

Assumption 3.2.10. *The travel satisfaction or costs of any traveler's utility is represented in monetary units. Also, we have* $u_0(a_{0j}) = c_j$ *for any vehicle* $j \in \mathcal{J}$*.*

Although Assumption 3.2.10 allows us to simplify the mathematical modeling, it is also natural in a realistic market of shared mobility to assume that all valuations and transactions between travelers and vehicles are done using money. Intuitively, $u_0(a_{0j}) = c_j$ for any vehicle $j \in \mathcal{J}$ ensures that for any assignment the vehicle's operating cost is covered.

Assumption 3.2.11. *The total operating cost of all vehicles* $\sum_{j \in \mathcal{J}} c_j$ *remains fixed.*

Assumption 3.2.11 implies that the operating cost of all vehicles cannot be altered in the long run while accommodating the travelers' desired origin-destination requests. In other words, the traveler-service assignments cannot really alter the total operating cost of all vehicles.

Problem 3.2.12. *The optimization problem formulation of the shared mobility market is*

$$\max_{a_{ij}} W(\mathbf{a}) = \max_{a_{ij}} \sum_{i \in \mathcal{I}} u_i(a_{ij}), \tag{3.38}$$

subject to:

$$\sum_{j \in \mathcal{J}} a_{ij} \leq 1, \quad \forall i \in \mathcal{I}, \tag{3.39}$$

$$\sum_{i \in \mathcal{I}} a_{ij} \leq \varepsilon_j, \quad \forall j \in \mathcal{J}, \tag{3.40}$$

where (3.39) *ensures that each traveler* $i \in \mathcal{I}$ *is assigned to only one vehicle* $j \in \mathcal{J}$*, and* (3.40) *ensures that the vehicle maximum capacity is not exceeded while the vehicle shared by travelers.*

Remark 3.2.13. *We note that the solution of Problem 2.2.13 will always assign a vehicle that can satisfy the origins and destinations of all the travelers that are assigned to it.*

3.2.2 Main Results

Theorem 3.2.14. *Let* $\mathbf{a}^*$ *denote an optimal assignment of Problem 2.2.13. Then, the objective function* (2.53) *evaluated at* $\mathbf{a}^*$ *is mathematically equivalent to the classic maximization of the social welfare at* $\mathbf{a}^*$ *with utility function defined as*

$$u_i(a_{ij}) = v_i(a_{ij}) - t_i(a_{ij}) - c_{ij}(a_{ij}), \tag{3.41}$$

where $v_i(a_{ij})$ *is the satisfaction of traveler* $i \in \mathcal{I}$*,* $t_i(a_{ij})$ *is the monetary payment made by traveler* $i \in \mathcal{I}$ *for using vehicle* $j \in \mathcal{J}$*, and* $c_{ij}(a_{ij})$ *is the operating cost of vehicle* $j \in \mathcal{J}$ *assigned to traveler* $i \in \mathcal{I}$*.*

Proof. By Assumption 3.2.10, we can write the objective function (2.53) of Problem 2.2.13 as follows

$$\max_{a_{ij}} \sum_{i \in \mathcal{I}} u_i(a_{ij}) = \max_{a_{ij}} \sum_{i \in \mathcal{I} \setminus \{0\}} u_i(a_{ij}) + \sum_{j \in \mathcal{J}} c_{0j}(a_{0j}), \tag{3.42}$$

where the term $\sum_{j \in \mathcal{J}} c_{0j}(a_{0j})$ represents the total operating cost of all the vehicles that are unassigned to travelers, and can be written as

$$\sum_{j \in \mathcal{J}} c_{0j}(a_{0j}) = \sum_{j \in \mathcal{J}} c_j - \sum_{j \in \widehat{\mathcal{J}}} c_j. \tag{3.43}$$

Substituting (3.43) into (3.42) yields

$$\max_{a_{ij}} \sum_{i \in \mathcal{I}} u_i(a_{ij}) = \max_{a_{ij}} \sum_{i \in \mathcal{I} \setminus \{0\}} u_i(a_{ij}) + \sum_{j \in \mathcal{J}} c_j - \sum_{j \in \widehat{\mathcal{J}}} c_j. \tag{3.44}$$

Since $\sum_{j\in\mathcal{J}} c_j$ is constant by Assumption 3.2.11, it can be neglected from the maximization problem. Hence, by optimality, we have $\sum_{j\in\widehat{\mathcal{J}}} c_j = \sum_{j\in\mathcal{J}} \sum_{i\in\mathcal{I}\setminus\{0\}} c_{ij}(a_{ij})$, and since the series is finite, (3.44) becomes

$$\max_{a_{ij}} \sum_{i\in\mathcal{I}\setminus\{0\}} u_i(a_{ij}) - \sum_{i\in\mathcal{I}\setminus\{0\}} \sum_{j\in\mathcal{J}} c_{ij}(a_{ij}) = \\ \max_{a_{ij}} \sum_{i\in\mathcal{I}\setminus\{0\}} \left(v_i(a_{ij}) - t_i(a_{ij}) - c_{ij}(a_{ij})\right), \quad (3.45)$$

where in the last equation we have used the fact that

$$\sum_{i\in\mathcal{I}\setminus\{0\}} u_i(a_{ij}) = \sum_{i\in\mathcal{I}\setminus\{0\}} \sum_{j\in\mathcal{J}} u_i(a_{ij}), \quad (3.46)$$

and the result immediately follows. □

Proposition 3.2.15. *Let the traveler-vehicle profit allocation* $(\pi_{ij}(a_{ij}), \rho_{ij}(a_{ij}))_{i\in\mathcal{I},j\in\mathcal{J}}$ *under the traveler-vehicle assignment* $\mathbf{a}$ *of Problem 2.2.13 form a space, denoted by* $\mathcal{S}$*. Then,* $\mathcal{S}$ *is convex.*

Proof. It is straightforward to see that the space of stable solutions $\mathcal{S}$ is defined by a set of linear constraints. Therefore, the space of stable solutions $\mathcal{S}$ is convex. □

Theorem 3.2.16 (Stability)**.** *If* $(\pi_{ij}(a_{ij}), \rho_{ij}(a_{ij}))_{i\in\mathcal{I},j\in\mathcal{J}}$ *is stable, then* $\mathbf{a}$ *is an optimal assignment of Problem 2.2.13.*

Proof. Let $\mathbf{a}$ and $\mathbf{a}'$ denote two different traveler-vehicle assignments of Problem 2.2.13. It is sufficient to consider the case where $(\pi_{ij}(a_{ij}), \rho_{ij}(a_{ij}))_{i\in\mathcal{I},j\in\mathcal{J}}$ is stable under $\mathbf{a}$ and only feasible under $\mathbf{a}'$. Then, we want to show that $\mathbf{a}'$ is not optimal. So, by Definition 3.2.8, we have

$$\begin{aligned} \pi_{ij}(a_{ij}) + \rho_{ij}(a_{ij}) &= u_i(a_{ij}) - c_{ij}(a_{ij}) \\ &\geq u_i(a'_{ij}) - c_{ij}(a'_{ij}). \end{aligned} \quad (3.47)$$

We take the summation over $i \in \mathcal{I} \setminus \{0\}$ and $j \in \widehat{\mathcal{J}}$ of (3.47) as follows

$$\sum_{i\in\mathcal{I}\setminus\{0\}} \sum_{j\in\widehat{\mathcal{J}}} \left(\pi_{ij}(a_{ij}) + \rho_{ij}(a_{ij})\right) \geq \sum_{i\in\mathcal{I}\setminus\{0\}} \sum_{j\in\widehat{\mathcal{J}}} \left(u_i(a'_{ij}) - c_{ij}(a'_{ij})\right). \quad (3.48)$$

So, the RHS of (3.48) becomes

$$\sum_{i\in\mathcal{I}\setminus\{0\}}\sum_{j\in\widehat{\mathcal{J}}}\left(u_i(a'_{ij})-c_{ij}(a'_{ij})\right)=\sum_{i\in\mathcal{I}\setminus\{0\}}\sum_{j\in\mathcal{J}}\left(u_i(a'_{ij})-(1-a'_{0j})\cdot c_{ij}(a'_{ij})\right). \tag{3.49}$$

By using conditions (ii) and (iii) from Definition 3.2.7, the LHS of (3.48) becomes

$$\sum_{i\in\mathcal{I}\setminus\{0\}}\sum_{j\in\widehat{\mathcal{J}}}\left(\pi_{ij}(a_{ij})+\rho_{ij}(a_{ij})\right)=\sum_{i\in\mathcal{I}\setminus\{0\}}\sum_{j\in\mathcal{J}}\left(u_i(a_{ij})-(1-a_{0j})\cdot c_{ij}(a_{ij})\right). \tag{3.50}$$

Thus, substituting (3.49) and (3.50) into (3.48) yields

$$\sum_{i\in\mathcal{I}\setminus\{0\}}\sum_{j\in\mathcal{J}}\left(u_i(a_{ij})-(1-a_{0j})\cdot c_{ij}(a_{ij})\right)\geq\sum_{i\in\mathcal{I}\setminus\{0\}}\sum_{j\in\mathcal{J}}\left(u_i(a'_{ij})-(1-a'_{0j})\cdot c_{ij}(a'_{ij})\right), \tag{3.51}$$

which simplifies to, for any assignment a'_{ij},

$$\sum_{i\in\mathcal{I}}u_i(a_{ij})\geq\sum_{i\in\mathcal{I}}u_i(a'_{ij}), \tag{3.52}$$

since the summation over the $j\in\mathcal{J}$ is redundant. Hence, the social welfare under assignment $\mathbf{a}$ is greater or equal than the social welfare under $\mathbf{a}'$. Therefore, we conclude that if $(\pi_{ij}(a_{ij}),\rho_{ij}(a_{ij}))_{i\in\mathcal{I},j\in\mathcal{J}}$ is stable, then the assignment $\mathbf{a}$ is necessarily optimal.

□

Theorem 3.2.17. *If there are two optimal assignments of Problem 2.2.13, denoted by* $\mathbf{a}$ *and* $\tilde{\mathbf{a}}$*, respectively, then the resulted traveler-vehicle profit allocation, denoted by* $(\pi_{ij}(a_{ij}),\rho_{ij}(a_{ij}))_{i\in\mathcal{I},j\in\mathcal{J}}$*, is feasible and stable under both assignments.*

Proof. Let $\mathbf{a}$ and $\tilde{\mathbf{a}}$ denote two optimal assignment. What we have to show is that if $(\pi_{ij}(a_{ij}),\rho_{ij}(a_{ij}))_{i\in\mathcal{I},j\in\mathcal{J}}$ is stable under assignment $\mathbf{a}$ and feasible under $\tilde{\mathbf{a}}$, then it is also stable under $\tilde{\mathbf{a}}$. We follow the same arguments up until (3.48) to get

$$\sum_{i\in\mathcal{I}\setminus\{0\}}\sum_{j\in\widehat{\mathcal{J}}}\left(\pi_{ij}(a_{ij})+\rho_{ij}(a_{ij})\right)\geq\sum_{i\in\mathcal{I}\setminus\{0\}}\sum_{j\in\widehat{\mathcal{J}}}\left(u_i(a'_{ij})-c_{ij}(a'_{ij})\right). \tag{3.53}$$

Hence, we observe that if $\tilde{\mathbf{a}}$ is an optimal assignment, then by Definition 3.2.8, (3.53) will hold at equality. Thus, the feasibility equation (3.36) is satisfied. Therefore, under the optimal assignment $\tilde{\mathbf{a}}$, we conclude that $(\pi_{ij}(a_{ij}),\rho_{ij}(a_{ij}))_{i\in\mathcal{I},j\in\mathcal{J}}$ is stable. □

Proposition 3.2.18. *If there are two arbitrary travelers with the same needs that are assigned to different vehicles, then there is no difference in their utility.*

Proof. Suppose there are two travelers $i, i' \in \mathcal{I}$ with the same needs and two vehicles $j, j' \in \mathcal{J}$. We want to show that in our market both travelers will receive the same utility even under different assignments. So, we assume that there are two assignments $\mathbf{a}$ and $\mathbf{a}'$, where in $\mathbf{a}$ traveler $i \in \mathcal{I}$ is assigned to vehicle $j \in \mathcal{J}$ while in $\mathbf{a}'$ traveler $i \in \mathcal{I}$ is assigned to vehicle j'. Similarly, for traveler i'. For an optimal $\mathbf{a}$, the stability conditions of $(\pi_{ij}(a_{ij}), \rho_{ij}(a_{ij}))_{i\in\mathcal{I}, j\in\mathcal{J}}$ are

$$\pi_{ij}(a_{ij}) + \sum_{\ell\in\mathcal{I}\setminus\{i\}} (\pi_{\ell j}(a_{\ell j}) + \rho_{ij}(a_{ij})) = u_i(a_{ij}) + \sum_{\ell\in\mathcal{I}\setminus\{i\}} (u_i(a_{\ell j}) - c_{\ell j}(a_{\ell j})) , \quad (3.54)$$

we have then

$$\pi_{ij}(a_{ij}) + \sum_{\ell\in\mathcal{I}\setminus\{i'\}} (\pi_{\ell j}(a_{\ell j}) + \rho_{ij'}(a_{ij'})) \geq u_i(a_{ij'}) + \sum_{\ell\in\mathcal{I}\setminus\{i'\}} (u_i(a_{\ell j'}) - c_{\ell j'}(a_{\ell j'})) . \quad (3.55)$$

Similarly, for traveler i', we have the following:

$$\pi_{i'j}(a_{i'j}) + \sum_{\ell\in\mathcal{I}\setminus\{i'\}} (\pi_{\ell j}(a_{\ell j}) + \rho_{ij'}(a_{ij'})) \geq u_i(a_{i'j'}) + \sum_{\ell\in\mathcal{I}\setminus\{i'\}} (u_i(a_{\ell j'}) - c_{\ell j'}(a_{\ell j'})) , \quad (3.56)$$

then, we have

$$\pi_{i'j}(a_{i'j}) + \sum_{\ell\in\mathcal{I}\setminus\{i\}} (\pi_{\ell j}(a_{\ell j}) + \rho_{ij}(a_{ij})) = u_i(a_{i'j}) + \sum_{\ell\in\mathcal{I}\setminus\{i\}} (u_i(a_{\ell j}) - c_{\ell j}(a_{\ell j})) . \quad (3.57)$$

In a similar way, we can argue that since $\mathbf{a}'$ is optimal, the stability conditions of $(\pi_{ij}(a_{ij}), \rho_{ij}(a_{ij}))_{i\in\mathcal{I}, j\in\mathcal{J}}$ are

$$\pi_{ij}(a_{ij}) + \sum_{\ell\in\mathcal{I}\setminus\{i\}} (\pi_{\ell j}(a_{\ell j}) + \rho_{ij'}(a_{ij'})) = u_i(a_{ij'}) + \sum_{\ell\in\mathcal{I}\setminus\{i\}} (u_i(a_{\ell j'}) - c_{\ell j'}(a_{\ell j'})) , \quad (3.58)$$

which yields

$$\pi_{ij}(a_{ij}) + \sum_{\ell\in\mathcal{I}\setminus\{i'\}} (\pi_{\ell j}(a_{\ell j}) + \rho_{ij}(a_{ij})) \geq u_i(a_{ij}) + \sum_{\ell\in\mathcal{I}\setminus\{i'\}} (u_i(a_{\ell j}) - c_{\ell j}(a_{\ell j})) . \quad (3.59)$$

Similarly, for traveler i', we have the following:

$$\pi_{i'j}(a_{i'j}) + \sum_{\ell \in \mathcal{I} \setminus \{i'\}} (\pi_{\ell j}(a_{\ell j}) + \rho_{ij}(a_{ij})) = u_i(a_{i'j}) + \sum_{\ell \in \mathcal{I} \setminus \{i'\}} (u_i(a_{\ell j}) - c_{\ell j}(a_{\ell j})) \,, \quad (3.60)$$

which gives us

$$\pi_{i'j}(a_{i'j}) + \sum_{\ell \in \mathcal{I} \setminus \{i\}} (\pi_{\ell j}(a_{\ell j}) + \rho_{ij'}(a_{ij'})) \geq u_i(a_{i'j'}) + \sum_{\ell \in \mathcal{I} \setminus \{i\}} (u_i(a_{\ell j'}) - c_{\ell j'}(a_{\ell j'})) \,. \quad (3.61)$$

Recall that both travelers $i, i' \in \mathcal{I}$ have the same needs. Thus, $u_i(a_{ij}) = u_{i'}(a_{i'j})$ and $u_i(a_{ij'}) = u_{i'}(a_{i'j'})$. Therefore, from (3.54) - (3.61), it follows that $\pi_{ij}(a_{ij}) = \pi_{i'j}(a_{i'j})$. □

3.3 An Analytical Study of a Two-Sided Mobility Game

In this chapter, we consider a mobility system of travelers and providers, and propose a "mobility game" to study when a traveler is matched to a provider. Each traveler seeks to travel using the services of only one provider, who manages one specific mode of transportation (e.g., car, bus, train, bike). The services of each provider are capacitated and can serve up to a fixed number of travelers at any instant of time. Thus, our problem falls under the category of many-to-one assignment problems, where the goal is to find the conditions that guarantee the stability of assignments. We formulate a linear program of maximizing the social welfare of travelers and providers and show how it is equivalent to the original problem and relate its solutions to stable assignments. We also investigate our results under informational asymmetry and provide a "mechanism" that elicits the information of travelers and providers. Finally, we investigate and validate the advantages of our method by providing a numerical simulation example.

3.3.1 Modeling Framework

We consider a mobility system of two finite, disjoint, and non-empty groups of agents of which one represents the travelers and the other the providers. We denote the set of travelers by $\mathcal{I}$, $|\mathcal{I}| = I \in \mathbb{N}$ and the set of providers by $\mathcal{J}$, $|\mathcal{J}| = J \in \mathbb{N}$. In a typical mobility system, we expect to have more travelers than available providers, so $I \gg J$. Each provider represents a company (e.g., Uber, Lyft, Amtrak, DART, Lime) that manages a fleet of vehicles (cars, trains, busses, bicycles). We focus our study in static settings, in this chapter, thus, each provider can serve up to a fixed number of travelers within a fixed time period. For example, in a generic city neighborhood, on any given weekday morning, there are at most a certain number of ride-sharing vehicles available (Uber/Lyft). Formally, for each provider $j \in \mathcal{J}$, we impose a physical *traveler capacity*, denoted by $\varepsilon_j \in \mathbb{N}$. Naturally, each provider can serve a different number of travelers, so we expect ε_j to vary significantly. For example, a train company can provide travel services per hour to hundreds of travelers compared to a bikeshare company in a city. Next, travelers seek to travel using the services of at most one provider. We do not focus our modeling in routing or path-allocation (such problems have been studied extensively [124, 34]), rather we are interested in an optimal collective assignment of travelers to providers. Both travelers and providers have preferences and can be characterized by their type; thus, this is a two-sided mobility game.

Remark 3.3.1. *Without loss of generality, we expect the aggregate travel demands of all the travelers to be exactly met by the aggregate capacities of all the providers' mobility services. Thus, we have* $\sum_{j \in \mathcal{J}} \varepsilon_j = |\mathcal{I}| = I$.

Remark 3.3.2. *Intuitively, via a smartphone app, travelers book in advance for their travel needs and report their preferences and request a travel recommendation (which provider to use). The app collects all requests from specific neighborhoods at a fixed time, and then assigns each traveler to a provider by taking into account both the traveler's as well as the provider's preferences.*

Definition 3.3.3. *The* traveler-provider assignment *is a vector* $\mathbf{X} = (x_{11}, \ldots, x_{ij}, \ldots, x_{IJ}) = (x_{ij})_{i \in \mathcal{I}, j \in \mathcal{J}}$*, where* x_{ij} *is a binary variable of the form:*

$$x_{ij} = \begin{cases} 1, & \textit{if } i \in \mathcal{I} \textit{ is assigned to } j \in \mathcal{J}, \\ 0, & \textit{otherwise.} \end{cases} \tag{3.62}$$

We call x_{ij} *the* mobility outcome *of each traveler* i *and each provider* j *and denote by* $\mathcal{X}$ *the set of such outcomes.*

Definition 3.3.4. *For any traveler* $i \in \mathcal{I}$, $\theta_i = \max_{j\in\mathcal{J}} \{\theta_{ij}\} \in \Theta_i$, *where* $\theta_{ij} \in [0, 1]$, *is traveler* i*'s* personal predisposition *of provider* $j \in \mathcal{J}$.

We denote by θ_{-i} for the personal predisposition profile of all travelers except traveler i. Intuitively, a traveler might have a great affinity towards a taxicab service and a lower affinity towards a bus service. So, we expect different travelers to have different preferences on the mode of transportation to use.

Next, we represent the preferences of each traveler with a utility function consisted of two parts: the traveler's valuation of the mobility outcome and the associated payment required for the realization of the outcome. In other words, any traveler is expected to pay a toll or ticket fee for the services of a provider.

Definition 3.3.5. *Each traveler* i*'s preferences are summarized by a utility function* $u_i : \mathcal{X} \times \Theta_i \to \mathbb{R}$ *that determines the monetary value of the overall payoff realized by traveler* i *from their assignment to provider* j*. Let* $t_{ij} \in [\underline{t}, \bar{t}] \subset \mathbb{R}$ *denote traveler* i*'s mobility payment. Thus, traveler* i *receives a total utility in the form*

$$u_i(x_{ij}, \theta_i) = v_i(x_{ij}, \theta_i) - t_{ij}, \tag{3.63}$$

where $v_i : \mathcal{X} \times \Theta_i \to \mathbb{R}_{\geq 0}$ *is a linear valuation function that represents the maximum amount of money that traveler* i *is willing to pay for the mobility outcome* x_{ij}.

Remark 3.3.6. *If for any traveler* $i \in \mathcal{I}$, *we have* $x_{ij} = 0$ *for all* $j \in \mathcal{J}$, *then* $u_i = 0$. *Naturally, this means that for any traveler* i *with* $x_{ij} = 0$ *for all* $j \in \mathcal{J}$ *we have* $t_{ij} = 0$.

On similar lines, we define the providers' utility function.

Definition 3.3.7. *A provider* j*'s utility is given by*

$$u_j(x_{ij}, \delta_j) = t_{ij} - c_j(x_{ij}, \delta_j), \tag{3.64}$$

where $\delta_j \in (0, 1]$ represents the type of provider j, and c_j is a linear cost function related to the operation of the mobility services provided by $j \in \mathcal{J}$. We denote by δ_{-j} for the type profile of all providers except provider j.

Remark 3.3.8. *Intuitively, δ_j can be interpreted as the "operational value" of provider j for the mobility services it provides and operates. In other words, the monetary value of the entire process of its service to serve a traveler on a given location and time.*

In both (3.63) and (3.64), the "payment component" t_{ij} is not expected to dominate either the traveler's or the provider's utility function. This is because t_{ij} have an alternate sign in (3.63) and (3.64), so a high value (or low) can lead to negative utility for the travelers (or the providers) leading to a unfavorable match between traveler i and provider j. We will see later in the chapter how we can ensure unfavorable matchings do not happen.

Definition 3.3.9. *Under the assignment x_{ij} of traveler i and provider j, their* mobility (i, j)-matching payoff *is given by*

$$a_{ij}(x_{ij}) = u_i(x_{ij}, \theta_i) + u_j(x_{ij}, \delta_j), \tag{3.65}$$

where a_{ij} measures the combined payoff or benefit measured in monetary units of traveler i being assigned to provider j.

Remark 3.3.10. *By Remark 3.3.6, if $x_{ij} = 0$, then $a_{ij} = 0$.*

Definition 3.3.11. *The* utility assignment matrix **A** *is constructed with $|\mathcal{I}|$ rows and $|\mathcal{J}|$ columns and each entry represents the (i, j)-matching utility a_{ij} between traveler i and provider j for all $i \in \mathcal{I}$ and all $j \in \mathcal{J}$.*

Based on Definition 3.3.11, we can construct matrix **A** as follows:

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1J} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2J} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{I1} & a_{I2} & a_{I3} & \dots & a_{IJ} \end{bmatrix}. \tag{3.66}$$

The mobility game of travelers and providers is a collection of four objects, namely set of agents, vector of assignments, matrix of utilities, and a vector of capacities. Formally, we state the next definition.

Definition 3.3.12. *The mobility game can be fully characterized by the tuple* $\mathcal{M} = \langle \mathcal{I} \cup \mathcal{J}, \mathbf{X} = (x_{ij})_{i\in\mathcal{I}, j\in\mathcal{J}}, \mathbf{A}, (\varepsilon_j)_{j\in\mathcal{J}} \rangle$.

Definition 3.3.13. *A* feasible *assignment is a vector* $\mathbf{X} = (x_{ij})_{i\in\mathcal{I}, j\in\mathcal{J}}$, $x_{ij} \in \{0, 1\}$ *that satisfies constraints*

$$\sum_{j\in\mathcal{J}} x_{ij} \leq 1, \quad \forall i \in \mathcal{I}, \tag{3.67}$$

$$\sum_{i\in\mathcal{I}} x_{ij} \leq \varepsilon_j, \quad \forall j \in \mathcal{J}, \tag{3.68}$$

where (3.67) *ensures that each traveler* $i \in \mathcal{I}$ *is assigned to only one mobility service* $j \in \mathcal{J}$, *and* (3.68) *ensures that the traveler capacity of each provider* j *is not exceeded while its services are shared by multiple travelers. An* optimal *assignment is a feasible assignment* $(x_{ij})_{i\in\mathcal{I}, j\in\mathcal{J}}$ *such that*

$$\sum_{i\in\mathcal{I}} \sum_{j\in\mathcal{J}} a_{ij}(x_{ij}) \geq \sum_{i\in\mathcal{I}} \sum_{j\in\mathcal{J}} a_{ij}(x'_{ij}), \tag{3.69}$$

for all feasible assignments x'_{ij}.

Definition 3.3.14. *A feasible assignment* $\mathbf{X} = (x_{ij})_{i\in\mathcal{I}, j\in\mathcal{J}}$, $x_{ij} \in \{0, 1\}$ *is* stable *if there exist non-negative vectors* $\phi = (\phi_i)_{i\in\mathcal{I}}$ *and* $\psi = (\psi_j)_{j\in\mathcal{J}}$ *such that*

$$\sum_{i\in\mathcal{I}} \phi_i + \sum_{j\in\mathcal{J}} \varepsilon_j \cdot \psi_j = \sum_{i\in\mathcal{I}} \sum_{j\in\mathcal{J}} a_{ij}(x_{ij}) \tag{3.70}$$

with $\phi_i + \psi_j \geq a_{ij}$ *for all* $i \in \mathcal{I}$ *and all* $j \in \mathcal{J}$.

We will see later in Section 4.1.1 the mathematical and physical interpretation of ϕ and ψ.

Definition 3.3.15. *Let* $(t^*_{ij})_{i\in\mathcal{I}, j\in\mathcal{J}}$ *denote the mobility payments associated with the stable assignment, denoted by* $(x^*_{ij})_{i\in\mathcal{I}, j\in\mathcal{J}}$. *Then the equilibrium* $(x^*_{ij}, t^*_{ij})_{i\in\mathcal{I}, j\in\mathcal{J}}$ *is called an* ideal-mobility *equilibrium.*

From Definition 3.3.14, it is easy to see that Definition 3.3.15 implies that an ideal-mobility equilibrium in mobility game $\mathcal{M}$ ensures that (i) providers are assigned to travelers up to their capacity (thus, maximizing revenue), and (ii) travelers receive the best-possible utility being assigned to a provider (thus, maximizing welfare).

Assumption 3.3.16. *Every aspect of the mobility game $\mathcal{M}$ is considered known information to every traveler and provider.*

Assumption 3.3.16 seems strong but it will prove instrumental in Section 4.1.1. Our analysis will focus on how to show existence, optimality, and stability of traveler-provider assignments and then in Subsection 3.3.2.2, we will relax Assumption 3.3.16 and show how we can elicit the private information of both travelers and providers.

3.3.1.1 The Optimization Problem

In the mobility game $\mathcal{M}$, we are interested to know what are its stable assignments (alternatively called stable equilibria), whether they exist and under what conditions.

Problem 3.3.17. *The maximization problem of $\mathcal{M}$ is*

$$\max_{x_{ij}} \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} a_{ij}(x_{ij}), \tag{3.71}$$

$$\textit{subject to: } (3.67), (3.68),$$

where $x_{ij} \in \{0, 1\}$ for all $i \in \mathcal{I}$ and all $j \in \mathcal{J}$.

We can relax the binary variable constraint to a non-negativity constraint variable in Problem 3.3.17. We will show in the next section that this does not affect the optimal solutions of Problem 3.3.17 as we can ensure all optimal solutions of the equivalent linear program are binary valued.

Problem 3.3.18 (Linear Program)**.** *The linear program formulation of mobility game* $\mathcal{M}$ *is*

$$\max_{x_{ij}} \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} a_{ij}(x_{ij}) \tag{3.72}$$

$$\text{subject to: } (3.67), (3.68), \text{ and}$$

$$x_{ij} \geq 0, \quad \forall i \in \mathcal{I}, \quad \forall j \in \mathcal{J}, \tag{3.73}$$

where (3.73) *transforms the (binary) assignment problem to a (continuous) linear program of which* x_{ij} *can be interpreted as the probability that traveler* i *is matched to provider* j*.*

3.3.2 Analysis and Properties of the Mobility Game

3.3.2.1 Existence, Optimality, and Stability of Assignments

In this subsection, we show that for Problems 3.3.17 and 3.3.18 at least one optimal solution exists (thus, ensuring stability).

Theorem 3.3.19. *The stable assignments of mobility game* $\mathcal{M}$ *are the same with the optimal solutions of Problem 3.3.17. Furthermore, the set of optimal solutions of Problem 3.3.17 is non-empty.*

Proof. By relaxing the binary constraint of Problem 3.3.17, we get a linear program (Problem 3.3.18). Its set of all real-valued solutions is a polytope whose vertices have all integer-valued coordinates. Since the solutions are also guaranteed to be non-negative, the set of solutions is non-empty [209]. Thus, Problem 3.3.18 has at least one solution with integer components (in our case 0-1 components). Hence, the set of all optimal solution of Problem 3.3.17 is non-empty [209]. By Definition 3.3.13, assignments are stable as long as no agent in $\mathcal{I} \cup \mathcal{J}$ has an incentive (e.g., higher utility) to break their matching pair. So, finding a stable assignment is equivalent to finding the best in terms of aggregate utility among all possible feasible assignments. Mathematically, (3.69) naturally leads to a maximization problem. Therefore, the existence of a stable assignment of mobility game $\mathcal{M}$ is guaranteed. □

Next, we derive the dual of Problem 3.3.18.

Problem 3.3.20. *The dual of Problem 3.3.18 is given below:*

$$\min_{\phi,\psi} \sum_{i\in\mathcal{I}} \phi_i + \sum_{j\in\mathcal{J}} \varepsilon_j \cdot \psi_j, \tag{3.74}$$

subject to:

$$\phi_i + \psi_j \geq a_{ij}, \quad \forall i \in \mathcal{I}, \quad \forall j \in \mathcal{J}, \tag{3.75}$$

$$\phi_i \geq 0, \quad \forall i \in \mathcal{I}, \tag{3.76}$$

$$\psi_i \geq 0, \quad \forall j \in \mathcal{J}, \tag{3.77}$$

where ϕ is a $|\mathcal{I}|$-dimensional vector and ψ is a $|\mathcal{J}|$-dimensional vector.

Our objective is to establish a method for the mobility game $\mathcal{M}$'s stable assignments by solving Problem 3.3.18. In turn, we want to solve Problem 3.3.20 to find the stable assignments. This is possible only if we can guarantee strong duality (satisfying the conditions of complementary slackness). Formally, a feasible assignment x_{ij} and a feasible solution (ϕ, ψ) are optimal if and only if

$$\sum_{i\in\mathcal{I}} \sum_{j\in\mathcal{J}} a_{ij}(x_{ij}) = \sum_{i\in\mathcal{I}} \phi_i + \sum_{j\in\mathcal{J}} \varepsilon_j \cdot \psi_j. \tag{3.78}$$

The conditions that guarantee optimality are given by the theorem of complementary slackness, i.e.,

$$\phi_i + \psi_j - a_{ij} = 0, \quad \forall i \in \mathcal{I}, \quad \forall j \in \mathcal{J}, \tag{3.79}$$

$$\sum_{j\in\mathcal{J}} (x_{ij} - 1) \cdot \phi_i = 0, \quad \forall i \in \mathcal{I}, \tag{3.80}$$

$$\sum_{i\in\mathcal{I}} (x_{ij} - \varepsilon_j) \cdot \psi_j = 0, \quad \forall j \in \mathcal{J}. \tag{3.81}$$

Lemma 3.3.21. *The set of solutions of Problem 3.3.20 is non-empty and convex.*

Proof. We have already established that Problem 3.3.18 has at least one solution. Thus, it follows easily that Problem 3.3.20 has at least one solution too. Any solution of Problem 3.3.20 has a specific structure due to the geometry of the constraint set (3.75) - (3.77). Since at least one solution will be in 0-1 components, the constraints will force this solution to be in at a corner of a polyhedra. Thus, the set of solutions of Problem 3.3.20 is non-empty and has to be convex. □

Remark 3.3.22. *Intuitively, a dual solution (ϕ, ψ) can be seen as a method to share the "gains of mobility" among travelers and providers at an ideal-mobility equilibrium (see Definition 3.3.15). For example, component of vector ϕ describes the realized gain of traveler i when assigned to provider j (thus enjoying the mobility services of provider j). A component of vector ψ describes the per unit gain of provider j.*

Corollary 3.3.23. *The set of solutions of Problem 3.3.20 is a compact subset of $\mathbb{R}^{|\mathcal{I}|} \times \mathbb{R}^{|\mathcal{J}|}$.*

Proof. By Lemma 3.3.21 and Remark 3.3.22, it is straightforward to show that the set of solutions of Problem 3.3.20 is compact. □

Corollary 3.3.24. *There always exists at least one profile of mobility payments $(t_{ij})_{i\in\mathcal{I},j\in\mathcal{J}}$ under assignment $(x_{ij})_{i\in\mathcal{I},j\in\mathcal{J}}$.*

Proof. By definition of the mobility game $\mathcal{M}$ for any (feasible) assignment $(x_{ij})_{i\in\mathcal{I},j\in\mathcal{J}}$, there must be an associated profile of mobility payments $(t_{ij})_{i\in\mathcal{I},j\in\mathcal{J}}$. □

Next, we show that the existence of an optimal profile of mobility payments $(t_{ij})_{i\in\mathcal{I},j\in\mathcal{J}}$ can be guaranteed by the formulation of the dual program of Problem 3.3.18 and the computation of its solutions.

Theorem 3.3.25. *There exists an optimal profile of mobility payments $(t^*_{ij})_{i\in\mathcal{I},j\in\mathcal{J}}$ under stable assignment $(x^*_{ij})_{i\in\mathcal{I},j\in\mathcal{J}}$. Furthermore, we must have $\phi_i = u_i$ and $\psi_j = u_j$ for all $i \in \mathcal{I}$ and all $j \in \mathcal{J}$.*

Proof. Suppose $\mathbf{x}^* = (x^*_{ij})_{i\in\mathcal{I},j\in\mathcal{J}}$ is a stable assignment for mobility game $\mathcal{M}$. Under $\mathbf{x}^*$, we can calculate $\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}} a_{ij}(x_{ij})$, which by the theorem of strong duality and the definition of stability, (3.78) holds true. This is because (3.79) - (3.81) are equivalent to the conditions that ensure stability. Thus, there exist vectors (ϕ^*, ψ^*) from Problem 3.3.20 that feasible and optimal. By Definition 3.3.15, it follows that the mobility payments associated with the optimal assignments of travelers and providers are essentially the same with the optimal solutions of Problems 3.3.18 and 3.3.20. Therefore, by the established existence of solutions to Problems 3.3.18 and 3.3.20 as long as there exists a stable assignment $(x^*_{ij})_{i\in\mathcal{I},j\in\mathcal{J}}$ an optimal profile of mobility payment $(t^*_{ij})_{i\in\mathcal{I},j\in\mathcal{J}}$ must exist. By Remark 3.3.22 and Definition 3.3.14 at an optimal assignment we have $\phi_i = u_i$ and $\psi_j = u_j$ for all $i \in \mathcal{I}$ and all $j \in \mathcal{J}$. □

3.3.2.2 Asymmetric Information in the Mobility Game

So far, we have implicitly assumed that both the travelers and providers have complete information of the entire information structure of the mobility game $\mathcal{M}$. In other words, each traveler knows every other travelers' and providers' information, i.e., travelers know each others' utilities and valuations, providers know each other providers' types and cost functions. In a realistic setting, this implicit assumption is unreasonably restrictive. Thus, for the rest of the chapter, we focus on a "mechanism" that *induces the mobility game* $\mathcal{M}$ by eliciting the private information of all the travelers and providers. First, we relax Assumption 3.3.16 and consider that the types of travelers, i.e., $\theta = (\theta_i)$ and of providers, i.e., $\delta = (\delta_j)$ are private information, i.e., known only to themselves. Next, we denote by $\mathbf{X}_{-i}$ the assignment of travelers in $\mathcal{I} \setminus \{i\}$ to providers in $\mathcal{J}$. Similarly, we denote by $\mathbf{X}_{-j}$ the assignment of travelers in $\mathcal{I}$ to providers in $\mathcal{J} \setminus \{j\}$. Furthermore, we assume that travelers are charged by the mechanism, say $t_i \in \mathbb{R}$, and providers are compensated by the mechanism, say $t_j \in \mathbb{R}$. The proposed mechanism ensures to collect all funds from the travelers and compensate accordingly the providers.

Algorithm 2: Pricing Mechanism

Data: $\mathcal{I}, \mathcal{J}, (\theta_i)_{i\in\mathcal{I}}, (\delta_j)_{j\in\mathcal{J}}$

Result: $\mathbf{x}^*$ and $(t_{ij})_{i\in\mathcal{I},j\in\mathcal{J}}$

Define the valuation functions of every traveler and provider and use them to construct matrix $\mathbf{A}$. Solve for the optimal solution $\mathbf{x}^*$ of Problem 3.3.17;

for $i \in \mathcal{I}$ **do**

Solve for the optimal solution $\mathbf{X}^*_{-i}$ of Problem 3.3.17;

Set the mobility payment for each traveler i:

$$t_i = \sum_{\ell\in\mathcal{I}\setminus\{i\}} \sum_{j\in\mathcal{J}} u_\ell(x_{ij}, \theta_{-i}) - \sum_{\ell\in\mathcal{I}\setminus\{i\}} \sum_{j\in\mathcal{J}} u_\ell(x_{ij}, \theta_\ell)$$

end

for $j \in \mathcal{J}$ **do**

Solve for the optimal solution $\mathbf{x}^*_{-j}$ of Problem 3.3.17;

Set the mobility payment for each provider j:

$$t_j = \sum_{i\in\mathcal{I}} \sum_{\kappa\in\mathcal{J}\setminus\{j\}} u_\kappa(x_{ij}, \delta_{-j}) - \sum_{i\in\mathcal{I}} \sum_{\kappa\in\mathcal{J}\setminus\{j\}} u_\kappa(x_{ij}, \delta_\kappa)$$

end

Theorem 3.3.26 (Voluntary Participation)**.** *No traveler $i \in \mathcal{I}$ and no provider $j \in \mathcal{J}$ can gain for better individual utility by matching externally compared to the utility gained by participating in the induced mobility game $\mathcal{M}$.*

Proof. It is sufficient to show that no agent in $\mathcal{I} \cup \mathcal{J}$ can gain negative utility by participating in the induced game $\mathcal{M}$, i.e., we must have $u_i, u_j \geq 0$ for all $i, j \in \mathcal{I} \cup \mathcal{J}$. First, note that the maximization of $\sum_{i\in\mathcal{I}} \sum_{j\in\mathcal{J}} a_{ij}(x_{ij})$ is the highest possible value we can achieve. Removing even one agent, does not increase this value under any scenario. Thus, by definition, both payments t_i and t_j are non-negative. At equilibrium, the utilities of any traveler and provider are equivalent to the solutions (ϕ, ψ) of Problem 3.3.20 (as we showed in Theorem 3.3.25). Since (ϕ, ψ) ensures non-negativity it follows that $u_i, u_j \geq 0$ for all $i, j \in \mathcal{I} \cup \mathcal{J}$. □

Theorem 3.3.27 (Truthfulness)**.** *Misreporting does not benefit any traveler or provider.*

Proof. Let us assume that all agents in $\mathcal{I} \cup \mathcal{J}$ voluntary participate in the mechanism. We consider two cases when traveler i misreports their true type, i.e., $\hat{\theta}_i \geq \theta_i$ and $\hat{\theta}_i \leq \theta_i$, where $\hat{\theta}_i$ is traveler i's report. If traveler i reports $\hat{\theta}_i$ that is lower than their true type, then traveler i cannot improve their utility as t_i is necessarily non-negative and a lower misreporting $\hat{\theta}_i \leq \theta_i$ can only lead to lower utilities. Suppose now that traveler i reports $\hat{\theta}_i$ that is higher than their true type. The exact value of $\hat{\theta}_i$ can be chosen by the following maximization problem

$$\max_{\hat{\theta}_i} u_i(x_{ij}, \hat{\theta}_i) = \max_{\hat{\theta}_i} v_i(x_{ij}, \hat{\theta}_i) - t_i, \tag{3.82}$$

where t_i is given by Algorithm 2. Note though that the maximization of $v_i(x_{ij}, \hat{\theta}_i) - \sum_{\ell \in \mathcal{I} \setminus \{i\}} \sum_{j \in \mathcal{J}} u_\ell(x_{ij}, \hat{\theta}_{-i}) - \sum_{\ell \in \mathcal{I} \setminus \{i\}} \sum_{j \in \mathcal{J}} u_\ell(x_{ij}, \hat{\theta}_\ell)$ is equivalent to the maximization of $\sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} v_i(x_{ij}, \hat{\theta}_i)$ with respect to the assignment x_{ij}. After all that is the goal of traveler i, namely by misreporting their type to lead the mechanism to a better assignment. Let $\hat{x}^*_{ij}$ and x^*_{ij} denote the optimal assignment with $\hat{\theta}_i$ and θ_i, respectively. Hence, we have

$$\arg\max_{\hat{x}^*_{ij}} \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} v_i(\hat{x}^*_{ij}, \hat{\theta}_i) \leq \arg\max_{x^*_{ij}} \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} v_i(x^*_{ij}, \theta_i) \tag{3.83}$$

for each traveler i. It follows immediately from (3.83) that traveler i can maximize their utility by minimizing the mobility payments as defined in Algorithm 2. This is only possible when $\hat{\theta}_i = \theta_i$. Thus, traveler i cannot improve their utility by misreporting. Therefore, we conclude that, with the proposed pricing mechanism (Algorithm 2), under no circumstance can traveler i improve their utility by misreporting about their type θ_i. In other words, any traveler i has a strategy to always truthfully report their type to the mechanism. We can follow the same arguments to show this for the providers. Therefore, the proof is completed. □

Proposition 3.3.28. *If traveler i is matched to provider j while having misreported their type to the mechanism, then traveler i does not gain a better utility compared to the utility gained under the true type.*

Proof. We show this only for the travelers as the arguments are similar for the providers. By construction of the mobility payments in Algorithm 2 non-negativity of the payments for each traveler is guaranteed, i.e., $t_i \geq 0$. By definition, the valuation of each traveler is non-negative under any assignment. Thus, the utility defined in (3.63) is also non-negative. At equilibrium, we have

$$\max_{x_{ij}} \sum_{i \in \mathcal{I}} v_i(x_{ij}, \theta_i) \leq \sum_{i \in \mathcal{I}} v_i(x_{ij}^*). \tag{3.84}$$

Thus, it follows that

$$\begin{aligned} u_i(x_{ij}, \theta_i) &= v_i(x_{ij}, \theta_i) - t_i \\ &= \sum_{i \in \mathcal{I}} v_i(x_{ij}^*) - \sum_{\ell \in \mathcal{I} \setminus \{i\}} v_\ell(x_{ij}, \theta_\ell) \geq 0. \end{aligned} \tag{3.85}$$

Therefore, no traveler can hope for better utility by misreporting. □

Proposition 3.3.29 (Social Efficiency)**.** *The proposed mechanism satisfies social efficiency as it ensures the maximization of the aggregate social welfare of both travelers and providers.*

Proof. By construction of the mobility payments in Algorithm 2, it follows immediately that the optimal solution maximizes the social welfare. □

3.3.3 Simulation Results

In this section, we present a numerical example, its solution and discuss its physical interpretation. Consider a mobility system of four providers $\mathcal{J} = \{\text{bike}, \text{car}, \text{bus}, \text{train}\}$ and twenty travelers $\mathcal{I} = \{1, 2, \ldots, 20\}$. Each provider has traveler capacities, namely we have $\varepsilon_{bike} = 1$, $\varepsilon_{car} = 4$, $\varepsilon_{bus} = 5$, and $\varepsilon_{train} = 10$. Moreover, we partition the set of travelers $\mathcal{I}$ into four types, i.e., students, commuters, tourists, consumers with sizes $|\mathcal{I}_{\text{students}}| = 3$, $|\mathcal{I}_{\text{commuters}}| = 5$, $|\mathcal{I}_{\text{tourists}}| = 4$, $|\mathcal{I}_{\text{consumers}}| = 8$. Each type of travelers can represent the personal predisposition $\theta = (\theta_i)_{i\in\mathcal{I}}$. Next, with a slight abuse of notation, we generate a random utility assignment matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 0.5 \\ 2.5 & 2 & 1.5 \\ 2.5 & 4 & 1.5 \\ 2.5 & 5 & 6.5 \end{bmatrix}, \tag{3.86}$$

where each row represents a type of travelers and each column represents a provider. The entry a_{ij} of $\mathbf{A}$ represents the overall utility of assignment x_{ij}.

We solve Problem 3.3.18 and compute with an optimal solution that maximizes the aggregate utilities of each traveler and each provider according to the travelers' preferences and maximizing the capacities of each provider. The computational complexity of the proposed method is relatively low as long as the number of travelers and providers remain small. This is reasonable to expect as at any given moment there can only less than five different travel options (so, the number of providers is always small). By ensuring we partition the travelers' requests according to origin, destination, and type, we can make certain that the number of travelers does not make the optimization problems untrackable.

We can see from Fig. 3.1 that an efficient allocation of the providers' resources and services to different types of travelers can be attained using a game-theoretic framework. The assignment shown in Fig. 3.1 is stable, maximizes the social welfare, and maximizes the capacity of the providers (thus, maximizing their utilities too).

3.4 Mobility Equity and Economic Sustainability Using Game Theory

In this chapter, we consider a multi-modal mobility system of travelers each with an individual travel budget, and propose a game-theoretic framework to assign each traveler to a "mobility service" (each one representing a different mode of transportation). We are interested in equity and sustainability, thus we maximize the worst-case revenue of the mobility system while ensuring "mobility equity," which we define it

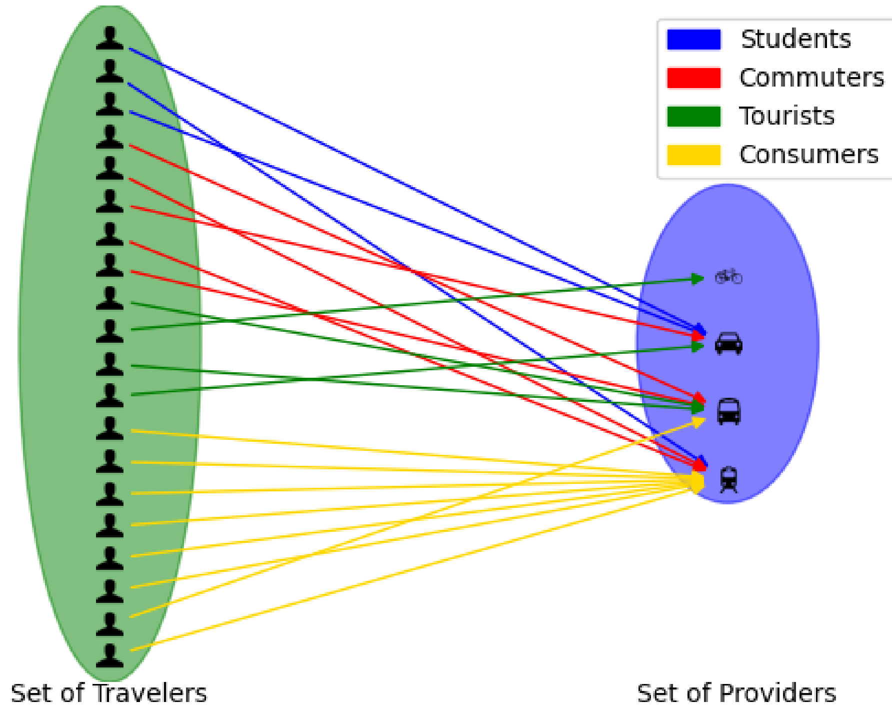

Figure 3.1: Optimal assignment of travelers to providers with 20 travelers and 4 providers.

in terms of accessibility. In the proposed framework, we ensure that all travelers are truthful and voluntarily participate under informational asymmetry, and the solution respects the individual budget of each traveler. Each traveler may seek to travel using multiple services (e.g., car, bus, train, bike). The services are capacitated and can serve up to a fixed number of travelers at any instant of time. Thus, our problem falls under the category of many-to-one assignment problems, where the goal is to find the conditions that guarantee the stability of assignments. We formulate a linear program of maximizing worst-case revenue under the constraints of mobility equity, and we fully characterize the optimal solution. Finally, we conclude our work by providing a numerical example to illustrate the proposed framework.

3.4.1 Modeling Framework

In this section, we present the mathematical formulation of our game-theoretic framework.

We consider a mobility system where $I \in \mathbb{N}_{\geq 2}$ travelers, indexed by $i \in \mathcal{I}$, $|\mathcal{I}| = I$, are interested in a non-empty set of $J \in \mathbb{N}$ mobility services, indexed by $j \in \mathcal{J}$, made available by a central authority in a city. For our purposes, we call this authority "social planner." In addition, expect $I < J$. Any $j \in \mathcal{J}$ represents the service that can be offered to a traveler i. So, for example, a taxicab service, say some $j \in \mathcal{J}$, can satisfy the travel needs of up to five travelers; thus, any service j can be divided to multiple travelers based on the service j's physical capacity. Each traveler $i \in \mathcal{I}$ has a private valuation v_{ij} associated with each of the services $j \in \mathcal{J}$, which is not known to the social planner.

Travelers are constrained by a *travel budget*, $b_i \in \mathbb{R}_{\geq 0}$, for any traveler $i \in \mathcal{I}$. Thus, we can only charge travelers payments that do not exceed their individual budgets. We write $\mathcal{B} = \{b_1, b_2, \ldots, b_I\}$. For the purposes of this work, we assume that the budgets of each traveler are known to the social planner. Our reasoning here is twofold: A probabilistic distribution for unknown private budgets leads to an impossibility result for socially-efficient mechanisms [66, 67]. In addition, based on transportation literature, it is reasonable to expect travelers to submit their travel budget on a mobility app [87, 236].

For each service $j \in \mathcal{J}$, we model the social planner's beliefs on the realization of the private valuations for service j as real values from some subset of real values.

Definition 3.4.1. *For each traveler $i \in \mathcal{I}$, the traveler i's valuation profile of all mobility services is $v_i = (v_{i1}, v_{i2}, \ldots, v_{iJ})$, $v_{ij} \in \mathbb{R}$. We write*

$$v_{-ij} = (v_{1j}, \ldots, v_{(i-1)j}, v_{(i+1)j}, \ldots, v_{Ij}) \tag{3.87}$$

for the valuation profile of all travelers except i for service j and denote by $v_{-i} = (v_{-i1}, \ldots, v_{-iJ})$ the profile of valuations of all services of all travelers except traveler i.

Then, $v = (v_i, v_{-i}) \in \mathcal{V} \subset \mathbb{R}^{I\times J}$ *is the valuation profile of all travelers for all mobility services.*

For an arbitrary traveler i, the valuation v_{ij} can represent the realization of a satisfaction function that captures, for example, the maximum amount of money that traveler i is willing to pay for mobility service j.

Remark 3.4.2. *The travelers can use multiple services to satisfy their mobility needs, i.e., to reach their destination, via a smartphone app. The social planner then compiles all travelers' origin-destination requests and other information (e.g., preferred travel time, value of time, and maximum willingness-to-pay) in order to provide a travel recommendation to each traveler.*

Remark 3.4.3. *In the modeling framework, we consider that the travelers' budgets are known to the social planner as it is reasonable to expect travelers to submit their travel budget on a mobility app [87, 236]. However, the valuations of any traveler for each different mobility service is considered private information. Realistically, we cannot expect any traveler to provide truthfully their preferences for any service.*

Mathematically, the allocation of the finite number of mobility services to travelers can be described by a vector of binary variables.

Definition 3.4.4. *The traveler-service assignment is a vector* $\mathbf{a} = (a_{ij}(v))_{i\in\mathcal{I},j\in\mathcal{J}}$*, where* a_{ij} *is a binary variable of the form:*

$$a_{ij}(v) = \begin{cases} 1, & \textit{if } i \in \mathcal{I} \textit{ is assigned to } j \in \mathcal{J}, \\ 0, & \textit{otherwise.} \end{cases} \tag{3.88}$$

Note that the assignment $a_{ij}(v)$ between traveler i and service j depends on the valuation v_i of traveler i and the valuations of all other travelers, i.e., v_{-i}.

Furthermore, it is possible in our framework for a traveler to reject all assignments with any service. However, we show in Theorem 3.94 how to avoid such an unfortunate outcome by providing the right incentives to travelers to use at least one service.

Naturally, each service can accommodate up to a some number of travelers, different for each type of services. So, we expect the "physical traveler capacity" of each service to vary significantly.

Definition 3.4.5. *Each service $j \in \mathcal{J}$ is associated with a current* traveler capacity, *denoted by $\varepsilon_j \in \mathbb{N}$ and $\varepsilon_j \leq \bar{\varepsilon}_j$, where $\bar{\varepsilon}_j$ denotes the maximum traveler capacity of service j.*

For example, a bus can provide travel services to a hundred travelers (seated and standing) compared to a bike-sharing company's bike (since one traveler per bike).

Definition 3.4.6. *A* feasible *assignment is a vector $\mathbf{a} = (a_{ij})_{i\in\mathcal{I},j\in\mathcal{J}}$, $a_{ij} \in \{0,1\}$ that satisfies*

$$\sum_{j\in\mathcal{J}} a_{ij}(v) \leq \delta_i, \quad \forall i \in \mathcal{I}, \quad \forall v \in \mathcal{V}, \tag{3.89}$$

$$\sum_{i\in\mathcal{I}} a_{ij}(v) \leq \bar{\varepsilon}_j, \quad \forall j \in \mathcal{J}, \quad \forall v \in \mathcal{V}, \tag{3.90}$$

where (3.89) ensures that each traveler $i \in \mathcal{I}$ is assigned to at most $\delta_i \in \mathbb{N}$ mobility service $j \in \mathcal{J}$, and (3.90) ensures that the traveler capacity of each service j is not exceeded while it is shared by multiple travelers.

Next, we represent the preferences of each traveler with a utility function consisted of two parts: the traveler's valuation of the mobility outcome and the associated payment required for the realization of that outcome. In other words, any traveler is expected to pay a toll or ticket fee for the mobility service used.

Definition 3.4.7. *Each traveler i's preferences are summarized by a utility function $u_i : \mathcal{V} \times \mathbb{R} \to \mathbb{R}$ that determines the monetary value of the overall payoff realized by traveler i from their assignment to service j. Thus, traveler i receives a total utility*

$$u_i((v_i, v_{-i}), p_i) = \sum_{j\in\mathcal{J}} v_{ij} a_{ij}(v_i, v_{-i}) - p_i(v_i, v_{-i}), \tag{3.91}$$

where $p_i \in \mathbb{R}$ *denotes traveler* i*'s mobility payment.*

Note that each traveler's goal is to choose a strategy/action that will maximize their own utility only.

We now formally present the definition of "mobility equity" of our game-theoretic framework.

Definition 3.4.8. *A mobility system* $\langle \mathcal{I}, \mathcal{J}, \mathcal{V}, (u_i)_{i\in\mathcal{I}}, (p_i)_{i\in\mathcal{I}} \rangle$ *admits an equilibrium that is* mobility equitable *if (i) travelers truthfully report their private information, (ii) travelers voluntarily participate, and (iii) travelers can afford travel.*

Next, we formally define the relation that ensures "economic sustainability" for our game-theoretic framework.

Definition 3.4.9. *Let linear function* $w : \mathcal{V} \times \mathbb{R} \to \mathbb{R}_{\geq 0}$ *that depends on the valuations, assignments, and individual budgets of all the travelers denote the worst-case revenue. Mathematically, we have*

$$\sum_{i\in\mathcal{I}} \sum_{j\in\mathcal{J}} w_i(v_{ij}, a_{ij}, b_i) \leq \sum_{i\in\mathcal{I}} p_i(v), \quad \forall v \in \mathcal{V}. \tag{3.92}$$

Our intuition behind Definition 3.4.9 is conceptually based on what the United Nations Development Programme has developed as part of their Sustainable Development Goals. In particular, our goal in this work is to ensure long-term economic growth in the worst possible cases (thus, maximizing (3.92)) under the constraints of Definition 3.4.8.

We now formally define the constraints that will ensure mobility equity in our framework's solutions based on Definition 3.4.8.

Definition 3.4.10. *For the travelers to have no incentive to misreport their valuations to the social planner, we need*

$$\sum_{j\in\mathcal{J}} v_{ij} a_{ij}(\tilde{v}_i, v_{-i}) - p_i(\tilde{v}_i, v_{-i}) - \sum_{j\in\mathcal{J}} v_{ij} a_{ij}(v_i, v_{-i}) + p_i(v_i, v_{-i}) \leq 0, \tag{3.93}$$

for all $v = (v_i, v_{-i}) \in \mathcal{V}$*, any* $\tilde{v}_i \in \mathcal{V}$*, and for all travelers* $i \in \mathcal{I}$ *using any mobility service* $j \in \mathcal{J}$*. We call* $\tilde{v}_i$ *traveler* i*'s reported valuation that deviates from the true valuation* v_i*. If* (3.93) *holds, then we say that the mechanism induces* truthfulness.

Definition 3.4.11. *The travelers in the mobility system* voluntarily participate *(VP) if, for any traveler* $i \in \mathcal{I}$*,*

$$p_i(v_i, v_{-i}) \leq \sum_{j \in \mathcal{J}} v_{ij} a_{ij}(v_i, v_{-i}), \quad \forall v \in \mathcal{V}. \tag{3.94}$$

We say then that the proposed mechanism induces voluntary participation from all travelers.

Definition 3.4.12. *The mechanism induces on individual level* budget fairness *(BF), if for any traveler* $i \in \mathcal{I}$*, we have*

$$p_i(v) \leq b_i, \quad \forall v \in \mathcal{V}. \tag{3.95}$$

3.4.2 The Optimization Problem

Problem 3.4.13. *The maximization problem is formulated as follows*

$$\max_{a_{ij}} \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} w_i(v_{ij}, a_{ij}, b_i), \tag{3.96}$$

$$\text{subject to: } (3.89), (3.90), (3.92), (3.93), (3.94), (3.95),$$

where $a_{ij} \in \{0, 1\}$ *for all* $i \in \mathcal{I}$ *and all* $j \in \mathcal{J}$*.*

We note here that Problem 3.4.13 is a special case of the many-to-many assignment problem that is known to be very hard to solve analytically. Thus, we relax the integer constraint and focus our analysis on deriving the optimal solutions of a linearized version of Problem 3.3.17. Thus, we introduce a non-negativity constraint variable as follows.

Problem 3.4.14. *The linear program formulation is*

$$\max_{a_{ij}} \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} w_i(v_{ij}, a_{ij}, b_i) \tag{3.97}$$

subject to: (3.89), (3.90), (3.92), (3.93), (3.94), (3.95), *and*

$$a_{ij} \geq 0, \quad \forall i \in \mathcal{I}, \quad \forall j \in \mathcal{J}, \tag{3.98}$$

where (3.98) *transforms the (binary) assignment problem to a (continuous) linear program.*

Remark 3.4.15. *Intuitively, in Problem 3.3.18, if $a_{ij} > 0$ it implies that traveler $i \in \mathcal{I}$ is assigned to service $j \in \mathcal{J}$.*

Problem 3.4.14 is a constrained linear maximization problem that admits at least one solution under certain conditions. A solution of Problem 3.4.14 ensures the assignments between the travelers and services are mobility equitable and economic sustainable. In particular, we maximize the worst-case revenue of the mobility system under the constraints of truthfulness, VP, and BF. Next, inspired from Myerson's auction [165] and the Vickrey-Clarke-Groves (VCG) auction mechanism, we introduce two key variables that can helps us solve Problem 3.3.18, i.e., *nominal assignments* and *reservation payments*.

Definition 3.4.16. *For any traveler $i \in \mathcal{I}$, there is a reservation payment for each mobility service $j \in \mathcal{J}$, denoted by $r_{ij} \in \mathbb{R}_{\geq 0}$, representing the minimum necessary mobility payment of traveler i to get assigned to mobility service j.*

Definition 3.4.17. *The final assignment $a_{ij}(v)$ evaluated at the realized valuation profile $v \in \mathcal{V}$ is computed as the sum of the nominal assignment $\bar{a}_{ij}$ and the adapted assignment $\tilde{a}_{ij}(v)$, i.e., we have*

$$a_{ij}(v_i, v_{-i}) = \bar{a}_{ij} + \tilde{a}_{ij}(v_i, v_{-i}). \tag{3.99}$$

We provide the exact methodology of computing $\bar{a}_{ij}$ and $\tilde{a}_{ij}(v)$ in the next section.

3.4.3 Analysis and Properties of the Mechanism

In this section, we show formally that the proposed mechanism satisfies the desired properties of mobility equity as defined in Definition 3.4.8, i.e., truthfulness, voluntary participation, and budget fairness for all travelers. We start our exposition by presenting the derivation of the dual program of Problem 3.3.18.

Lemma 3.4.18. *The dual problem of Problem 3.3.18 is*

$$\min \sum_{v \in \mathcal{V}} \left[\sum_{i \in \mathcal{I}} \xi_1^i(v)\delta_i + \sum_{j \in \mathcal{J}} \xi_2^j(v)\bar{\varepsilon}_j + \sum_{i \in \mathcal{I}} \xi_4^i(v) b_i \right] \tag{3.100}$$

$$\text{subject to: } \xi_1^i(v) + \xi_2^j(v) + \sum_{\tilde{v}_i \in \mathcal{V}} \tilde{v}_{ij}\xi_4^i(v, \tilde{v}_i)$$

$$-v_{ij} \sum_{\tilde{v}_i} \xi_4^i(v, \tilde{v}_i) - v_{ij}\xi_5^i(v) \geq 0, \quad \forall v \in \mathcal{V}, \tag{3.101}$$

$$\sum_{\tilde{v}_i \in \mathcal{V}} \xi_4^i(v, \tilde{v}_i) - \sum_{\tilde{v}_i \in \mathcal{V}} \xi_4^i(v_{-i}, \tilde{v}_i, v_i) - \xi_3(v)$$

$$+\xi_5^i(v) + \xi_6^i(v) = 0, \quad \forall v \in \mathcal{V}, \tag{3.102}$$

$$\sum_{v \in \mathcal{V}} \xi_3(v) = 1, \tag{3.103}$$

$$\xi_1(v), \xi_2^j(v), \xi_3, \xi_4^i(v, \tilde{v}_i), \xi_5^i(v), \xi_6^i(v) \geq 0, \tag{3.104}$$

where $\xi_1^i, \xi_2^j, \xi_3, \xi_4^i, \xi_5^i$, *and* ξ_6^i *are the dual variables for constraints* (3.89), (3.90), (3.92), (3.93), (3.94), *and* (3.95), *respectively.*

Proof. The computations here are straightforward following standard techniques from [204], hence we omit them due to space limitations. □

Based on Lemma 3.4.18, we can now compute the nominal assignments and reservation payments as follows: we formulate the optimization problem for the assignments

$$\max_{a_{ij}} \sum_{i\in\mathcal{I}} \sum_{j\in\mathcal{J}} v_{ij} a_{ij} \tag{3.105}$$

$$\text{subject to: (3.89), (3.90), and}$$

$$v_{ij} a_{ij}(\tilde{v}_i, v_{-i}) - v_{ij} a_{ij}(v_i, v_{-i}) \leq 0, \tag{3.106}$$

$$\forall \tilde{v}_i \in \mathcal{V}, \quad \forall i \in \mathcal{I}, \quad \forall j \in \mathcal{J},$$

$$\sum_{j\in\mathcal{J}} v_{ij} a_{ij}(v_i, v_{-i}) \leq b_i, \quad \forall i \in \mathcal{I}, \tag{3.107}$$

$$a_{ij} \geq 0, \quad \forall i \in \mathcal{I}, \quad \forall j \in \mathcal{J}, \tag{3.108}$$

where solving (3.105) gives us the valuation profile $v^{\text{worst}} = (v_{ij}^{\text{worst}})_{i\in\mathcal{I}, j\in\mathcal{J}}$ at the worst case and the associated nominal assignment $\bar{a}_{ij}$. Next, we derive ξ_1^i, ξ_2^j, ξ_5^i, and ξ_6^i from Lemma 3.4.18 and then compute

$$r_{ij} = \xi_1^i + \xi_2^j + \xi_5^i v_{ij}^{\text{worst}} + \xi_6^i v_{ij}^{\text{worst}}. \tag{3.109}$$

The next step now is to present the pricing mechanism for any traveler $k \in \mathcal{I}$ of our proposed framework. But first, we define $\gamma_{ij} = \arg\min_{\tilde{v}\in\mathcal{V}} \sum_{j\in\mathcal{J}} \bar{a}_{ij} \tilde{v}_{ij}$. Then, we have

$$\begin{aligned} p_k(v) = & \sum_{j\in\mathcal{J}} \tilde{a}_{kj}(v) r_{kj} + \sum_{j\in\mathcal{J}} \bar{a}_{kj} r_{kj} - \sum_{j\in\mathcal{J}} \bar{a}_{kj} \xi_5^k \gamma_{kj} \\ & + \sum_{i\in\mathcal{I}\setminus\{k\}} \sum_{j\in\mathcal{J}} \tilde{a}_{ij;k}(v_{-k})(v_{ij} - r_{ij}) - \sum_{i\in\mathcal{I}\setminus\{k\}} \sum_{j\in\mathcal{J}} \tilde{a}_{ij}(v)(v_{ij} - r_{ij}), \end{aligned} \tag{3.110}$$

where $\tilde{a}_{ij;k}$ represents a "temporary" assignment of travelers to mobility services expecting traveler k (we see how to estimate this variable in (3.121)). We formally present how to compute such an assignment in Theorem 3.4.20. The term $\sum_{i\in\mathcal{I}} \sum_{j\in\mathcal{J}} \tilde{a}_{ij}(v)(v_{ij} - r_{ij})$ represents the "social welfare" of all travelers based on the valuations of each mobility service j and the reservation mobility payments r_{ij}. We motivate our mobility pricing mechanism (3.110) as follows: with the help of the reservation payments we parameterize the totals of social welfare in terms of the travelers' valuations. So, the first three terms capture the parameterized social welfare for all services from the point of view of one traveler. Then the other two terms represent the social welfare excluding traveler k's contribution. Using these reservation payments, we then introduce a mobility payment p_k for traveler k that charges the minimum required payment for traveler k to get assigned to mobility service j while keeping all other travelers' reported valuations fixed.

Next, we show that our mechanism induces truthfulness from all travelers, i.e., no traveler has an incentive to misreport or lie to the social planner.

Theorem 3.4.19. *The proposed framework induces all travelers to report their valuations $v \in \mathcal{V}$ truthfully to the social planner under the pricing mechanism (3.110).*

Proof. Consider traveler k with a true valuation v_{kj} for each service $j \in \mathcal{V}$. By reporting v'_{kj}, traveler k is assigned service j with $\tilde{a}_{kj}(v'_k, v_{-k})$. We formulate the following optimization problem

$$(\tilde{a}_{ij}(v))_{i\in\mathcal{I},j\in\mathcal{J}} = \arg\max_{\tilde{a}\in\mathcal{A}} \sum_{i\in\mathcal{I}} \sum_{j\in\mathcal{J}} \tilde{a}_{ij}(v_{ij} - r_{ij}), \tag{3.111}$$

where $\mathcal{A}$ is the set of positive values for $\tilde{a}$ that satisfies the following two constraints:

$$\sum_{i\in\mathcal{I}} \tilde{a}_{ij} \leq 1 - \sum_{i\in\mathcal{I}} \bar{a}_{ij}, \quad \forall j \in \mathcal{J}, \tag{3.112}$$

$$\sum_{j\in\mathcal{J}} \tilde{a}_{ij}\tilde{v}_{ij} \leq b_i - \sum_{j\in\mathcal{J}} \bar{a}_{ij} r_{ij} + \sum_{j\in\mathcal{J}} \bar{a}_{kj}\xi_5^i \gamma_{ij}, \quad \forall i \in \mathcal{I}, \tag{3.113}$$

where (3.113) must hold for all $\tilde{v} \in \mathcal{V}$, and $\gamma_{ij} = \arg\min_{\tilde{v}\in\mathcal{V}} \sum_{j\in\mathcal{J}} \bar{a}_{ij}\tilde{v}_{ij}$. Since $\mathcal{A}$ does not depend on any specific valuation profile, we have $(\tilde{a}_{ij}(v'_k, v_{-k}))_{ij} \in \mathcal{A}$. Thus, we have

$$\sum_{i\in\mathcal{I}} \sum_{j\in\mathcal{J}} \tilde{a}_{ij}(v_k, v_{-k})(v_{ij} - r_{ij}) \geq \sum_{i\in\mathcal{I}} \sum_{j\in\mathcal{J}} \tilde{a}_{ij}(v'_k, v_{-k})(v_{ij} - r_{ij}). \tag{3.114}$$

Using Definition 3.4.7, we now compare the utilities of traveler k under the two different valuations. So, we have

$$u_k(v_k, v_{-k}) = \sum_{j\in\mathcal{J}} a_{kj}(v_k, v_{-k})v_{kj} - p_k(v_k, v_{-k}), \tag{3.115}$$

which, by Definition 3.4.17 and (3.110), we can expand as follows

$$u_k(v_k, v_{-k}) = \sum_{j\in\mathcal{J}} \tilde{a}_{kj}(v_k, v_{-k})v_{kj} + \sum_{j\in\mathcal{J}} \bar{a}_{kj}v_{kj} - \sum_{j\in\mathcal{J}} \tilde{a}_{kj}r_{kj} - \sum_{j\in\mathcal{J}} \bar{a}_{kj}r_{kj} + \sum_{j\in\mathcal{J}} \bar{a}_{kj}\xi_5^k\gamma_{kj} - \sum_{i\in\mathcal{I}\setminus\{k\}} \sum_{j\in\mathcal{J}} \tilde{a}_{ij;k}(v_{-k})(v_{ij} - r_{ij}) + \sum_{i\in\mathcal{I}\setminus\{k\}} \sum_{j\in\mathcal{J}} \tilde{a}_{ij}(v_k, v_{-k})(v_{ij} - r_{ij}) \quad (3.116)$$

$$= \sum_{i\in\mathcal{I}} \sum_{j\in\mathcal{J}} \tilde{a}_{ij}(v_k, v_{-k})(v_{ij} - r_{ij}) - \sum_{i\in\mathcal{I}\setminus\{k\}} \sum_{j\in\mathcal{J}} \tilde{a}_{ij;k}(v_{-k})(v_{ij} - r_{ij}) + \sum_{j\in\mathcal{J}} \bar{a}_{kj}v_{kj} - \sum_{j\in\mathcal{J}} \bar{a}_{kj}r_{kj} + \sum_{j\in\mathcal{J}} \bar{a}_{kj}\xi_5^k\gamma_{kj} \quad (3.117)$$

$$\geq \sum_{i\in\mathcal{I}} \sum_{j\in\mathcal{J}} \tilde{a}_{ij}(v_k', v_{-k})(v_{ij} - r_{ij}) - \sum_{i\in\mathcal{I}\setminus\{k\}} \sum_{j\in\mathcal{J}} \tilde{a}_{ij;k}(v_{-k})(v_{ij} - r_{ij}) + \sum_{j\in\mathcal{J}} \bar{a}_{kj}v_{kj} - \sum_{j\in\mathcal{J}} \bar{a}_{kj}r_{kj} + \sum_{j\in\mathcal{J}} \bar{a}_{kj}\xi_5^k\gamma_{kj} \quad (3.118)$$

$$= \sum_{j\in\mathcal{J}} \tilde{a}_{kj}(v_k', v_{-k})v_{kj} + \sum_{j\in\mathcal{J}} \bar{a}_{kj}v_{kj} - \sum_{j\in\mathcal{J}} \tilde{a}_{kj}(v_k', v_{-k})r_{kj} - \sum_{j\in\mathcal{J}} \bar{a}_{kj}r_{kj} - \sum_{i\in\mathcal{I}\setminus\{k\}} \sum_{j\in\mathcal{J}} \tilde{a}_{ij}(v_{-k})(v_{ij} - r_{ij}) + \sum_{i\in\mathcal{I}\setminus\{k\}} \sum_{j\in\mathcal{J}} \tilde{a}_{ij}(v_k, v_{-k})(v_{ij} - r_{ij}), \quad (3.119)$$

where the last equality (3.119) follows by simple rearrangement using (3.110); thus, (3.119) is equal to $u_k(v_k', v_{-k})$. Therefore, the result follows. □

Theorem 3.4.20. *The proposed framework ensures that no traveler pays more than their budget for their assignment, i.e., under the pricing mechanism (3.110), for any traveler $i \in \mathcal{I}$, we have $p_i(v) \leq b_i$, for all $v \in \mathcal{V}$.*

Proof. The mobility payment (3.110) of any traveler k can be written as

$$p_k(v) = \sum_{j\in\mathcal{J}} \tilde{a}_{kj}(v)r_{kj} + \sum_{j\in\mathcal{J}} \bar{a}_{kj}r_{kj} - \sum_{j\in\mathcal{J}} \bar{a}_{kj}\xi_5^k\gamma_{kj} + \sum_{i\in\mathcal{I}\setminus\{k\}} \sum_{j\in\mathcal{J}} \tilde{a}_{ij;k}(v_{-k})(v_{ij} - r_{ij}) - \sum_{i\in\mathcal{I}} \sum_{j\in\mathcal{J}} \tilde{a}_{ij}(v)(v_{ij} - r_{ij}). \quad (3.120)$$

Next, we formulate the following optimization problem:

$$(\tilde{a}_{ij;k}(v_{-k}))_{i\in\mathcal{I}\backslash\{k\},j\in\mathcal{J}} = \arg\max_{\tilde{a}\in\mathcal{A}_k} \sum_{i\in\mathcal{I}\backslash\{k\}} \sum_{j\in\mathcal{J}} \tilde{a}_{ij}(v_{ij} - r_{ij}), \tag{3.121}$$

where $\tilde{a}_{ij;k}(v_{-k})$ takes positive values from set $\mathcal{A}_k$ with constraints:

$$\sum_{i\in\mathcal{I}\backslash\{k\}} \tilde{a}_{ij} \leq 1 - \sum_{i\in\mathcal{I}} \bar{a}_{ij}, \quad \forall j \in \mathcal{J}, \tag{3.122}$$

$$\sum_{j\in\mathcal{J}} \tilde{a}_{ij}\tilde{v}_{ij} \leq b_i - \sum_{j\in\mathcal{J}} \bar{a}_{ij} r_{ij}, \tag{3.123}$$

where (3.123) holds for all $\tilde{v} \in \mathcal{V}$ and $i \in \mathcal{I}\backslash\{k\}$. Thus, we have $(\tilde{a}_{ij;k}(v_{-k}))_{i\in\mathcal{I}\backslash\{k\},j\in\mathcal{J}} \in \mathcal{A}_k$, which yields

$$\sum_{i\in\mathcal{I}\backslash\{k\}} \tilde{a}_{ij;k}(v_{-k}) \leq 1 - \sum_{i\in\mathcal{I}} \bar{a}_{ij}, \quad \forall j \in \mathcal{J}, \tag{3.124}$$

$$\sum_{j\in\mathcal{J}} \tilde{a}_{ij;k}(v_{-k})\tilde{v}_{ij} \leq b_i - \sum_{j\in\mathcal{J}} \bar{a}_{ij} r_{ij}, \tag{3.125}$$

where (3.125) holds for all $\tilde{v} \in \mathcal{V}$ and for all $i \in \mathcal{I} \setminus \{k\}$. If we construct an assignment $\alpha = (\alpha_{ij})$ such that

$$\alpha_{ij} = \begin{cases} \tilde{a}_{ij;k}(v_{-k}), & \forall i \in \mathcal{I} \setminus \{k\}, \quad \forall j \in \mathcal{J}, \\ 0, & i = k, \quad \forall j \in \mathcal{J}, \end{cases} \tag{3.126}$$

then we have

$$\begin{aligned} \sum_{i\in\mathcal{I}} \alpha_{ij} &= \sum_{i\in\mathcal{I}\backslash\{k\}} \tilde{a}_{ij;k}(v_{-k}) \\ &\leq 1 - \sum_{i\in\mathcal{I}} \bar{a}_{ij}, \quad \forall j \in \mathcal{J}, \end{aligned} \tag{3.127}$$

$$\begin{aligned} \sum_{j\in\mathcal{J}} \alpha_{ij}\tilde{v}_{ij} &= \sum_{j\in\mathcal{J}} \tilde{a}_{ij;k}(v_{-k})\tilde{v}_{ij} \\ &\leq b_i - \sum_{j\in\mathcal{J}} \bar{a}_{ij} r_{ij}, \end{aligned} \tag{3.128}$$

where (3.128) holds for all $\tilde{v} \in \mathcal{V}$ and for all $i \in \mathcal{I} \setminus \{k\}$. Similarly, for traveler k, it follows from (3.105)-(3.108) that

$$\sum_{j \in \mathcal{J}} \alpha_{kj} \tilde{v}_{kj} = 0 \leq b_k - \sum_{j \in \mathcal{J}} \bar{a}_{kj} r_{kj}. \tag{3.129}$$

Thus, it follows immediately that $\alpha \in \mathcal{A}$ which results to the following inequality:

$$\begin{aligned} \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \tilde{a}_{ij}(v)(v_{ij} - r_{ij}) &\geq \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \alpha_{ij}(v_{ij} - r_{ij}) \\ &= \sum_{i \in \mathcal{I} \setminus \{k\}} \sum_{j \in \mathcal{J}} \tilde{a}_{ij;k}(v_{-k})(v_{ij} - r_{ij}). \end{aligned} \tag{3.130}$$

From (3.120), we subtract $\sum_{j \in \mathcal{J}} \tilde{a}_{kj}(v_k, v_{-k}) v_{kj}$ to obtain

$$\begin{aligned} p_k(v) = \sum_{j \in \mathcal{J}} \tilde{a}_{kj}(v) r_{kj} + \sum_{j \in \mathcal{J}} \bar{a}_{kj} r_{kj} - \sum_{j \in \mathcal{J}} \bar{a}_{kj} \xi_5^k \gamma_{kj} \\ + \sum_{i \in \mathcal{I} \setminus \{k\}} \sum_{j \in \mathcal{J}} \tilde{a}_{ij;k}(v_{-k})(v_{ij} - r_{ij}) - \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \tilde{a}_{ij}(v)(v_{ij} - r_{ij}). \end{aligned} \tag{3.131}$$

So, from (3.130) and (3.131), we get

$$p_k(v) \leq \sum_{j \in \mathcal{J}} \tilde{a}_{kj}(v) v_{kj} + \sum_{j \in \mathcal{J}} \bar{a}_{kj} r_{kj} - \sum_{j \in \mathcal{J}} \bar{a}_{kj} \xi_5^k \gamma_{kj}, \tag{3.132}$$

which leads to $p_k(v) \leq b_k$. □

Theorem 3.4.21. *The proposed framework induces all travelers to voluntary participate under the pricing mechanism* (3.110), *and thus satisfy the last necessary property for mobility equity.*

Proof. By Theorem 3.4.19, we have

$$\begin{aligned} u_i(v_k, v_{-k}) = \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \tilde{a}_{ij}(v_k, v_{-k})(v_{ij} - r_{ij}) \\ - \sum_{i \in \mathcal{I} \setminus \{k\}} \sum_{j \in \mathcal{J}} \tilde{a}_{ij;k}(v_{-k})(v_{ij} - r_{ij}) + \sum_{j \in \mathcal{J}} \bar{a}_{kj} v_{kj} - \sum_{j \in \mathcal{J}} \bar{a}_{kj} r_{kj} + \sum_{j \in \mathcal{J}} \bar{a}_{kj} \xi_5^k \gamma_{kj}, \end{aligned} \tag{3.133}$$

which leads to

$$u_i(v_k, v_{-k}) \geq \sum_{j \in \mathcal{J}} \bar{a}_{kj} v_{kj} - \sum_{j \in \mathcal{J}} \bar{a}_{kj} r_{kj} + \sum_{j \in \mathcal{J}} \bar{a}_{kj} \xi_5^k \gamma_{kj}, \tag{3.134}$$

where we have used (3.130). From Lemma 3.4.18 it follows straightforwardly that

$$\sum_{j\in\mathcal{J}} \bar{a}_{ij} r_{ij} - \sum_{j\in\mathcal{J}} \bar{a}_{ij} v'_{ij} \leq 0, \quad v'_{ij} \in \mathcal{V}, \quad \forall i \in \mathcal{I}, \tag{3.135}$$

$$\bar{a}_{ij} r_{ij} = \bar{a}_{ij} v_{ij}^{\text{worst}}, \quad \forall i \in \mathcal{I}, \quad \forall j \in \mathcal{J}. \tag{3.136}$$

Thus, we have

$$\sum_{j\in\mathcal{J}} \bar{a}_{kj} v_{kj} \geq \sum_{j\in\mathcal{J}} \bar{a}_{kj} z_{kj} \tag{3.137}$$

$$= \sum_{j\in\mathcal{J}} \bar{a}_{kj} r_{kj} \tag{3.138}$$

$$\geq \sum_{j\in\mathcal{J}} \bar{a}_{kj} r_{kj} - \sum_{j\in\mathcal{J}} \bar{a}_{kj} \xi_5^k \gamma_{kj}. \tag{3.139}$$

Note that $\bar{a}_{kj}$, ξ_5^k, and γ_{ij} are non-negative. Thus, we have, for any traveler $i \in \mathcal{I}$, $u_i(v_k, v_{-k}) \geq 0$. □

3.4.4 Implementation

In this subsection, we outline how our proposed framework could be potentially implemented. We consider a typical major metropolitan area with an extensive road and public transit infrastructure; a good example is Boston. Several key areas in Boston are connected by roads, buses, light rail, and bikes, thus any traveler has easy access to any of the four available modes of transportation, namely car, bus, light rail, or bike. By applying the MaaS concept, a social planner (a central computer) can offer travel services (e.g., navigation, location, booking, payment) to all passing travelers at certain travel hub locations (e.g., train stations with bus stops and taxi waiting line). Information can be shared among all travelers via a "mobility app," which allows travelers to access the services offered by the social planner. Using this app, travelers will be able to pay for their travel needs while providing their individual budget and valuations. This can be done using a "preferences" questionnaire in the mobility app. Each mode of transportation offers different benefits in utility; for example, a car is more convenient than a bus and is expected to be in high demand. This justifies our modeling choice of each traveler having valuations for each mobility service. Our framework guarantees that by the design of the payments (3.110) no traveler has an incentive to misreport these preferences (Theorem 3.4.19). In addition, travelers are incentivized to use the mobility app multiple times for their travels and interact with each other more than once (Theorem 3.4.21). Hence, our framework provides an efficient and fair way for travelers to travel using different modes of transportation from one place to another while competing with many other travelers and pay a ticket or a toll always within their individual budget using the mobility app (Theorem 3.4.20).

3.5 Summary

In this chapter, we formulated the routing of strategic travelers that use CAVs in a transportation network as a social resource allocation mechanism design problem. Considering a Nash-implementation approach, we showed that our proposed informationally decentralized mechanism efficiently allocates travel time to all travelers that seek to commute in the network. Our mechanism induces a game which at least one equilibrium prevents congestion (a significant rebound effect), while also attaining the properties of individually rationality, budget balanced, strongly implementability. Ongoing work includes conducting a simulation-based analysis under different traffic scenarios to showcase the practical implications of our mechanism. Extending and enhancing the traveler-behavioral model, motivated by a social-mobility survey can be a worthwhile undertaking as a future research direction allowing the study of the relationship of emerging mobility and the intricacies of human decision-making.

In this chapter, we provided an answer to how one can ensure a socially-acceptable assignment between travelers and the shared vehicles' operators. We focused on the behavioral decision-making of both the travelers and the vehicles' operators and designed a shared mobility market consisted of travelers and vehicles in a transportation network. We formulated a binary linear program and derived necessary and sufficient conditions for its solution to be an assignment between travelers and vehicles that cannot be improved any further. Consequently, we showed that our optimal assignment maximizes the social welfare of all travelers, and ensures the feasibility and stability of the traveler-vehicle profit allocation while respecting the decision-making of both the travelers and the vehicles' operators.

In this chapter, we provided a theoretical study of a two-sided game for a mobility system of travelers and providers focusing on how to discretely assign travelers to providers when both have preferences. We formulated a binary program and its equivalent linear program and showed that at least one optimal solution exists and derived the conditions for such solution to be stable. We then allowed informational asymmetry in the proposed mobility game and provided a pricing mechanism to ensure we can elicit the private information of all travelers and providers. We showed that our mechanism guarantees economic efficiency in terms of maximizing the social welfare, and ensures voluntary participation, thus making sure that all agents have a unique dominant strategy.

In this chapter, we have provided a game-theoretic framework for a multi-modal mobility system where travelers can travel using different modes of transportation and each has a different and unique travel budget. Our goal in this chapter was to ensure economic sustainability by maximizing the worst-case revenue of the mobility system under the constraints of mobility equity, which we defined explicitly as truthfulness, voluntary participation, and budget fairness. We proved that our framework ensures budget fairness in the sense that no budget is violated. Under informational asymmetry, we showed that no traveler has an incentive to misreport and they voluntarily participate. Thus, our framework satisfies mobility equity by ensuring access to mobility to all travelers.

3.5.1 Limitations

One important limitation of our framework is the lack of a transportation network's dynamics. In addition, our framework although trackable it cannot solve this problem online. Realistically, we cannot expect travelers to access a mobility app and wait for travel recommendations in a static transportation setting. Future work will take into consideration the dynamics of a multi-modal transportation network (traffic network, road capacities). A potential direction for future research should also relax the assumption that travelers act rationally and accept the optimal and beneficial to them travel recommendations. We can explore this problem using prospect theory and analyze the optimal assignments between travelers and assignments under a behavioral model that takes into consideration risks and uncertainties. Another interesting direction for future research is to expand the current framework by explicitly designing the valuation functions and relaxing the linearity assumption. Nonlinearity in a mechanism like ours can complicate the derivation of the best possible equilibrium in a mobility system.

Chapter 4

A TRAVELER-CENTRIC MOBILITY GAME: EFFICIENCY AND STABILITY UNDER RATIONALITY AND PROSPECT THEORY

4.1 A Traveler-centric Mobility Game: Efficiency and Stability Under Rationality and Prospect Theory

In this chapter, we study a routing and travel-mode choice problem for mobility systems with a multimodal transportation network as a "mobility game" with coupled action sets. We develop a game-theoretic framework to study the impact on efficiency of the travelers' behavioral decision making. In our framework, we introduce a mobility "pricing mechanism" in which we model traffic congestion using linear cost functions while also considering the waiting times at different transport hubs. We show that travelers' selfish actions lead to a pure-strategy Nash equilibrium. We then perform a Price of Anarchy analysis to establish that the mobility system's inefficiencies remain relatively low as the number of travelers increases. We deviate from the standard game-theoretic analysis of decision making by extending our modeling framework to capture the subjective behavior of travelers using prospect theory. Finally, we provide a simulation study as a proof of concept for our proposed mobility game.

In this chapter, we propose a game-theoretic framework for the travelers' routing and travel-mode choices in a multimodal transportation network. We study the existence of a NE and the resulting inefficiencies of the travelers' decision making. One of the most significant aims of our work is to show that although we cannot guarantee equilibrium uniqueness, we can provide an upper bound for the inefficiency that arises from the individual strategic interactions of the travelers. In particular, our modeling framework (called mobility game), considers the impacts of "negative congestion externalities" and waiting costs in the travelers' decision making. That way, we offer an improved look at the socioeconomic factors that can affect the efficient and sustainable distribution of travel demand in a transportation network with multiple different modes of transportation (e.g., car, bus, light rail, bike). Moreover, we study the travelers' decision making under two behavioral models: (1) rational choice theory, where decision makers are selfish and seek to maximize only their own utility; and (2) prospect theory, where the decision-makers' biases and subjectivity is taken into account when decisions are made under risk.

The features that distinguish our work from the state of the art are as follows:

1. we model the interactions between travelers using a mobility game, a combination of a routing game and a a potential game with travel-mode choices and coupled action sets (see Section 4.1.1);
2. we take into account the traffic congestion cost factors using linear cost functions for the waiting time of travelers at different transport hubs; each transport hub allows a traveler to choose any of available modes of transportation to utilize for their travel needs (see Section 4.1.1);
3. we introduce a mobility token-based pricing mechanism to control travel demand and study the inefficiencies at a NE by showing that a NE exists (Theorem 4.1.16) and deriving a bound that remains small enough as the number of travelers increases (Theorem 4.1.21); and
4. we incorporate a behavioral model (prospect theory) for the travelers' decision making under the uncertainty of the mobility tokens (Theorem 4.1.25).

The remainder of the chapter is structured as follows. First, in Section 4.1.1, we present the mathematical formulation of our mobility game, which forms the basis of our theoretical study in this chapter. Then, in Section 4.1.1, we derive the properties of our mobility game showing NE existence and bounding the Price of Anarchy (PoA) in Subsection 4.1.2. Then we prove that a NE exists under prospect theory in Subsection 4.1.4. Finally, we draw conclusions and offer a discussion of future research.

4.1.1 Modeling Framework

We consider a mobility system of two finite, disjoint, and non-empty sets, (1) the set of travelers $\mathcal{I}$, $|\mathcal{I}| = I \in \mathbb{N}_{\geq 2}$, and (2) the set of mobility services by $\mathcal{J}$, $|\mathcal{J}| = J \in \mathbb{N}$. For example, $j \in \mathcal{J}$ can represent either a car, a bus, a light rail vehicle, or a bike. We consider that in our mobility game, $I < J$. The set of all mobility services $\mathcal{J}$ can be partitioned to a finite number of disjoint subsets, each representing a specific type of a mobility service, i.e., $\mathcal{J} = \bigcup_{h=1}^{H} \mathcal{J}_h$, where $H \in \mathbb{N}$ is the total number of subsets of $\mathcal{J}$. For example, if there are only two modes of transportation, say cars and buses, then $\mathcal{J} = \mathcal{J}_1 \cup \mathcal{J}_2$, where $\mathcal{J}_1$ represents the subset of all available cars, and $\mathcal{J}_2$ represents the subset of all available buses.

Definition 4.1.1. *The set of all different types of services is* $\mathcal{H} = \{1, \dots, H\}$, $H \in \mathbb{N}$, *where each element* $h \in \mathcal{H}$ *represents a possible travel option. We denote the type of service* j *used by traveler* i *by* $h_i \in \mathcal{H}$.

For example, suppose $H = 4$. Then each element $h \in \mathcal{H}$ can be associated one-to-one to the elements of the set $\{\text{car}, \text{bus}, \text{light rail}, \text{bike}\}$.

Naturally, each service can accommodate up to a finite number of travelers, that is different for each type of service. So, we expect the “physical traveler capacity” of each service to vary significantly.

Definition 4.1.2. *Each service* $j \in \mathcal{J}$ *is characterized by a current* physical traveler capacity, *i.e.,* $\varepsilon_j \in \{0, 1, 2, \dots, \bar{\varepsilon}_j\}$, *where* $\bar{\varepsilon}_j \in \mathbb{N}$ *denotes the maximum traveler capacity of service* j.

For example, one bus can provide travel services up to eighty travelers (seated and standing) compared to a bike-sharing company's bike (one bike per traveler).

Travelers seek to travel in a transportation network represented by a directed multigraph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where each node in $\mathcal{V}$ represents a city area (neighborhood) with a "transport hub," i.e., a central place where travelers can use different modes of transportation. Each edge $\mathcal{E}$ represents a sequence of city roads with public transit lanes. For our purposes, we think of $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ as a representation of a smart city network with a road and public transit infrastructure. In network $\mathcal{G}$, any traveler $i \in \mathcal{I}$ seeks to travel from an origin $o \in \mathcal{V}$ to a destination $d \in \mathcal{V}$ while making optional stops at a self-chosen transport hub $v_i \in \mathcal{V}$. So, on one hand, all travelers are associated with the same origin-destination pair (o, d). On the other hand, travelers can make a stop along their route. Next, each type of mobility services $h \in \mathcal{H}$ is associated with a sequence of edges, i.e., a route that connects at least two nodes (or transport hubs). We say that there exists a set of routes for each traveler i where each route connects their origin-destination pair (o, d) and can be traveled by any mobility service. Formally, we have $\mathcal{P}^{(o,d)} \subset 2^{\mathcal{E}}$ to denote the set of routes available to traveler i in origin-destination (o, d), where each route in $\mathcal{P}^{(o,d)}$ consists of a set of edges connecting o to d.

Each traveler $i \in \mathcal{I}$ seeks to travel in network $\mathcal{G}$ using one of the available mobility services $j \in \mathcal{J}$ of type $h \in \mathcal{H}$. Thus, any traveler can choose the type of mobility service they prefer for their specific travel needs. Note that any $j \in \mathcal{J}$ can use any edge. Thus, travelers compete with each other for the available services in the transportation network. For the purposes of this work, we restrict our attention to all available modes of transportation that use the road infrastructure. In addition, each transport hub (including the ones at (o, d) allows travel by any mobility service (any mode of transportation), thus a traveler's travel preferences or needs can be satisfied by many and different mobility services (as one expects from a multimodal transportation network).

Selfish behavior, however, may lead to inefficiencies. Therefore, as part of our efforts to *control* the inefficiencies that arise from the travelers' selfishness (and thus control the emergence of rebound effects), in our mobility game, we introduce the idea of a “mobility pricing mechanism” to incentivize travelers to use services in public transit for their travel needs. Informally, each transport hub starts with a budget, collects payments for services, and then provides monetary incentives (pricing mechanism) to travelers to ensure a *socially-efficient* utilization of services in the network. By “socially-efficient,” we mean that the endmost collective travel outcomes must achieve two objectives: (i) respect and satisfy the travelers’ preferences regarding mobility, and (ii) ensure the alleviation of congestion in the system. We formalize the idea of our mobility pricing mechanism in the following definitions.

Definition 4.1.3. *Each traveler i starts with a* mobility wallet *represented in monetary units by $\theta_i \in [0, \bar{\theta}_i]$, where $\bar{\theta}_i \in \mathbb{R}_{>0}$ is the maximum amount of traveler i's monetary units. Traveler i uses their wallet θ_i to pay for their travels in-network $\mathcal{G}$.*

The mobility wallet for each traveler represents the financial resources available to them, i.e., the amount of money they have to spend on transportation. In addition, by introducing mobility wallets, we can realistically model the travelers’ cost-constrained decision-making, where different transportation options have different costs.

Definition 4.1.4. *Any traveler i is required to pay a fee, called “mobility payment,” for using a mobility service in network $\mathcal{G}$. This mobility payment is given by function $\pi_i : \mathcal{H} \times \mathbb{N} \to \mathbb{R}_{\geq 0}$, where $0 \leq \pi_i(h_i, \varepsilon_j) \leq \theta_i$.*

Note that $\pi_i(h_i, \varepsilon_j)$ has the same monetary units as θ_i in Definition 4.1.3. Intuitively, a traveler i pays π_i for using mobility service j of type h_i. The mobility payment π_i of any traveler i varies extensively for each type of service h_i and increases fast as ε_j tends to $\bar{\varepsilon}_j$. To ensure our exposition is compact, we omit the arguments of $\pi_i(h_i, \varepsilon_j)$, and simply write π_i.

In mobility game $\mathcal{M}$, each traveler i pays for using a mobility service j of type h_i on route ρ_i with origin-destination pair (o, d) making an optional stop at transport hub v_i. At each transport hub, available funds can be offered to incentivize travelers to use public transit services. Our mobility game can be thought of as a static game that is played repeatedly [249], thus travelers are assumed to take different actions multiple times. Therefore, the pricing mechanism needs to consider both the payments of all travelers for each type of service and the available funds at each transport hub. We formalize this idea for the allocation of mobility payments for each traveler i by stating the following definition.

Definition 4.1.5. *Suppose traveler $i \in \mathcal{I}$ chooses route $\rho_i \in \mathcal{P}^{(o,d)}$ and makes a stop at transport hub v_i along that route using some service $j \in \mathcal{J}$ of type $h_i \in \mathcal{H}$. Then, the set of co-travelers at $v_i \in \mathcal{V}$ is $\mathcal{S}_{v_i} = \{k \in \mathcal{I} \mid v_k = v_i\}$.*

In words, $\mathcal{S}_{v_i}$ groups all travelers including traveler i who have made a stop at transport hub v_i. Next, we formally define the available budget at transport hub v_i.

Definition 4.1.6. *Let $b(v_i) \in \mathbb{R}$ be the number of funds available for transactions at traveler i's transport hub $v_i \in \mathcal{V} \setminus \{(o, d)\}$ over all types of services $h \in \mathcal{H}$.*

Intuitively, $b(v_i)$ represents the available funds (e.g., after covering all expenses), in the same monetary units as θ_i and π_i, that transport hub v_i may allocate to travelers. Practically, even though our proposed mobility game is not dynamic, $b(v_i)$ can be computed based on historical data (e.g., along similar lines presented in [14]), and thus capture the "demand" of services at a particular v_i. Each traveler $i \in \mathcal{I}$ starts with a mobility wallet θ_i and pays π_i while they make a stop at transport hub v_i. Note that we are interested in modeling the travelers' decision-making in regards to commuting with cars and public transportation. That is why, in our model, each traveler is required to pass through a specific vertex, such as a transfer station (e.g., business district). Our justification is that many public transportation trips involve transferring between different modes of transportation or commuters must pass through a specific locations as part of their daily trip.

We can capture the travelers' preferences of different outcomes using a "utility function." Travelers are expected to act as utility maximizers. Thus, we can influence the travelers' behavior by introducing a *control input* in the utility function. In our mobility game, we consider a *mobility pricing mechanism*, as a control input, that aims to reward or penalize each traveler i (either by increasing a traveler's utility or decreasing it). We offer here an informal description of our pricing mechanism. The total excess amount of mobility funds is $b(v_i) - \sum_{i \in \mathcal{S}_{v_i}} \pi_i$. The total excess amount of mobility funds at transport hub v_i excluding traveler i is $b(v_i) - \sum_{k \in \mathcal{S}_{v_i} \setminus \{i\}} \pi_k$. Given the available mobility funds already present at v_i, we can redistribute the "mobility wealth" based on the types of services and roads utilized by the travelers as follows: we consider a quadratic-based pricing mechanism $\tau(v_i, \pi_i)$, defined formally next, which is the same for all travelers. Under this pricing mechanism, we observe the following two interesting properties: for high values of π_i, τ is strictly decreasing; for low values of π_i, τ is strictly increasing. Thus, if traveler i pays a high payment π_i (e.g., which implies traveler i uses a car), then disincentive is also high to use this mobility service. Thus, this serves as an indirect incentive for a traveler to use public transit or a different transport hub (this becomes clear in (4.4)). Furthermore, the pricing mechanism τ can take negative values, and actually strictly decreases fast as π_i takes high values for any traveler i. So, travelers can get penalized if they choose a "high-demand" type of service (thus, leading to a high valued $\sum_{k \in \mathcal{S}_{v_i} \setminus \{i\}} \pi_k$). Even if a traveler has the means to pay (i.e., the traveler has a large mobility wallet), the pricing mechanism can penalize the traveler with hefty fees, thus all travelers have the incentive to minimize the penalties and choose public transit services or less congested transport hubs. For example, when a traveler uses a bike, their mobility payment will be low and so they can earn (instead of paying for the service). This incentivizes a sustainable allocation of services to all travelers. We offer the formal definition of the pricing mechanism next for the allocation of the mobility funds and payments.

Definition 4.1.7. *The pricing mechanism is a multivariable function* $\tau \mapsto \mathbb{R}$ *that depends on a traveler* i*'s transport hub* v_i *and mobility payment* π_i*, and is explicitly* *given by*

$$\tau(v_i, \pi_i) = \left(b(v_i) - \sum_{k \in \mathcal{S}_{v_i} \setminus \{i\}} \pi_k\right)^2 - \left(b(v_i) - \sum_{i \in \mathcal{S}_{v_i}} \pi_i\right)^2. \tag{4.1}$$

Recall that the term $b(v_i)$ captures the demand of a transport hub v_i based on what types of services in general travelers have been using (e.g., if a transport hub has a lot of money, it means travelers use cars significantly).

Remark 4.1.8. *If we expand (4.1) and simplify, we obtain the following relation*

$$\tau(v_i, \pi_i) = 2\pi_i \left(b(v_i) - \frac{\pi_i}{2} - \sum_{k \in \mathcal{S}_{v_i} \setminus \{i\}} \pi_k\right). \tag{4.2}$$

The behavior of (4.1) is rather interesting. Obviously, as traveler i's payment increases, then (4.1) decreases. However, given that $b(v_i) > \sum_{k \in \mathcal{S}_{v_i} \setminus \{i\}} \pi_k$, for small values of π_i, τ increases up to a maximum point, and then starts to decrease. This characteristic of (4.1) serves as a strong incentive for travelers to choose services that are "cheap" (bikes) or uncongested (buses) since then π_i will be small. Otherwise, τ can take very high negative values as π_i increases.

As long as $b(v_i)$ is higher than $\sum_{k \in \mathcal{S}_{v_i} \setminus \{i\}} \pi_k$, then the pricing mechanism (4.1) redistributes wealth back to each traveler i based on what is available on the self-chosen transport hub v_i and how much travelers pay by taking into consideration traveler i's contribution at transport hub v_i.

Since the travelers' objective is to maximize their payoff, we need a way to "incentivize" travelers to avoid decisions that may lead to an empty mobility wallet. Thus, we introduce an empty wallet "disincentive" for an arbitrary traveler i.

Definition 4.1.9. *Given the current amount of mobility wallet θ_i of any traveler i, the* disincentive *of having an empty wallet is a decreasing function $g : [0, \bar{\theta}_i] \to \mathbb{R}$ given by*

$$g(\theta_i) = \frac{\bar{\theta}_i}{\theta_i + \eta_i \pi_i}, \tag{4.3}$$

where $\eta_i \in (0, 1)$ is a socioeconomic characteristic of traveler i and affects the impact of how much they choose to spend or save in terms of their mobility wallet.

Definition 4.1.9 establishes mathematically a disincentive as a function where $\bar{\theta}_i$ is proportional to the sum of the current mobility wallet θ_i and the weighted mobility payment $\eta_i \pi_i$. We expect each traveler to avoid as much as possible an empty wallet; hence, (4.3) ensures to "penalize" travelers with a low mobility wallet θ_i while choosing to spend $\eta_i \pi_i \approx \theta_i$. Thus, (4.3) grows fast as θ_i decreases. We offer now the formal definition of a traveler's action set.

Definition 4.1.10. *For an arbitrary traveler $i \in \mathcal{I}$, the action set is $\mathcal{A}_i = \mathcal{P}^{(o,d)} \times \mathcal{V} \times \mathbb{R}_{\geq 0}$, where $\mathcal{P}^{(o,d)}$ is the set of routes that connects traveler i's origin-destination pair (o, d), $\mathcal{V}$ is the set of nodes in network $\mathcal{G}$ that includes all possible transports hubs v_i, and $\pi_i \in \mathbb{R}_{\geq 0}$ is the mobility payment of traveler i.*

Note that, by Definition 4.1.10, the action set $\mathcal{A}_i$ of an arbitrary traveler i is a coupled set with discrete values (route, transport hub, source/destination pair), and continuous values (mobility payment). Thus, the action profile $a_i \in \mathcal{A}_i$ of traveler $i \in \mathcal{I}$ is a vector of discrete and continuous values. We write $\mathcal{A} = \mathcal{A}_1 \times \mathcal{A}_2 \times \cdots \times \mathcal{A}_I$ for the Cartesian product of all the travelers' action sets. We write $a_{-i} = (a_1, a_2, \ldots, a_{i-1}, a_{i+1}, \ldots, a_I)$ for the action profile that excludes traveler $i \in \mathcal{I}$. Next, for the aggregate action profile, we write $a = (a_i, a_{-i})$, $a \in \mathcal{A}$. We also denote by a^*, a^{Nash} an action profile at a social optimum and at a NE, respectively.

Next, we introduce a travel time latency function to capture the congestion cost that travelers may experience.

Definition 4.1.11. *Let the total number of services of all types $h = 1, \ldots, H$ on road $e \in \mathcal{E}$ with at least one traveler be $J_e = \sum_{h \in \mathcal{H}} \omega_h |\mathcal{J}_{e,h}|$, where $\mathcal{J}_{h,e}$ is the set of all services on road e of type h, and $(\omega_h)_{h \in \mathcal{H}}$, $\omega_h \in [0, 1]$ are weight parameters that depend on the type h to capture the different impact of services on the traffic. Then,* *the travel time latency function is a strictly increasing linear function* $c_e : \mathbb{N} \to \mathbb{R}$*, with explicit form* $c_e(J_e) = \xi_1 J_e + \xi_2$*, where* ξ_1, ξ_2 *are constants.*

Notice that we assume linearity in the travel time latency functions c_e, which is not unique in the literature [202, 200, 71]. The justification behind linearity is that it is the simplest yet most useful way for a mathematical analysis to capture the travel costs in terms of distance or road capacity and the traffic congestion costs. The choice of the constants ξ_1 and ξ_2 play an important role, namely ξ_2 can represent the length of road e and ξ_1 normalizes the number of services on road e so that both components of c_e have the same units.

We can now formally define the utility of any traveler $i \in \mathcal{I}$.

Definition 4.1.12. *The utility* $u_i : \mathcal{A} \to \mathbb{R}$ *of traveler* $i \in \mathcal{I}$ *is what traveler* i *receives under the risk-neutral setting given by*

$$u_i(a) = \tau(v_i, \pi_i) - g(\theta_i) - \zeta_1 \left(\sum_{e \in \rho_i : \rho_i \in \mathcal{P}^{(o,d)}} c_e(J_e) \right) - \zeta_2 \left(\frac{|\mathcal{S}_{v_i}|}{\sigma(v_i, h_i)} \right), \tag{4.4}$$

where $\sigma(v_i, h_i)$ *is the rate of travel service at transport hub* v_i *for type of service* h_i *(how many travelers per hour can travel using type of service* h_i *from transport hub* v_i*), and* $\zeta_1, \zeta_2 \in \mathbb{R}$ *are unit parameters that transform time to money (that way the units of* (4.4) *are consistent).*

Note that both constants ζ_1, ζ_2 get absorbed by the constants of function c_e (as defined in Definition 4.1.11) and parameter σ, respectively. So, we can safely omit them from the mathematical analysis. In (4.4), the first term represents the pricing mechanism and is the amount of mobility funds redistributed to traveler i for paying π_i. The second term is the disincentive as defined in Definition 4.1.9, and the third term is the congestion cost of traveler i due to traffic on road e. Finally, the last term in (4.4) is a waiting cost for joining transport hub v_i, where the number of travelers at transport hub v_i is proportional to the rate of travel service at transport hub v_i.

Next, we characterize fully and formally the mobility game.

Definition 4.1.13. *The mobility game is fully characterized by the tuple*

$$\mathcal{M} = \langle \mathcal{I}, \mathcal{J}, (\mathcal{A}_i)_{i\in\mathcal{I}}, (u_i)_{i\in\mathcal{I}} \rangle, \tag{4.5}$$

a collection of sets of travelers, mobility services, actions, and a profile of utilities.

The mobility game $\mathcal{M}$ is a non-cooperative repeated routing game with a multimodal transportation network and coupled action sets. Travelers have a travel-mode choice to make that will satisfy their travel needs. The benefit of ensuring that our mobility game $\mathcal{M}$ is a repeated game is that it eliminates the possibility of unassigned travelers. At this point also, we clarify the *information structure* of the mobility game $\mathcal{M}$ ("who knows what?" [135]). All travelers have complete knowledge of the mobility system (network, travel time latencies, waiting costs, and utility functions). Each traveler knows their own information (action and utility) as well as the information of other travelers. For the purposes of our work, we observe that a NE is most fitting to apply as a solution concept as it requires complete information.

Before we continue to the analysis of our mobility game $\mathcal{M}$, we summarize the notation that has been introduced so far in our paper.

Analysis and Properties

In this section, our goal is to establish the existence of at least one NE in the mobility game $\mathcal{M}$, derive an upper bound for the PoA, and perform a prospect theory analysis.

We start our exposition by summarizing two necessary preliminary concepts and results of game theory that we use throughout the paper.

Definition 4.1.14. *A game $\mathcal{M}$ is an* exact potential game *if there exists a potential function $\Phi : \mathcal{A} \to \mathbb{R}$ such that for all $i \in \mathcal{I}$, for all a_{-i}, and for all $a_i, a_i' \in \mathcal{A}_i$, we have*

$$\Phi(a_i, a_{-i}) - \Phi(a_i', a_{-i}) = u_i(a_i, a_{-i}) - u_i(a_i', a_{-i}). \tag{4.6}$$

Table 4.1: A summary of our notation.

Symbol	Description
$\mathcal{I}$	Set of travelers
$\mathcal{J}$	Set of mobility services
$\mathcal{J}_h$	Set of mobility services of type h
$\mathcal{H}$	Set of different types of services
ε_j	Physical traveler capacity for service $j \in \mathcal{J}$
$\bar{\varepsilon}_j$	Maximum traveler capacity of service $j \in \mathcal{J}$
$\mathcal{G}$	Network with set of edges $\mathcal{E}$ and set of nodes $\mathcal{V}$
v_i	Node in network $\mathcal{G}$ that represents a transport hub
$\mathcal{P}^{(o,d)}$	Set of routes that connect the origin o to destination d
θ_i	Mobility wallet for traveler $i \in \mathcal{I}$
$\bar{\theta}_i$	Maximum amount of travelers i's monetary units for the wallet θ_i
π_i	Mobility payment of traveler $i \in \mathcal{I}$
ρ_i	Route chosen by traveler $i \in \mathcal{I}$
$\mathcal{S}_{v_i}$	Set of co-travelers at transport hub v_i for traveler $i \in \mathcal{I}$
$b(v_i)$	Amount of funds available for transactions at transport hub for traveler $i \in \mathcal{I}$
τ	Pricing mechanism
g	Empty wallet disincentive
η_i	Socioeconomic characteristic of traveler $i \in \mathcal{I}$
$\mathcal{A}_i$	Set of actions for traveler $i \in \mathcal{I}$
$\mathcal{A}$	Cartesian product of all action sets
J_e	Total number of services of all types on road $e \in \mathcal{E}$
c_e	Travel time latency function
σ	Rate of travel service

Definition 4.1.15. *An action profile* $a^{Nash} = (a_i^{Nash}, a_{-i}^{Nash})$ *is called a pure-strategy* Nash equilibrium *for game* $\mathcal{M}$ *if, for all* $i \in \mathcal{I}$, *we have* $u_i(a_i^{Nash}, a_{-i}^{Nash}) \geq u_i(a_i', a_{-i}^{Nash})$, *for all* $a_i' \in \mathcal{A}_i$.

The potential function Φ is a useful tool in showing whether a game has a NE and analyzing its properties. This is because, by construction, the effect on the utility of any traveler's action is expressed by one function common for all travelers.

4.1.2 Existence of a Nash Equilibrium

In this subsection, we prove that for the mobility game $\mathcal{M}$, as defined in Definition 4.1.13 (please also see Table 4.1), there exists at least one NE. The key idea of our proof is the use of a potential function, as defined in Definition 4.1.14, that captures the changes in utility of an arbitrary traveler that deviates in their action.

Theorem 4.1.16. *The mobility game $\mathcal{M}$ admits a pure-strategy NE.*

Proof. As it is standard in the existence of NE results for potential games (see Chapter 2 [126]), we start by stating the explicit form of the potential function for the mobility game $\mathcal{M}$, i.e.,

$$\Phi(a) = \sum_{v\in\mathcal{V}} \left(b(v) - \sum_{i\in\mathcal{S}_v} \pi_i \right)^2 - \sum_{i\in\mathcal{I}} \frac{\bar{\theta}_i}{\theta_i + \eta_i \pi_i} - \sum_{e\in\mathcal{E}} \sum_{k=1}^{J_e} c_e(k) - \sum_{v\in\mathcal{V}} \sum_{i\in\mathcal{I}} \frac{|\mathcal{S}_v|(|\mathcal{S}_v|+1)}{2\sigma(v,h_i)}, \tag{4.7}$$

Our goal now is to verify Definition 4.1.14, thus showing that $\mathcal{M}$ is a potential game. Mathematically, for an arbitrary traveler i and for any two actions $a_i = (\rho_i \in \mathcal{P}^{(o,d)}, v_i, \pi_i)$ and $a'_i = (\rho'_i \in \mathcal{P}^{(o,d)}, v'_i, \pi'_i)$, we need to show

$$\Phi(a_i, a_{-i}) - \Phi(a'_i, a_{-i}) = u_i(a_i, a_{-i}) - u_i(a'_i, a_{-i}). \tag{4.8}$$

Note here that any traveler i that deviates in their action a_i to a'_i by changing their route ρ_i to ρ'_i that does not require an additional service j on any new roads $e \in \rho'_i$, then traveler i's impact to congestion is negligent. Thus, traveler i can change routes and still travel along an existing service j in road $e \in \rho'_i$ if that service j has not reached its maximum capacity $\bar{\varepsilon}_j$. If traveler i changes their route from ρ_i to ρ'_i and the traveler capacities of all services on that route are not maxed out, then the number of services J_e does not change in the roads that remain the same along both routes (any $e \in \rho_i \cap \rho'_i$). However, the number of services J_e increases by one in any road $e \in \rho'_i \setminus \rho_i$ since we require an additional service j in road e for traveler i. Thus, we have $\sum_{e\in\rho'_i} c_e(J_e) = \sum_{e\in\rho'_i\cap\rho_i} c_e(J_e) + \sum_{e\in\rho'_i\setminus\rho_i} c_e(J_e+1)$.

If traveler i changes their transport hub v_i to v'_i, then their new waiting cost is $\frac{|\mathcal{S}_{v'_i}|+1}{\sigma(v'_i,h)}$, where $\mathcal{S}_{v'_i} = \{k \in \mathcal{I} \setminus \{i\} \mid v_k = v'_i\}$. We make a similar argument for π_i and π'_i, and thus, we can write

$$u_i(a'_i, a_{-i}) = \left(b(v'_i) - \sum_{k\in\mathcal{S}_{v'_i}\setminus\{i\}} \pi_k\right)^2 - \left(b(v'_i) - \pi'_i - \sum_{k\in\mathcal{S}_{v'_i}\setminus\{i\}} \pi_k\right)^2 \\ - \frac{\bar{\theta}_i}{\theta_i + \eta_i\pi'_i} - \frac{|\mathcal{S}_{v'_i}|}{\sigma(v'_i, h_i)} - \left[\sum_{e\in\rho'_i\cap\rho_i} c_e(J_e) + \sum_{e\in\rho'_i\setminus\rho_i} c_e(J_e+1)\right], \quad (4.9)$$

where the first component represents the squared remaining cost that traveler i has to pay for deviating to the alternative transport hub v'_i without taking into account their π'_i. Thus, it accounts for the base cost of the deviation in the transport hub and mobility payment and the sum of prices paid by other the travelers. The second component represents the squared remaining cost that traveler i has to pay for the alternative transport hub v'_i when actually they do consider their π'_i, along with the sum of prices paid by other travelers. We note here that the difference between these two terms highlights the impact of traveler i's price π'_i and v'_i on their u_i when they deviate and choose an alternative action. If traveler i chooses a higher payment, the remaining cost decreases, and this difference will have a negative impact on their utility. Conversely, if traveler i chooses a lower payment, the remaining cost increases, and this difference will have a positive impact on their utility. Next, we subtract (4.9) from (4.4) to get

$$u_i(a_i,a_{-i}) - u_i(a_i',a_{-i}) = \left(b(v_i) - \sum_{k\in\mathcal{S}_{v_i}\setminus\{i\}} \pi_k\right)^2 - \left(b(v_i) - \sum_{i\in\mathcal{S}_{v_i}} \pi_i\right)^2$$
$$- \left(b(v_i') - \sum_{k\in\mathcal{S}_{v_i'}\setminus\{i\}} \pi_k\right)^2 + \left(b(v_i') - \pi_i' - \sum_{k\in\mathcal{S}_{v_i'}\setminus\{i\}} \pi_k\right)^2 - \sum_{e\in\rho_i\setminus\rho_i'} c_e(J_e)$$
$$+ \sum_{e\in\rho_i'\setminus\rho_i} c_e(J_e+1) - \frac{|\mathcal{S}_{v_i}|}{\sigma(v_i,h_i)} + \frac{|\mathcal{S}_{v_i'}|+1}{\sigma(v_i',h_i)} + \frac{\eta_i\bar{\theta}_i(\pi_i-\pi_i')}{(\theta_i+\eta_i\pi_i)(\theta_i+\eta_i\pi_i')}, \quad (4.10)$$

where $\sum_{e\in\rho_i\setminus\rho_i'} c_e(J_e) = \sum_{e\in\rho_i} c_e(J_e) - \sum_{e\in\rho_i'\cap\rho_i} c_e(J_e)$. Now, we denote all four components of (4.7) as follows: $\phi_1 = -\sum_{v\in\mathcal{V}} \left(b(v) - \sum_{i\in\mathcal{S}_v} \pi_i\right)^2$, $\phi_2 = -\sum_{i\in\mathcal{I}} \frac{\bar{\theta}_i}{\theta_i+\eta_i\pi_i}$, $\phi_3 = -\sum_{e\in\mathcal{E}} \sum_{k=1}^{J_e} c_e(k)$, and $\phi_4 = -\sum_{v\in\mathcal{V}} \sum_{i\in\mathcal{I}} \frac{|\mathcal{S}_v|(|\mathcal{S}_v|+1)}{2\sigma(v,h_i)}$. Step by step, we compute the difference of all four different ϕ's under a_i and a_i' as follows

$$\phi_1(a_i,a_{-i}) - \phi_1(a_i',a_{-i}) = \left(b(v_i') + \sum_{i\in\mathcal{S}_{v_i'}} \pi_i'\right)^2 - \sum_{v\in\mathcal{V}} \left(b(v) - \sum_{i\in\mathcal{S}_v} \pi_i\right)^2$$
$$- \sum_{v\in\mathcal{V}\setminus\{v_i'\}} \left(b(v) - \sum_{i\in\mathcal{S}_v} \pi_i\right)^2 = \left(b(v_i) - \sum_{k\in\mathcal{S}_{v_i}\setminus\{i\}} \pi_k\right)^2 - \left(b(v_i) - \sum_{i\in\mathcal{S}_{v_i}} \pi_i\right)^2$$
$$- \left(b(v_i') - \sum_{k\in\mathcal{S}_{v_i'}\setminus\{i\}} \pi_k\right)^2 + \left(b(v_i') - \pi_i' - \sum_{k\in\mathcal{S}_{v_i'}\setminus\{i\}} \pi_k\right)^2, \quad (4.11)$$

where we note that in $\sum_{v\in\mathcal{V}}$ both v_i and v_i' are included, so if we expand the summations that involve the unrelated nodes in $\mathcal{V}$, then most terms cancel out (what remains are only the terms that involve the deviations of traveler i, thus we get (4.11)). Next, we have

$$\phi_2(a_i,a_{-i}) - \phi_2(a_i',a_{-i}) = -\sum_{i\in\mathcal{I}} \frac{\bar{\theta}_i}{\theta_i+\eta_i\pi_i} + \left[\sum_{k\in\mathcal{I}\setminus\{i\}} \frac{\bar{\theta}_k}{\theta_k+\eta_k\pi_k} + \frac{\bar{\theta}_i}{\theta_i+\eta_i\pi_i'}\right]$$
$$= \sum_{i\in\mathcal{I}} \frac{\eta_i\bar{\theta}_i(\pi_i-\pi_i')}{(\theta_i+\eta_i\pi_i)(\theta_i+\eta_i\pi_i')}. \quad (4.12)$$

$$\phi_3(a_i, a_{-i}) - \phi_3(a'_i, a_{-i}) = -\sum_{e\in\mathcal{E}}\sum_{k=1}^{J_e} c_e(k) + \sum_{e\in\mathcal{E}\setminus\{e\in\rho'_i\}}\sum_{k=1}^{J_e-1} c_e(k) + \sum_{e\in\mathcal{E}\setminus\{e\in\rho_i\}}\sum_{k=1}^{J_e+1} c_e(J_e)$$
$$= -\sum_{e\in\rho_i\setminus\rho'_i} c_e(J_e) + \sum_{e\in\rho'_i\setminus\rho_i} c_e(J_e+1), \tag{4.13}$$

$$\phi_4(a_i, a_{-i}) - \phi_4(a'_i, a_{-i}) = \sum_{v\in\mathcal{V}}\sum_{i\in\mathcal{I}} \frac{|\mathcal{S}_v|(|\mathcal{S}_v|+1)}{2\sigma(v,h_i)} - \sum_{v\in\mathcal{V}\setminus\{v_i\}\cup\{v'_i\}}\sum_{k\in\mathcal{I}\setminus\{i\}}\left[\frac{|\mathcal{S}_v|(|\mathcal{S}_v|+1)}{2\sigma(v,h_k)}\right]$$
$$- \frac{|\mathcal{S}_{v_i}|(|\mathcal{S}_{v_i}|-1)}{2\sigma(v_i,h_i)} - \frac{(|\mathcal{S}_{v'_i}|+1)(|\mathcal{S}_{v'_i}|+2)}{2\sigma(v'_i,h_i)} = \frac{|\mathcal{S}_{v_i}|}{\sigma(v_i,h_i)} - \frac{|\mathcal{S}_{v'_i}|+1}{\sigma(v'_i,h_i)}, \tag{4.14}$$

We define $\Phi(a_i, a_{-i}) - \Phi(a'_i, a_{-i}) = \sum_{k=1}^{4}[\phi_k(a_i, a_{-i}) - \phi_k(a'_i, a_{-i})]$. We take the sum of (4.11) - (4.14) and thus, we obtain $\Phi(a_i, a_{-i}) - \Phi(a'_i, a_{-i}) = u_i(a_i, a_{-i}) - u_i(a'_i, a_{-i})$. This proves that the mobility game $\mathcal{M}$ is a potential game, and so following from key results (see [126]), we conclude that $\mathcal{M}$ admits a pure-strategy NE. □

Note that this existence result is not straightforward as the action set of any traveler is a coupled set composed of countable and uncountable subsets.

Corollary 4.1.17. *If the mobility game $\mathcal{M}$ is played repeatedly, then the travelers' actions converge to a pure-strategy NE in finite time.*

Proof. We aim to show that if the mobility game $\mathcal{M}$ is played repeatedly, the travelers' actions will converge to a pure-strategy NE in finite time. This proof relies on Theorem 4.1.16 and Theorem 2.6 (pp. 33) from [126]. The mobility game $\mathcal{M}$ can be classified as a repeated routing game with complete information since each traveler $i \in \mathcal{I}$ has full knowledge of the mobility system (travel time latencies, network congestion, and other relevant parameters) when making their decisions. We demonstrate convergence to a pure-strategy NE by considering the decision-making process of the travelers; so we outline the iterative process: (i) Traveler $i \in \mathcal{I}$ chooses their initial action a_i based on the current state of the mobility system. (ii) After observing traveler i's action, all other travelers $k \in \mathcal{I}, k \neq i$ choose their actions a_k accordingly, taking into account the updated state of the mobility system. (iii) The mobility system's state is updated again, reflecting the actions of all travelers in the current round. (iv) Traveler i now evaluates their action a_i and decides whether to deviate to a'_i based on the updated state of the mobility system. (v) These steps are repeated for all travelers until no traveler has an incentive to change their action, leading to a convergence to a pure-strategy NE. we observe that travelers compete and as a result, each traveler's actions are influenced by the actions of others. So, continuous deviations eventually will lead to a pure strategy NE by an iterative process (see Theorem 2.6 from [126]). In conclusion, by showing that the mobility game $\mathcal{M}$ is a repeated routing game with complete information and that the iterative decision-making process of the travelers leads to a stable equilibrium, we have demonstrated that the travelers' actions will converge to a pure-strategy NE in finite time. □

4.1.3 Price of Anarchy and Stability Analysis

An existence result (Theorem 4.1.16) leads to the problem of multiple NE and raises questions about the efficiency of each equilibrium. For example, an important concern is the efficiency of the equilibrium that the travelers will reach (as it is guaranteed by Corollary 4.1.17). To address this concern, we provide an analysis based on the *Price of Anarchy* (PoA) [122], which is one of the most widely-used metrics to measure the inefficiency in a system and provides an understanding of how the travelers' decision-making affect the overall performance of the system. We provide the formal definition of the PoA.

Definition 4.1.18. *Let the* social welfare *of the mobility game* $\mathcal{M}$ *be represented by* $J(a) = \sum_{i \in \mathcal{I}} u_i(a)$. *Then, the PoA is the ratio of the maximum optimal social welfare over the minimum social welfare at a NE, i.e.,*

$$PoA = \frac{\max_{a \in \mathcal{A}} \sum_{i \in \mathcal{I}} u_i(a)}{\min_{a \in \mathcal{A}^{Nash}} \sum_{i \in \mathcal{I}} u_i(a)} \geq 1, \tag{4.15}$$

where $\mathcal{A}^{Nash}$ *is the set of NE, which is guaranteed to be non-empty according to Theorem 4.1.16.*

Next, we show that, for the mobility game $\mathcal{M}$, (4.15) is as low as possible at an arbitrary NE. Thus, it follows that our PoA result yields an upper bound for the inefficiencies at a NE of the mobility game $\mathcal{M}$.

But first, we prove two useful lemmata that are necessary for our work.

Lemma 4.1.19. *Let $a_i^* = (\rho_i^*, v_i^*, \pi_i^*)$ denote the optimal action of traveler $i \in \mathcal{I}$, define $\tilde{b}^2 = \sum_{v\in\mathcal{V}} \left(\sum_{h\in\mathcal{H}} b(v,h)\right)^2 = \sum_{v\in\mathcal{V}} (b(v))^2$, and at a NE:*

$$J_3(a) = \sum_{i\in\mathcal{I}} \Bigg[\Bigg(b(v_i^*) - \sum_{i\in\mathcal{S}_{v_i^*}} \pi_i\Bigg)^2 - \Bigg(b(v_i^*) - \pi_i^* - \sum_{k\in\mathcal{S}_{v_i^*}\setminus\{i\}} \pi_k\Bigg)^2 - \frac{\bar{\theta}_i}{\theta_i + \eta_i \pi_i^*}\Bigg]. \quad (4.16)$$

Then, we have

$$\begin{aligned} \sum_{i\in\mathcal{I}} \Bigg[\Bigg(b(v_i^*) - \sum_{k\in\mathcal{S}_{v_i^*}\setminus\{i\}} \pi_k\Bigg)^2 - \Bigg(b(v_i^*) - \pi_i^* - \sum_{k\in\mathcal{S}_{v_i^*}\setminus\{i\}} \pi_k\Bigg)^2 - \frac{\bar{\theta}_i}{\theta_i + \eta_i \pi_i^*}\Bigg] \\ \leq J_3(a^*) - \sqrt{(\tilde{b}^2 + 2(J_3(a^{Nash}) - I\bar{\theta}_i))(\tilde{b}^2 + 2(J_3(a^*) - I\bar{\theta}_i))} - 4I\bar{\theta}_i - \tilde{b}^2 \\ - \tilde{b}\left(\sqrt{\tilde{b}^2 + 2(J_3(a^{Nash}) - I\bar{\theta}_i)} + \sqrt{\tilde{b}^2 + 2(J_3(a^*) - I\bar{\theta}_i)}\right). \quad (4.17) \end{aligned}$$

Proof. At social optimum, the pricing mechanism is given by

$$\begin{aligned} \tau^*(v_i^*, \pi_i^*) &= \Bigg(b(v_i^*) - \sum_{k\in\mathcal{S}^*_{v_i^*}\setminus\{i\}} \pi_k^*\Bigg)^2 - \Bigg(b(v_i^*) - \sum_{i\in\mathcal{S}^*_{v_i^*}} \pi_i^*\Bigg)^2 \\ &= \Bigg(b(v_i^*) - \sum_{k\in\mathcal{S}^*_{v_i^*}\setminus\{i\}} \pi_k^*\Bigg)^2 - \Bigg(b(v_i^*) - \pi_i^* - \sum_{k\in\mathcal{S}^*_{v_i^*}\setminus\{i\}} \pi_k^*\Bigg)^2 \\ &= 2\pi_i^* b(v_i^*) - (\pi_i^*)^2 - 2\pi_i^* \sum_{k\in\mathcal{S}^*_{v_i^*}\setminus\{i\}} \pi_k^*. \quad (4.18) \end{aligned}$$

Summing over all travelers now gives

$$\sum_{i\in\mathcal{I}}\left[2\pi_i^*b(v_i^*)-(\pi_i^*)^2-2\pi_i^*\sum_{k\in\mathcal{S}^*_{v_i^*}\setminus\{i\}}\pi_k^*\right]$$
$$=2\sum_{i\in\mathcal{I}}\pi_i^*b(v_i^*)-\sum_{v\in\mathcal{V}}\left(2\left(\sum_{i\in\mathcal{S}^*_{v^*}}\pi_i^*\right)^2\right)+\sum_{i\in\mathcal{I}}(\pi_i^*)^2$$
$$=2\sum_{v\in\mathcal{V}}b(v)\sum_{i\in\mathcal{S}^*_{v_i^*}}\pi_i^*-\sum_{v\in\mathcal{V}}\left(2\left(\sum_{i\in\mathcal{S}^*_{v^*}}\pi_i^*\right)^2\right)+\sum_{i\in\mathcal{I}}(\pi_i^*)^2. \quad (4.19)$$

So, we use the Cauchy-Schwarz inequality to bound (4.19), i.e.,

$$J_3(a^*)-\sum_{i\in\mathcal{I}}\frac{\bar{\theta}_i}{\theta_i+\eta_i\pi_i^*}\leq 2\sqrt{\sum_{v\in\mathcal{V}}(b(v))^2\sum_{v\in\mathcal{V}}\left(\sum_{i\in\mathcal{S}^*_{v^*}}\pi_i^*\right)^2}-2\sum_{v\in\mathcal{V}}\left(\sum_{i\in\mathcal{S}^*_{v^*}}\pi_i^*\right)^2+\sum_{i\in\mathcal{I}}(\pi_i^*)^2. \quad (4.20)$$

For any traveler i, it is always true that $\frac{\bar{\theta}_i}{\theta_i+\eta_i\pi_i^*}>0$, $\pi_i^*\in[0,\bar{\theta}_i]$, and also $\tilde{b}^2=\sum_{v\in\mathcal{V}}(b(v))^2$. Thus, (4.20) simplifies to

$$\sum_{v\in\mathcal{V}}\left(\sum_{i\in\mathcal{S}^*_{v^*}}\pi_i^*\right)^2-\tilde{b}\sqrt{\sum_{v\in\mathcal{V}}\left(\sum_{i\in\mathcal{S}^*_{v^*}}\pi_i^*\right)^2}-\frac{1}{2}\left(J_3(a^*)-I\cdot\bar{\theta}_i\right)\leq 0, \quad (4.21)$$

Note that (4.21) is a second-order polynomial with respect to $\sqrt{\sum_{v\in\mathcal{V}}\left(\sum_{i\in\mathcal{S}^*_{v^*}}\pi_i^*\right)^2}$. Thus, we compute the discriminant $\Delta^*=\tilde{b}^2+2\left(J_3(a^*)-I\cdot\bar{\theta}_i\right)$, where Δ^* denotes the discriminant at the social optimum, so clearly $\Delta^*\geq 0$. So, from (4.21), we get

$$2\sqrt{\sum_{v\in\mathcal{V}}\left(\sum_{i\in\mathcal{S}^*_{v^*}}\pi_i^*\right)^2}\leq\tilde{b}+\sqrt{\Delta^*}. \quad (4.22)$$

We need to follow the same steps to obtain a similar relation as (4.22) for a NE. Hence, we have

$$2\sqrt{\sum_{v\in\mathcal{V}}\left(\sum_{k\in\mathcal{S}_v}\pi_k\right)^2}\leq\tilde{b}+\sqrt{\Delta}, \quad (4.23)$$

where $\Delta = \tilde{b}^2 + 2\left(J_3(a^{\text{Nash}}) - I \cdot \bar{\theta}_i\right)$. Next, the LHS of (4.17), if expanded, can be simplified as follows:

$$\sum_{i \in \mathcal{I}} \left[\left(b(v_i^*) - \sum_{k \in \mathcal{S}_{v_i^*} \setminus \{i\}} \pi_k \right)^2 - \left(b(v_i^*) - \pi_i^* - \sum_{k \in \mathcal{S}_{v_i^*} \setminus \{i\}} \pi_k \right)^2 - \frac{\bar{\theta}_i}{\theta_i + \eta_i \pi_i^*} \right]$$

$$= 2 \sum_{i \in \mathcal{I}} \left(b(v_i^*) \pi_i^* - \pi_i^* \sum_{k \in \mathcal{S}_{v_i^*} \setminus \{i\}} \pi_k - \frac{1}{2} (\pi_i^*)^2 \right) - \sum_{i \in \mathcal{I}} \frac{\bar{\theta}_i}{\theta_i + \eta_i \pi_i^*}. \quad (4.24)$$

$$J_3(a^*) - 2 \sum_{i \in \mathcal{I}} \pi_i^* \sum_{k \in \mathcal{S}_{v_i^*} \setminus \{i\}} \pi_k + 2 \sum_{i \in \mathcal{I}} \pi_i^* \sum_{k \in \mathcal{S}^*_{v_i^*} \setminus \{i\}} \pi_k^*$$

$$= J_3(a^*) - 2 \sum_{i \in \mathcal{I}} \pi_i^* \left[\sum_{k \in \mathcal{S}_{v_i^*} \setminus \{i\}} \pi_k - \sum_{k \in \mathcal{S}^*_{v_i^*} \setminus \{i\}} \pi_k^* \right]$$

$$= J_3(a^*) - 2 \sum_{i \in \mathcal{I}} \pi_i^* \left(\sum_{k \in \mathcal{S}_{v_i^*}} \pi_k^* - \pi_i(v_i^*) - \sum_{k \in \mathcal{S}^*_{v_i^*}} \pi_k^* + \pi_i^* \right), \quad (4.25)$$

where $\pi_i(v_i^*)$ denotes traveler i's payment at an optimal v_i^*, and thus, (4.25) can be simplified by noting that

$$2 \sum_{i \in \mathcal{I}} \pi_i^* \left(\sum_{k \in \mathcal{S}_{v_i^*}} \pi_k^* - \pi_i(v_i^*) - \sum_{k \in \mathcal{S}^*_{v_i^*}} \pi_k^* + \pi_i^* \right)$$

$$= 2 \sum_{i \in \mathcal{I}} \left[(\pi_i^*)^2 - \pi_i^* \pi_i(v_i^*) \right] + 2 \sum_{v \in \mathcal{V}} \sum_{k \in \mathcal{S}^*_{v^*}} \pi_k^* \sum_{k \in \mathcal{S}_v} \pi_k - 2 \sum_{v \in \mathcal{V}} \left(\sum_{k \in \mathcal{S}^*_{v^*}} \pi_k^* \right)^2, \quad (4.26)$$

Using (4.26), we impose an upper bound to (4.25) as follows

$$J_3(a^*) - 2 \sum_{i \in \mathcal{I}} \pi_i^* \left(\sum_{k \in \mathcal{S}_{v_i^*}} \pi_k^* - \pi_i(v_i^*) - \sum_{k \in \mathcal{S}^*_{v_i^*}} \pi_k^* + \pi_i^* \right)$$

$$\leq J_3(a^*) - 4 I \bar{\theta}_i - 2 \sum_{v \in \mathcal{V}} \left[\sum_{k \in \mathcal{S}^*_{v^*}} \pi_k^* \sum_{k \in \mathcal{S}_v} \pi_k \right]. \quad (4.27)$$

We continue by upper bounding the summations in (4.27):

$$\sum_{v\in\mathcal{V}}\left[\sum_{k\in\mathcal{S}^*_{v^*}}\pi^*_k\sum_{k\in\mathcal{S}_v}\pi_k\right]\le\sqrt{\sum_{v\in\mathcal{V}}\left(\sum_{k\in\mathcal{S}^*_{v^*}}\pi^*_k\right)^2\sum_{v\in\mathcal{V}}\left(\sum_{k\in\mathcal{S}_v}\pi_k\right)^2}$$
$$\le\frac{1}{2}\left(\sqrt{\tilde{b}^2}+\sqrt{\Delta}\right)\left(\sqrt{\tilde{b}^2}+\sqrt{\Delta^*}\right)$$
$$=\frac{1}{2}\sqrt{\Delta\Delta^*}+\frac{\tilde{b}}{2}\left(\sqrt{\Delta}+\sqrt{\Delta^*}\right)+\frac{\tilde{b}^2}{2},\tag{4.28}$$

by the Cauchy-Schwartz inequality and relations (4.22) and (4.23). Finally, we substitute $\Delta=\tilde{b}^2+2(J_3(a^{\text{Nash}})-I\bar{\theta}_i)$ and $\Delta^*=\tilde{b}^2+2(J_3(a^*)-I\bar{\theta}_i)$ into (4.28) and with (4.27) we obtain the desired bound. □

Lemma 4.1.20. *We have*

$$\frac{J(a^*)}{J(a^{Nash})}\le 1-\sqrt{\left(\frac{\tilde{b}^2}{I}+\frac{2J(a^*)}{J(a^{Nash})}\right)\left(\frac{\tilde{b}^2}{I}+2\right)}-\frac{\tilde{b}^2}{I}-4\bar{\theta}_i$$
$$-\frac{\tilde{b}}{\sqrt{I}}\left(\sqrt{\frac{\tilde{b}^2}{I}+\frac{2J(a^*)}{J(a^{Nash})}}+\sqrt{\frac{\tilde{b}^2}{I}+2}\right).\tag{4.29}$$

Proof. We can show this result by expanding and rearranging (4.29) to obtain a simplified relation. So, we have

$$\frac{J(a^*)}{J(a^{\text{Nash}})}-\left(1+\frac{\tilde{b}^2}{I}-4\bar{\theta}_i\right)+\frac{\tilde{b}}{\sqrt{I}}\sqrt{\frac{\tilde{b}^2}{I}+2}\le-\left(\sqrt{\frac{\tilde{b}^2}{I}+2}+\frac{\tilde{b}}{\sqrt{I}}\right)\sqrt{\frac{\tilde{b}^2}{I}+\frac{2J(a^*)}{J(a^{\text{Nash}})}}.\tag{4.30}$$

We seek to solve for $\frac{J(a^*)}{J(a^{\text{Nash}})}$, so we remove the square roots by squaring both sides of (4.30), i.e.,

$$\frac{J(a^*)}{J(a^{\text{Nash}})}-2(D+E^2)\frac{J(a^*)}{J(a^{\text{Nash}})}+\left(D^2-\frac{\tilde{b}^2E^2}{I}\right)\le 0,\tag{4.31}$$

where $E=\sqrt{\frac{\tilde{b}^2}{I}+2}+\frac{\tilde{b}}{\sqrt{I}}$ and $D=\left(1+\frac{\tilde{b}^2}{I}-4\bar{\theta}_i\right)-\frac{\tilde{b}}{\sqrt{I}}\sqrt{\frac{\tilde{b}^2}{I}+2}$. We solve (4.31) by noting the positivity of the coefficients to obtain

$$\frac{J(a^*)}{J(a^{\text{Nash}})}\le E^2+D+E\sqrt{E^2+2D+\frac{\tilde{b}^2}{I}}.\tag{4.32}$$

We observe that $E^2 + 2D + \frac{\tilde{b}^2}{I} \le \left(E + \frac{D}{E} + \frac{\tilde{b}^2}{2EI}\right)^2$, and so, an upper bound exists for (4.32). Thus, we have $\frac{J(a^*)}{J(a^{\text{Nash}})} \le 2E^2 + 2D + \frac{\tilde{b}^2}{2I}$. We substitute back both E and D and get

$$\begin{aligned}\frac{J(a^*)}{J(a^{\text{Nash}})} &\le 2 + \frac{3\tilde{b}^2}{2I} + \frac{3\tilde{b}}{\sqrt{I}}\sqrt{\frac{\tilde{b}^2}{I} + 2} + \frac{5\tilde{b}^2}{2I} \\ &= 2 + \frac{4\tilde{b}^2}{I} + \frac{\tilde{b}}{\sqrt{I}}\sqrt{\frac{\tilde{b}^2}{I} + 2}. \end{aligned} \tag{4.33}$$

Hence, since $\frac{\tilde{b}^2}{I} + 2 \le \left(\frac{\tilde{b}}{\sqrt{I}} + \frac{\sqrt{I}}{\tilde{b}}\right)^2$, the result follows. □

We are ready now to state and prove our PoA result.

Theorem 4.1.21. *Any inefficiencies of any NE of the mobility game $\mathcal{M}$ remain low as close to a constant as the number of travelers $|\mathcal{I}| = I$ tends to infinity. Mathematically, we have*

$$PoA \le 2 + \frac{5}{I}\sum_{v\in\mathcal{V}}\left(\sum_{h\in\mathcal{H}} b(v,h)\right)^2. \tag{4.34}$$

Proof. From Definition 4.1.15, for some arbitrary traveler $i \in \mathcal{I}$, it is clear that $u_i(a_i^{\text{Nash}}, a_{-i}) \le u_i(a_i^*, a_{-i})$, so if we expand the RHS of it, we get

$$\begin{aligned} u_i(a_i^*, a_{-i}) = &\left(b(v_i^*) - \sum_{k\in\mathcal{S}_{v_i^*}\setminus\{i\}} \pi_k\right)^2 - \left(b(v_i^*) - \pi_i^* - \sum_{k\in\mathcal{S}_{v_i^*}\setminus\{i\}} \pi_k\right)^2 \\ &- \sum_{e\in\rho_i^*\setminus\rho_i} c_e(J_e+1) - \sum_{e\in\rho_i^*\cap\rho_i} c_e(J_e) - \frac{|\mathcal{S}_{v_i^*}|}{\sigma(v_i^*, h_i)} - \frac{\bar{\theta}_i}{\theta_i + \eta_i\pi_i^*}, \end{aligned} \tag{4.35}$$

where $b(v_i^*)$ is the budget at an optimal v_i^* and $\mathcal{S}_{v_i^*} = \{k \in \mathcal{I} \mid v_k^* = v_i^*\}$. Summing over all travelers in (4.35), and keeping note of $u_i(a_i^{\text{Nash}}, a_{-i}) \le u_i(a_i^*, a_{-i})$ yields

$$J(a^{\text{Nash}}) \le \sum_{i\in\mathcal{I}} u_i(a_i^*, a_{-i}). \tag{4.36}$$

At this point, we recall that the travel time latency functions are linear. Thus, we can write

$$J_1(a^{\text{Nash}}) = \sum_{i\in\mathcal{I}} \sum_{e\in\rho_i^{\text{Nash}}} c_e(J_e^{\text{Nash}}) = \sum_{e\in\mathcal{E}} J_e^{\text{Nash}}(\xi_1 J_e^{\text{Nash}} + \xi_2), \tag{4.37}$$

$$J_1(a^*) = \sum_{i\in\mathcal{I}} \sum_{e\in\rho_i^*} c_e(J_e^*) = \sum_{e\in\mathcal{E}} J_e^*(\xi_1 J_e^* + \xi_2), \tag{4.38}$$

where J_e^{Nash} and J_e^* denote the number of services on road e at a NE and social optimum, respectively. Inspired from [10], we impose an upper bound on each component of the RHS of (4.36), and thus we have the following

$$\begin{aligned}
\sum_{i\in\mathcal{I}} \left(\sum_{e\in\rho_i^*\setminus\rho_i} c_e(J_e+1) + \sum_{e\in\rho_i^*\cap\rho_i} c_e(J_e) \right) &\le \sum_{i\in\mathcal{I}} \sum_{e\in\rho_i^*} c_e(J_e+1) \\
&= \sum_{e\in\mathcal{E}} \xi_1 J_e^* J_e + J_e^*(\xi_1+\xi_2) \\
&\le \sqrt{\sum_{e\in\mathcal{E}} \xi_1 J_e^2 \xi_1 (J_e^*)^2} + \sum_{e\in\mathcal{E}} J_e^*(\xi_1 J_e^* + \xi_2) \\
&\le \sqrt{\sum_{e\in\mathcal{E}} (\xi_1 J_e^2 + \xi_2 J_e)(\xi_1 (J_e^*)^2 + \xi_2 J_e^*)} + \sum_{e\in\mathcal{E}} J_e^*(\xi_1 J_e^* + \xi_2) \\
&= \sqrt{J_1(a^{\text{Nash}}) \times J_1(a^*)} + J_1(a^*),
\end{aligned} \tag{4.39}$$

where we simplified the notation as $J_e^{\text{Nash}} = J_e$, used $c_e(J_e) \le c_e(J_e+1)$ for each $e \in \rho_i^* \cap \rho_i$, and applied the Cauchy-Schwarz inequality twice. Note that $\xi_1 J_e^* + \xi_2 \le \xi_1 (J_e^*)^2 + \xi_2 J_e^*$ at any $e \in \mathcal{E}$. Next, we introduce notation: $J_2(a^{\text{Nash}}) = \sum_{i\in\mathcal{I}} \frac{|\mathcal{S}_{v_i^{\text{Nash}}}|}{\sigma(v_i^{\text{Nash}}, h_i)}$ and $J_2(a^*) = \sum_{i\in\mathcal{I}} \frac{|\mathcal{S}_{v_i^*}|}{\sigma(v_i^*, h_i)}$. We have

$$\sum_{i\in\mathcal{I}} \frac{|\mathcal{S}_{v_i^*}|}{\sigma(v_i^*, h_i)} \le \sqrt{J_2(a^*) J_2(a^{\text{Nash}})} + J_2(a^*). \tag{4.40}$$

Now we introduce the following notation:

$$J_3(a^{\text{Nash}}) = \sum_{i\in\mathcal{I}} \Bigg[\Bigg(b(v_i^{\text{Nash}}) - \sum_{k\in\mathcal{S}_{v_i}^{\text{Nash}}\setminus\{i\}} \pi_k^{\text{Nash}} \Bigg)^2 - \Bigg(b(v_i^{\text{Nash}}) - \sum_{i\in\mathcal{S}_{v_i}^{\text{Nash}}} \pi_i^{\text{Nash}} \Bigg)^2 - \frac{\bar{\theta}_i}{\theta_i + \eta_i \pi_i^{\text{Nash}}} \Bigg], \tag{4.41}$$

where $b(v_i^{\text{Nash}})$ is the budget at a v_i^{Nash}, and

$$J_3(a^*) = \sum_{i \in \mathcal{I}} \left[\left(b(v_i^*) - \sum_{k \in \mathcal{S}^*_{v_i^*} \setminus \{i\}} \pi_k^* \right)^2 - \left(b(v_i^*) - \sum_{i \in \mathcal{S}^*_{v_i^*}} \pi_i^* \right)^2 - \frac{\bar{\theta}_i}{\theta_i + \eta_i \pi_i^*} \right]. \quad (4.42)$$

By Lemma 4.1.19, we have a bound for the first two components of (4.36), thus

$$\begin{aligned} \sum_{i \in \mathcal{I}} \left[\left(b(v_i^*) - \sum_{k \in \mathcal{S}_{v_i^*} \setminus \{i\}} \pi_k \right)^2 - \left(b(v_i^*) - \pi_i^* - \sum_{k \in \mathcal{S}_{v_i^*} \setminus \{i\}} \pi_k \right)^2 - \frac{\bar{\theta}_i}{\theta_i + \eta_i \pi_i^*} \right] \\ \leq J_3(a^*) - \sqrt{(\tilde{b}^2 + 2(J_3(a^{\text{Nash}}) - I\bar{\theta}_i))(\tilde{b}^2 + 2(J_3(a^*) - I\bar{\theta}_i))} - 4I\bar{\theta}_i - \tilde{b}^2 \\ - \tilde{b} \left(\sqrt{\tilde{b}^2 + 2(J_3(a^{\text{Nash}}) - I\bar{\theta}_i)} + \sqrt{\tilde{b}^2 + 2(J_3(a^*) - I\bar{\theta}_i)} \right), \end{aligned} \quad (4.43)$$

We combine all relations together (4.39), (4.40), and (4.43) and substitute them into (4.36) to get

$$\begin{aligned} J(a^{\text{Nash}}) \leq \sqrt{J_1(a^{\text{Nash}}) J_1(a^*)} + J_1(a^*) + \sqrt{J_2(a^*) J_2(a^{\text{Nash}})} + J_2(a^*) - \tilde{b}^2 + J_3(a^*) \\ + \sqrt{(\tilde{b}^2 + 2(J_3(a^{\text{Nash}}) - I\bar{\theta}_i)(\tilde{b}^2 + 2(J_3(a^*) - I\bar{\theta}_i))} \\ - \tilde{b} \left(\sqrt{\tilde{b}^2 + 2(J_3(a^{\text{Nash}}) - I\bar{\theta}_i)} + \sqrt{\tilde{b}^2 + 2(J_3(a^*) - I\bar{\theta}_i)} \right) \\ \leq J(a^*) - \tilde{b} \left(\sqrt{\tilde{b}^2 + 2J_3(a^{\text{Nash}})} + \sqrt{\tilde{b}^2 + 2J_3(a^*)} \right) \\ - \sqrt{\left(\tilde{b}^2 + 2 \left(\sum_{k=1}^{3} J_k(a^*) - I\bar{\theta}_k \right) \right) \left(\tilde{b}^2 + 2 \left(\sum_{k=1}^{3} J_k(a^{\text{Nash}}) - I\bar{\theta}_k \right) \right)} - 4I\bar{\theta}_i - \tilde{b}^2, \end{aligned} \quad (4.44)$$

where we have used the fact that for any four numbers $(\gamma_k \in \mathbb{R}_{>0})$, $k = 1, 2, 3, 4$, we have $\sqrt{\gamma_1 \gamma_2} + \sqrt{\gamma_3 \gamma_4} \leq \sqrt{(\gamma_1 + \gamma_3)(\gamma_2 + \gamma_4)}$. Then (4.44) leads to

$$\begin{aligned} J(a^*) - \sqrt{(\tilde{b}^2 + 2J(a^*))(\tilde{b}^2 + 2J(a^{\text{Nash}}))} \\ - \tilde{b} \left(\sqrt{\tilde{b}^2 + 2J_3(a^{\text{Nash}})} + \sqrt{\tilde{b}^2 + 2J_3(a^*)} \right) - 4I\bar{\theta}_i - \tilde{b}^2 \\ \leq J(a^*) - \sqrt{(\tilde{b}^2 + 2J(a^*))(\tilde{b}^2 + 2J(a^{\text{Nash}}))} \\ - \tilde{b} \left(\sqrt{\tilde{b}^2 + 2J(a^{\text{Nash}})} + \sqrt{\tilde{b}^2 + 2J(a^*)} \right) - 4I\bar{\theta}_i - \tilde{b}^2. \end{aligned} \quad (4.45)$$

So, we have the following after a simple rearrangement

$$J(a^*) \leq J(a^{\text{Nash}}) - \sqrt{(\tilde{b}^2 + 2J(a^*))(\tilde{b}^2 + 2J(a^{\text{Nash}}))}$$
$$- \tilde{b}\left(\sqrt{\tilde{b}^2 + 2J(a^{\text{Nash}})} + \sqrt{\tilde{b}^2 + 2J(a^*)}\right) - 4I\bar{\theta}_i - \tilde{b}^2. \quad (4.46)$$

We divide both sides of (4.46) by $J(a^{\text{Nash}})$ to obtain

$$\frac{J(a^*)}{J(a^{\text{Nash}})} \leq 1 - \sqrt{\left(\frac{\tilde{b}^2}{I} + \frac{2J(a^*)}{J(a^{\text{Nash}})}\right)\left(\frac{\tilde{b}^2}{I} + 2\right)} - \frac{\tilde{b}^2}{I} - 4\bar{\theta}_i$$
$$- \frac{\tilde{b}}{\sqrt{I}}\left(\sqrt{\frac{\tilde{b}^2}{I} + \frac{2J(a^*)}{J(a^{\text{Nash}})}} + \sqrt{\frac{\tilde{b}^2}{I} + 2}\right), \quad (4.47)$$

By Lemma 4.1.20, we reach the desired bound. □

Next, we discuss the intuition behind Theorem 4.1.21. The travelers of the mobility game $\mathcal{M}$ are considered selfish and non-cooperative. Thus, one important question is what will be the impact of selfishness on the efficiency of the mobility system. Since the existence of a NE is guaranteed by Theorem 4.1.16 and the mobility game $\mathcal{M}$ converges to at least one NE, we can compare the level of inefficiency at a NE to the social optimum (this is exactly what the PoA does). The bound we have derived in (4.34) under certain conditions ensures the mobility system's inefficiency is guaranteed to remain within a constant, and as the number of travelers increases, this bound becomes smaller and smaller. So, our mobility game $\mathcal{M}$ can ensure in a realistic setting (big city with road infrastructure) with a large number of travelers a sufficiently efficient operation of the mobility system as it could ideally be operated by a central authority "ordering" the travelers how to travel. Our bound in (4.34) is strong in the sense that it excludes any other possibility of an improvement in efficiency compared to what we can achieve at a NE.

It happens that we can also upper bound using the "potential function method" (as used in [171]) the *Price of Stability* (PoS) for our mobility game $\mathcal{M}$. The PoS is defined as a ratio comparing social optimum and the best possible social welfare at a NE, i.e.,

$$\text{PoS} = \frac{\max_{a \in \mathcal{A}} \sum_{i \in \mathcal{I}} u_i(a)}{\max_{a \in \mathcal{A}^{\text{Nash}}} \sum_{i \in \mathcal{I}} u_i(a)}. \tag{4.48}$$

Theorem 4.1.22. *With linear travel time latency functions and pricing (4.1.7), the PoS for the mobility game $\mathcal{M}$ is upper bounded, i.e.,*

$$PoS = \frac{J(a^*)}{J(\tilde{a}^{Nash})} \leq \frac{1}{2} + \max_{i \in \mathcal{I}} \left[\bar{\theta}_i(\bar{\theta}_i - 1)\right] + \frac{1}{I} \sum_{v \in \mathcal{V}} (b(v))^2. \tag{4.49}$$

Proof. Recall that the *social welfare* of the mobility game $\mathcal{M}$ is represented by $J(a) = \sum_{i \in \mathcal{I}} u_i(a)$. We aim to compare the $J(a)$ with the potential function $\Phi(a)$. So, we have

$$\Phi(a) = \sum_{v \in \mathcal{V}} \left(b(v) - \sum_{i \in \mathcal{S}_v} \pi_i \right)^2 - \sum_{i \in \mathcal{I}} \frac{\bar{\theta}_i}{\theta_i + \eta_i \pi_i} - \sum_{e \in \mathcal{E}} \sum_{k=1}^{J_e} (\xi_1 k + \xi_2) - \sum_{v \in \mathcal{V}} \sum_{i \in \mathcal{I}} \frac{|\mathcal{S}_v|(|\mathcal{S}_v| + 1)}{2\sigma(v, h_i)}, \tag{4.50}$$

where we have substituted $c_e(J_e) = \xi_1 J_e + \xi_2$. Next, we have

$$\Phi(a) = \sum_{v \in \mathcal{V}} (b(v))^2 - 2 \sum_{v \in \mathcal{V}} \left(b(v) \sum_{i \in \mathcal{S}_v} \pi_i \right) + \sum_{v \in \mathcal{V}} \left(\sum_{i \in \mathcal{S}_v} \pi_i \right)^2 - \sum_{i \in \mathcal{I}} \frac{\bar{\theta}_i}{\theta_i + \eta_i \pi_i} - \sum_{e \in \mathcal{E}} (\xi_1 J_e^2 + (\xi_1 + 2\xi_2) J_e) - \sum_{v \in \mathcal{V}} \sum_{i \in \mathcal{I}} \frac{|\mathcal{S}_v|(|\mathcal{S}_v| + 1)}{2\sigma(v, h_i)}. \tag{4.51}$$

We also have the following

$$J(a) = \sum_{i \in \mathcal{I}} u_i(a) = \sum_{i \in \mathcal{I}} 2\pi_i \left(b(v_i) - \frac{\pi_i}{2} - \sum_{k \in \mathcal{S}_{v_i} \setminus \{i\}} \pi_k \right) - \sum_{i \in \mathcal{I}} \frac{\bar{\theta}_i}{\theta_i + \eta_i \pi_i} - \sum_{i \in \mathcal{I}} \sum_{e \in \rho_i : \rho_i \in \mathcal{P}^{(o,d)}} (\xi_1 J_e + \xi_2) - \sum_{i \in \mathcal{I}} \frac{|\mathcal{S}_{v_i}|}{\sigma(v_i, h_i)}, \tag{4.52}$$

which leads to

$$J(a) = \sum_{i \in \mathcal{I}} 2\pi_i \left(b(v_i) - \frac{\pi_i}{2} - \sum_{k \in \mathcal{S}_{v_i} \setminus \{i\}} \pi_k \right) - \sum_{i \in \mathcal{I}} \frac{\bar{\theta}_i}{\theta_i + \eta_i \pi_i} - \sum_{e \in \mathcal{E}} (\xi_1 J_e^2 + \xi_2 J_e) - \sum_{i \in \mathcal{I}} \frac{|\mathcal{S}_{v_i}|}{\sigma(v_i, h_i)}, \tag{4.53}$$

By comparing now (4.51) and (4.53), we note that

$$\frac{1}{2}J(a) \leq \Phi(a) \leq J(a) + \sum_{v\in\mathcal{V}}(b(v))^2 + 2\sum_{i\in\mathcal{I}}\sum_{k\in\mathcal{S}_{v_i}\setminus\{i\}}\pi_i\pi_k \tag{4.54}$$

$$\leq J(a) + 2I\max_{i\in\mathcal{I}}\left[\bar{\theta}_i(\bar{\theta}_i-1)\right] + \sum_{v\in\mathcal{V}}(b(v))^2. \tag{4.55}$$

Our next step is to define a NE that happens to maximize the potential function, say $\tilde{a}^{\text{Nash}}$ to get

$$J(\tilde{a}^{\text{Nash}}) \leq \Phi(\tilde{a}^{\text{Nash}}) \leq \Phi(a^*) \leq J(a^*) + 2I\max_{i\in\mathcal{I}}\left[\bar{\theta}_i(\bar{\theta}_i-1)\right] + \sum_{v\in\mathcal{V}}(b(v))^2. \tag{4.56}$$

Therefore, if we divide by $J(\tilde{a}^{\text{Nash}})$, we get the following relation

$$\text{PoS} = \frac{J(a^*)}{J(\tilde{a}^{\text{Nash}})} \leq \frac{1}{2} + \max_{i\in\mathcal{I}}\left[\bar{\theta}_i(\bar{\theta}_i-1)\right] + \frac{1}{I}\sum_{v\in\mathcal{V}}(b(v))^2. \tag{4.57}$$

□

Theorem 4.1.22 helps us understand the mobility game's NE and provides a metric for how close any NE might be to the social optimum. We can ensure that as the number of travelers increases, the smaller the PoS becomes, guaranteeing that the closer the mobility game's NE is to the social optimum.

4.1.4 Prospect Theory Analysis

In this subsection, we introduce prospect theory and its main concepts [247, 15]. We then incorporate prospect theory to our game $\mathcal{M}$. One of the main questions prospect theory attempts to answer is how a decision-maker may evaluate different possible actions/outcomes under uncertain and risky circumstances. Thus, prospect theory is a descriptive behavioral model and focuses on three main behavioral factors:

1. *Reference dependence*: decision makers make decisions based on their utility, which is measured from the "gains" or "losses." However, the utility is a gain or loss relative to a reference point that may be unique to each decision-maker. It has

been shown in experimental studies [15] the reference dependence captures the tendency of a decision-maker to be more affected in their decisions by the *changes in attributes* than by the *absolute magnitudes*. For example, the shortest/average travel time between two locations.

2. *Diminishing sensitivity*: changes in value have a greater impact near the reference point than away from the reference point. For example, an individual is highly likely to discriminate between a 1 and 2 hours travel time but not very likely to notice the difference between 18 and 19 hours travel time.

3. *Loss aversion*: decision-makers are more conservative in gains and riskier in losses. For example, a traveler may prefer to secure a 45 min commute rather than risking for a 1.5 hours commute.

One way to mathematize the above behavioral factors (1) - (3) is to consider an action by a decision-maker as a "gamble" with objective utility value $z \in \mathbb{R}$ (e.g., money). We say that this decision maker *perceives* z subjectively using a *value function* [238, 3]

$$\nu(z) = \begin{cases} (z - z_0)^{\beta_1}, & \text{if } z \geq z_0, \\ -\lambda(z_0 - z)^{\beta_2}, & \text{if } z < z_0, \end{cases} \tag{4.58}$$

where z_0 represents a reference point, $\beta_1, \beta_2 \in (0, 1)$ are parameters that represent the diminishing sensitivity. Both β_1, β_2 shape (4.58) in a way that the changes in value have a greater impact near the reference point than away from the reference point. We observe that (4.58) is concave in the domain of gains and convex in the domain of losses. Moreover, $\lambda \geq 1$ reflects the level of loss aversion of decision makers (see Fig. 4.1).

Remark 4.1.23. *To the best of our knowledge, there does not exist a widely-agreed theory that determines and defines the reference dependence [110, 116, 18]. In engineering [100, 71], it is assumed that* $z_0 = 0$ *captures a decision maker's expected status-quo level of the resources.*

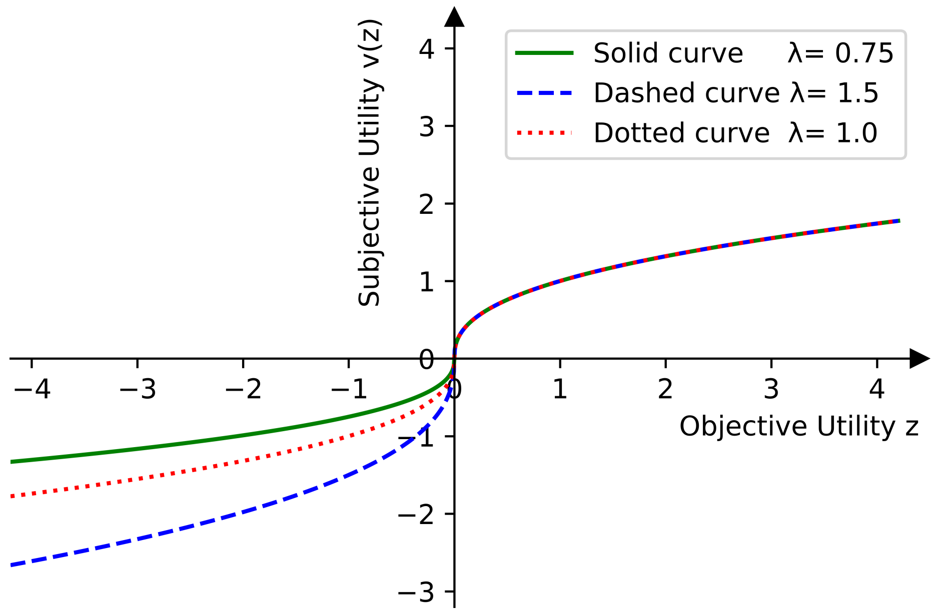

Figure 4.1: The value function for three different values of λ.

As we discussed earlier in this subsection, prospect theory models the subjective behavior of decision-makers under uncertainty and risk. Each objective utility $z \in \mathbb{R}$ is associated with a probabilistic occurrence, say $p \in [0, 1]$. Decision makers, though, are subjective and perceive p in different ways depending on its value. To capture this behavior, we introduce a strictly increasing function $w : [0, 1] \to \mathbb{R}$ with $w(0) = 0$ and $w(1) = 1$ called the *probability weighting function*. This function allows us to model how decision-makers may overestimate small probabilities of objective utilities, i.e., $w(p) > p$ if p is close to 0, or underestimate high probabilities, i.e., $w(p) < p$ if p is close to 1 (see Fig. 4.2). For the purposes of this work, we use the probability weighting function first introduced in [182],

$$w(p) = \exp\left(-(-\log(p))^{\beta_3}\right), \quad p \in [0, 1], \tag{4.59}$$

where $\beta_3 \in (0, 1)$ represents a *rational index*, i.e., the distortion of a decision-makers probability perceptions. Mathematically, β_3 controls the curvature of the weighting function (see Fig. 4.2). Although there are many different formulations for the probability weighting function, we use (4.59) defined in [182] as a single-parameter function and easier computationally to estimate.

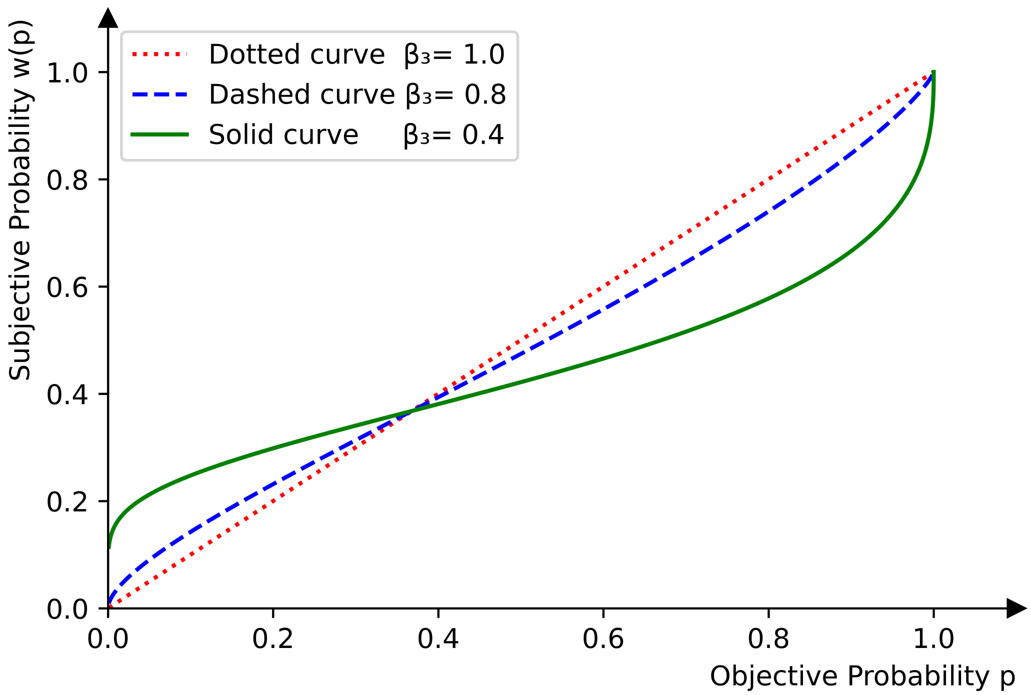

Figure 4.2: The probability weighting function for three different values of the rational index β_3.

Next, we define a *prospect* which is a tuple of the objective utility (gain or loss) and its probability of happening.

Definition 4.1.24. *Suppose that there are $K \in \mathbb{N}$ possible outcomes available to a decision-maker and $z_k \in \mathbb{R}$ is the kth gain/loss of objective utility. Then a prospect ℓ_k is a tuple of the utilities and their respective probabilities*

$$\ell_k = (z_0, z_1, z_2, \ldots, z_K; p_0, p_1, p_2, \ldots, p_K), \tag{4.60}$$

where $k = 1, 2, \ldots, K$. We denote the kth prospect more compactly as $\ell_k = (z_k, p_k)$. We have that $\sum_{k=0}^{K} p_k = 1$ and ℓ_k is well-ordered, i.e., $z_0 \leq x_1 \leq \cdots \leq z_K$. Under prospect theory, the decision-maker evaluates their "subjective utility" as $u(\ell) =$

$\sum_{0 \leq k \leq K} v(z_k) w(p_k)$, *where* $\ell = (\ell_k)_{k=1}^{K}$ *is the profile of prospects of* K *outcomes.*

In the remainder of this subsection, we apply the prospect theory to our game, clearly define the mobility outcomes (objective and subjective utilities), and then show that the prospect-theoretic mobility game $\mathcal{M}$ admits a NE.

Travelers may be uncertain about the available amount of mobility funds at any transport hub, that is why we define a *mobility prospect* to represent as a random variable Z with objective utilities $z_1, z_2, \ldots, z_K$ and their probabilities $p_1, p_2, \ldots, p_K$. Each z_k now represents the uncertain $b(v_i)$. In addition, the reference dependence of each traveler i is represented by $z_i^0 \in \mathbb{R}$. For any traveler i, the probability weighting function is $w_i : [0, 1] \rightarrow \mathbb{R}$ and the value function is $\nu_i(z_k, z_i^0) : \mathbb{R}^2 \rightarrow \mathbb{R}$, $k = 1, 2, \ldots, K$. Thus, we have

$$\mathbb{E}[Z] = \sum_{k=1}^{K} \nu_i(z_k, z_i^0) w_i(p_k), \tag{4.61}$$

where $w_i(p_k)$ is given by (4.59), and

$$\nu_i(z_k, z_i^0) = \begin{cases} (z_k - z_i^0)^\beta, & \text{if } z_k \geq z_i^0, \\ -\lambda(z_i^0 - z_k)^\beta, & \text{if } z_k < z_i^0, \end{cases} \tag{4.62}$$

where $\beta = \beta_1 = \beta_2$. We can justify $\beta_1 = \beta_2$ in the above definition as it has been verified to produce extremely good results, and the outcomes are consistent with the original data [238]. Next, we explicitly define the reference point for the mobility game $\mathcal{M}$ as follows $z_i^0 = \left(\sum_{k \in \mathcal{S}_{v_i} \setminus \{i\}} \pi_k\right)^2 - \left(\sum_{i \in \mathcal{S}_{v_i}} \pi_i\right)^2$ where z_i^0 represents the ideal redistribution of wealth to traveler i (since no transport hub v_i should make a profit, i.e., $b(v_i) = 0$). For the random variable Z, we assume a continuous distribution F with zero mean and a probability density function f, and explicitly have

$$Z = \left(F - \sum_{k \in \mathcal{S}_{v_i} \setminus \{i\}} \pi_k\right)^2 - \left(F - \sum_{i \in \mathcal{S}_{v_i}} \pi_i\right)^2. \tag{4.63}$$

So, by (4.61), we have $\mathbb{E}[Z] = \sum_{n\in\mathbb{R}} \nu_i(z(n), z_i^0) w_i(f(n))$, where $z(n)$ represents at each transport hub v_i of an arbitrary traveler i the realization of Z with $n \in \mathbb{R}$ available mobility funds. The total utility now under prospect theory for a traveler i is

$$u_i^{\mathrm{PT}}(a) = z_i^0 + \mathbb{E}[Z] - \frac{\bar{\theta}_i}{\theta_i + \eta_i \pi_i} - \sum_{e\in\rho_i:\rho_i\in\mathcal{P}^{(o,d)}} c_e(J_e) - \frac{|\mathcal{S}_{v_i}|}{\sigma(v_i, h_i)}. \tag{4.64}$$

Next, we show that our mobility game $\mathcal{M}$ under prospect theory is guaranteed to have at least one NE.

Theorem 4.1.25. *The mobility game $\mathcal{M}$ under prospect theory admits a pure-strategy NE.*

Proof. We expand $z(n)$ and subtract z_i^0 and simplify to get

$$z(n) = \left(n - \sum_{k\in\mathcal{S}_{v_i}\setminus\{i\}} \pi_k\right)^2 - \left(n - \sum_{i\in\mathcal{S}_{v_i}} \pi_i\right)^2$$

$$z(n) - z_i^0 = 2n\left[\sum_{k\in\mathcal{S}_{v_i}\setminus\{i\}} \pi_k - \sum_{i\in\mathcal{S}_{v_i}} \pi_i\right] = 2n\pi_i. \tag{4.65}$$

where $z_i^0 = \left(\sum_{k\in\mathcal{S}_{v_i}\setminus\{i\}} \pi_k\right)^2 - \left(\sum_{i\in\mathcal{S}_{v_i}} \pi_i\right)^2$. Substituting (4.65) into (4.64) yields

$$u_i^{\mathrm{PT}}(a) = z_i^0 + \sum_{n\in\mathbb{R}} \nu_i \cdot (2n\pi_i) \cdot w_i(f(n)) - \sum_{e\in\rho_i:\rho_i\in\mathcal{P}^{(o,d)}} c_e(J_e) - \frac{|\mathcal{S}_{v_i}|}{\sigma(v_i, h_i)} - \frac{\bar{\theta}_i}{\theta_i + \eta_i\pi_i}, \tag{4.66}$$

where v_i is given by (4.62). The next step is explicitly defining a new potential function under prospect theory. We have

$$\Psi(a) = \sum_{i\in\mathcal{I}} \sum_{n\in\mathbb{R}} \nu_i \cdot (2n\pi_i) \cdot w_i(f(n)) - \sum_{e\in\mathcal{E}} \sum_{k=1}^{J_e} c_e(k) - \sum_{v\in\mathcal{V}} \frac{|\mathcal{S}_v|(|\mathcal{S}_v| + 1)}{2\sigma(v, h_i)} - \sum_{i\in\mathcal{I}} \frac{\bar{\theta}_i}{\theta_i + \eta_i\pi_i} + \sum_{v\in\mathcal{V}} \left(\sum_{k\in\mathcal{S}_v} \pi_k\right)^2. \tag{4.67}$$

Next, we show that Ψ as given in (4.67) is an exact potential function. We notice that $\sum_{n\in\mathbb{R}} \nu_i \cdot (2n\pi_i) \cdot w_i(f(n))$ does not depend on a_{-i}, i.e., the actions of the other travelers except traveler i. Hence, following similar arguments as in Theorem 4.1.16, we obtain $u_i^{\mathrm{PT}}(a_i, a_{-i}) - u_i^{\mathrm{PT}}(a'_i, a_{-i}) = \Psi(a_i, a_{-i}) - \Psi(a'_i, a_{-i})$. Hence, Ψ is indeed an exact potential function for the mobility game $\mathcal{M}$ under prospect theory. Therefore, since any action profile that minimizes Ψ results in a NE, the mobility game $\mathcal{M}$ admits a NE under prospect theory. □

Corollary 4.1.26. *For the mobility game $\mathcal{M}$ under prospect theory, the sequence of best responses of an arbitrary traveler $i \in \mathcal{I}$ converges to a NE.*

Proof. It is sufficient to note that the action set $\mathcal{A}_i$ of any traveler i is compact. Thus, it follows from the results in [162] that the sequence of best responses of any traveler $i \in \mathcal{I}$ converges to a NE. □

Both Theorem 4.1.25 and Corollary 4.1.26 ensure that the mobility game $\mathcal{M}$ under the prospect-theoretic behavioral model admits a NE and prospect-based travelers will eventually converge to it. Both results establish that we can still ensure that an equilibrium can be reached under certain conditions for the cost and pricing functions.

4.2 Modeling Travel Behavior in Mobility Systems with an Atomic Routing Game and Prospect Theory

We now present a game-theoretic modeling framework for studying travel behavior in mobility systems by incorporating prospect theory. As part of our motivation, we conducted an experiment in a scaled smart city to investigate the frequency of errors in actual and perceived probabilities of a highway route under free-flow conditions. Based on these findings, we provide a game that captures how travelers distribute their traffic flows in a transportation network with splittable traffic, utilizing the Bureau of Public Roads function to establish the relationship between traffic flow and travel time cost. Given the inherent non-linearities, we propose a smooth approximation function that helps us estimate the prospect-theoretic cost functions. As part of our analysis, we characterize the best-fit parameters and derive an upper bound for the error. We then show a Nash Equilibrium existence. Finally, we present a numerical example and simulations to verify the theoretical results and demonstrate the effectiveness of our approximation.

4.2.1 Motivational Example

It is well established in the literature that travel behavior is a rather complex and complicated part of mobility that requires sophisticated models to capture the "actual" travel behavior and conditions. Travelers are frequently tasked to engage in a mobility system and make routing decisions under uncertainties. Such decisions are influenced by many factors (e.g., individual preferences, traffic accidents, and bad weather). Travelers then may fail to interpret the probabilities of congestion or traffic accident the right way and, thus, exhibit irrational behavior or become risk-averse. A key observation is that prospect-theoretic players aim to minimize their potential/expected losses.

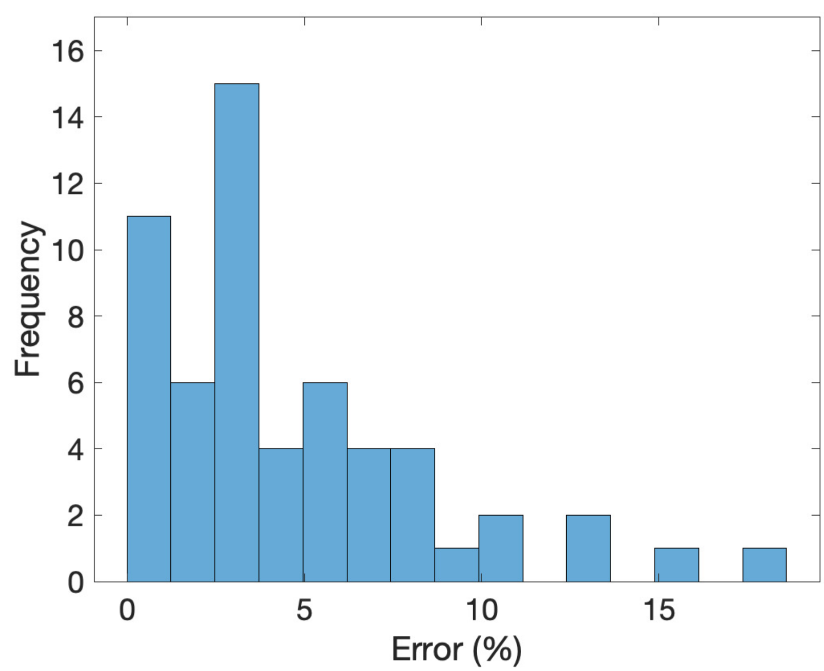

Figure 4.3: Frequency of errors of perceived/actual probabilities.

To better understand the factors that influence routing decisions under uncertainty, we designed and conducted an experiment with 20 participants using the *Intelligent Driver Model* (IDM) for all cars. We created artificial traffic congestion on different routes by controlling the speed of the preceding vehicles in a 1:25 scaled robotic testbed called *IDS Lab's Scaled Smart City* (IDS3C) [41]. During the experiment, the participants were asked to choose the route they preferred the car to take based on information provided on different traffic conditions: free flow and traffic delays. One of our key results from analyzing the data can be seen in Fig. 4.3. Here we have a histogram that shows the percentage error, and frequency means the number of cases. The total number of cases is about 60 because we had 3 different estimations for each participant. This shows that a majority of the participants failed to accurately perceive the actual probabilities of the traffic scenarios we presented to them, and most interestingly, other participants overestimated the actual probabilities by 15%. Our goal here is to understand what drives and influences a traveler's decision as they try to minimize their travel time cost. In addition, we aim to strongly motivate modeling that directly captures how the individuals' perceptions of probability affect their decision-making. We leverage the results of our experiment to motivate the modeling framework that follows in the Section.

4.2.2 Modeling Framework

We consider a routing game with a finite non-empty set of players $\mathcal{I}$, $|\mathcal{I}| = n \in \mathbb{N}$. Each player i represents a traveler with a connected and automated vehicle (CAV) who controls a significant amount of traffic, say $x_i \in \mathbb{R}_{\geq 0}$. The interpretation of this is that x_i represents the flow of traffic that player i contributes to a transportation network. We define traffic flow in this setting as the number of CAVs passing through each point in the network over time. This decision variable is non-negative as travelers make trips using the CAVs over time in the transportation network. This is in contrast to non-atomic routing games, where players only control an infinitesimal amount of traffic. We also assume that traffic is *splittable*. Players seek to travel in the transportation network represented by a directed multigraph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where each node in $\mathcal{V}$ may represent different city areas or neighborhoods (e.g., Braess' paradox network). Each edge $e \in \mathcal{E}$ may represent a road. For our purposes, we think of $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ as a representation of a smart city network with a road infrastructure that can support CAVs. Any player $i \in \mathcal{I}$ seeks to travel from an origin $o \in \mathcal{V}$ to a destination $d \in \mathcal{V}$. So, all players are associated with the same unique origin-destination pair $(o, d) \in \mathcal{V} \times \mathcal{V}$. Next, each player may use a sequence of edges that connects the OD pair (o, d). We have $\mathcal{R}^{(o,d)} \subset 2^{\mathcal{E}}$ to denote the set of routes available to any player $i \in \mathcal{I}$, where each route r_i consists of a sequence of edges connecting the origin-destination pair (o, d). We are interested in how such players may compete over the routes in the network for routing their traffic flows (e.g., this is a multiple-route traffic flow decision-making problem).

We say that each player $i \in \mathcal{I}$ seeks to route their traffic in $\mathcal{G}$ represented by a traffic flow x_i where each of its elements takes values in $\mathbb{R}_{\geq 0}$. For each $i \in \mathcal{I}$, the set of actions is

$$\mathcal{X}_i = \left\{ x_i \in \mathbb{R}_{\geq 0}^{|\mathcal{R}|} : \sum_{r_i \in \mathcal{R}} x_i^{r_i} = \bar{x}_i \right\}, \tag{4.68}$$

where $x_i = (x_i^{r_i^1}, x_i^{r_i^2}, \ldots, x_i^{r_i^{|\mathcal{R}|}})$, $\bar{x}_i \in \mathbb{R}_{\geq 0}$, is the total flow of player i, and r_i^k denotes the kth route in the network. We write $\mathcal{X} = \mathcal{X}_1 \times \mathcal{X}_2 \times \cdots \times \mathcal{X}_n$ for the Cartesian product of all the players' action sets. We also write $x_{-i} = (x_1, x_2, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n)$ for the action profile that excludes player $i \in \mathcal{I}$. Next, for the aggregate action profile, we write $x = (x_i, x_{-i})$, $x \in \mathcal{X}$.

Definition 4.2.1. *The flow on edge $e \in \mathcal{E}$ is the sum of the part of all players' traffic flows that have chosen a route that includes edge e, i.e., $f_e(x) = \sum_{i \in \mathcal{I}} \sum_{r_i \ni e} x_i^{r_i}$.*

In our routing game where each player $i \in \mathcal{I}$ chooses their own traffic demand vector x_i over a common set of paths $\mathcal{R}$, if player i chooses to send a traffic demand $x_i^{r_i}$ along route r_i, then this traffic demand will be distributed along all the edges in the route r_i. This is because the traffic demand on a particular route r_i is a single quantity that is distributed among the edges in the route. Therefore, if player i chooses to send a traffic demand $x_i(r_i)$ along route r_i, this traffic demand will be split among all the edges e in the path p_i that player i uses.

Next, we introduce a travel time latency function to capture the cost that players may experience. Intuitively, we capture the players' preferences for different outcomes using a "cost function," in which players are expected to act as cost minimizers. For each $e \in \mathcal{E}$, we consider non-negative cost functions $c_e : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$. We assume that the cost functions at each edge e are convex, continuous, and differentiable with respect to f_e. One standard way to define in an exact form c_e is by the BPR function, as it is a commonly used model for the relationship between flow and travel time. Mathematically, we have

$$c_e(f_e) = c_e^0 \left(1 + \frac{3}{20} \left(\frac{f_e}{f_e^{CRT}} \right)^4 \right), \tag{4.69}$$

where for any edge $e \in \mathcal{E}$, c_e^0 is the free-flow travel time and f_e^{CRT} is the *critical* capacity of traffic flow on road e. We note that the BPR function is non-linear, continuous, differentiable, strictly increasing, and strictly convex for $f_e \geq 0$.

Definition 4.2.2. *If the* maximum flow *on edge* $e \in \mathcal{E}$ *is* $f_e^{\max} \in \mathbb{R}_{>0}$*, then for the* critical flow*,* f_e^{CRT}*, on edge* $e \in \mathcal{E}$ *we have* $f_e^{CRT} < f_e^{\max}$*.*

Next, for some route r_i of any player i, its cost is the sum of the costs on the edges that constitute route r_i, i.e., $c_{r_i}(x) = \sum_{e \in r_i} c_e(f_e(x))$. Now, the total cost of some player i is

$$c_i(x) = \sum_{r_i \in \mathcal{R}} c_{r_i}(x) = \sum_{r_i \in \mathcal{R}} \left[\sum_{e \in r_i} c_e(f_e) \right], \tag{4.70}$$

which simplifies to $c_i(x) = \sum_{r_i \in \mathcal{R}} \sum_{e \in r_i} c_e(f_e)$.

Definition 4.2.3. *The game is fully characterized by the tuple* $\mathcal{M} = \langle \mathcal{I}, (\mathcal{X}_i)_{i \in \mathcal{I}}, (c_i)_{i \in \mathcal{I}} \rangle$*, a collection of sets of players, actions, and a profile of costs. This game is a simultaneous-move game where players make decisions at the same time and commute in* (o, d) *of network* $\mathcal{G}$*.*

The game $\mathcal{M}$ is a non-cooperative routing game with a transportation network and continuous action sets. Players are behaving according to prospect theory and aim to minimize their costs (e.g., travel time latencies). Naturally, players compete with each other over the available yet limited routes and how to utilize them in the transportation network. Indirectly, players make route choices that satisfy their travel needs (modeled through traffic flow).

Next, we clarify "who knows what?" in our routing game. All players have complete knowledge of the game and the network. Each player knows their own information (action and cost) as well as the information of other players. At equilibrium, we want to ensure that no player has an incentive to unilaterally deviate from their chosen decisions and change how they distribute their traffic flows over the available routes in the network. So, for the purposes of our work, we observe that a NE in pure strategies is most fitting to apply as a solution concept. We formally define a NE in terms of the players' traffic flows.

Definition 4.2.4. *A feasible flow profile* $x^* = (x_i^{r_i})_{i \in \mathcal{I}}^{r_i \in \mathcal{R}} \in \mathcal{X}$ *constitutes a NE if for each player* $i \in \mathcal{I}$, $c_i(x_i^*, x_{-i}^*) \leq c_i(x_i, x_{-i}^*)$, *for all* $x_i \in \mathcal{X}_i$.

In other words, a flow profile x^* is a NE if no player can reduce their total cost by unilaterally changing how they distribute their total traffic flow over the available routes in the network. In a NE, each player's specific x_i has the lowest possible cost among all possible distributions over the routes, given the choices made by other players.

4.2.2.1 Prospect Theory Analysis

In this subsection, we provide a brief introduction to prospect theory and its main concepts [247]. One of the main questions prospect theory attempts to answer is how a decision-maker may evaluate different possible actions/outcomes under uncertain and risky circumstances. Thus, prospect theory is a descriptive behavioral model and focuses on three main behavioral factors: (i) *Reference dependence*: decision makers make decisions based on their utility, which is measured from the "gains" or "losses." However, the utility is a gain or loss relative to a reference point that may be unique to each decision-maker. (ii) *Diminishing sensitivity*: changes in value have a greater impact near the reference point than away from the reference point. (iii) *Loss aversion*: decision-makers are more conservative in gains and riskier in losses. One way to mathematize the above behavioral factors (1) - (3), is to consider an action by a decision-maker as a "gamble" with objective utility value $z \in \mathbb{R}$. We say that this decision maker *perceives* z subjectively using a *value function* [238, 3]

$$v(z) = \begin{cases} (z - z_0)^{\beta_1}, & \text{if } z \geq z_0, \\ -\lambda(z_0 - z)^{\beta_2}, & \text{if } z < z_0, \end{cases} \tag{4.71}$$

where z_0 represents a reference point, $\beta_1, \beta_2 \in (0, 1)$ are parameters that represent the diminishing sensitivity. Both β_1, β_2 shape (4.71) in a way that the changes in value have a greater impact near the reference point than away from the reference point. We observe that (4.71) is concave in the domain of gains and convex in the domain of losses. Moreover, $\lambda \geq 1$ reflects the level of loss aversion of decision makers. To the best of our knowledge, there does not exist a widely-agreed theory that determines and defines the reference dependence [110]. In engineering [100, 71], it is assumed that $z_0 = 0$ captures a decision-makers expected status-quo level of the resources.

Prospect theory models the subjective behavior of decision-makers under uncertainty and risk. Each objective utility $z \in \mathbb{R}$ is associated with a probabilistic occurrence, say $p \in [0, 1]$. Decision makers, though, are subjective and perceive p in different ways depending on its value. To capture this behavior, we introduce a strictly increasing function $w : [0, 1] \to [0, 1]$ with $w(0) = 0$ and $w(1) = 1$ called the *probability weighting function*. This function allows us to model how decision-makers may overestimate small probabilities of objective utilities, i.e., $w(p) > p$ if p is close to 0, or underestimate high probabilities, i.e., $w(p) < p$ if p is close to 1. For the purposes of this work, we use the probability weighting function first introduced in [182], $w(p) = \exp\left(-(-\log(p))^{\beta_3}\right), \quad p \in [0, 1]$, where $\beta_3 \in (0, 1)$ represents a *rational index*, i.e., the distortion of a decision-maker's probability perceptions. Mathematically, β_3 controls the curvature of the weighting function.

Next, we define a *prospect* which is a tuple of the objective utility (gain or loss) and its probability of happening.

Definition 4.2.5. *Suppose that there are $K \in \mathbb{N}$ possible outcomes available to a decision-maker and $z_k \in \mathbb{R}$ is the kth gain/loss of objective utility. Then a prospect ℓ_k is a tuple of the utilities and their respective probabilities*

$$\ell_k = (z_0, z_1, z_2, \dots, z_K; p_0, p_1, p_2, \dots, p_K), \tag{4.72}$$

where $k = 1, 2, \dots, K$. We denote the kth prospect more compactly as $\ell_k = (z_k, p_k)$. We have that $\sum_{k=0}^{K} p_k = 1$ and ℓ_k is well-ordered, i.e., $z_0 \leq z_1 \leq \cdots \leq z_K$. Under prospect theory, the decision-maker evaluates their "subjective utility" as $u(\ell) = \sum_{0 \leq k \leq K} v(z_k) w(p_k)$, where $\ell = (\ell_k)_{k=1}^{K}$ is the profile of prospects of K outcomes.

In the remainder of this subsection, we apply the prospect theory to our modeling framework, clearly define the mobility outcomes (objective and subjective utilities), and then show that the prospect-theoretic game $\mathcal{M}$ admits a NE. Players may be uncertain about the value of the traffic disturbances as it is affected by unexpected factors, and so that is why we use Prelec's probability weighting function $w : [0,1] \to [0,1]$ to capture how different traveler populations "perceive" probabilities. In addition, we are interested in capturing how players may perceive their gains or losses regarding their travel time costs with respect to the costs at critical density. The *mobility prospect* is whether the f_e will reach its critical or jammed point. Formally, π_e is the probability that $f_e \in (0, f_e^{CRT})$, and $1 - \pi_e$ is the probability for $f_e \in (f_e^{CRT}, f_e^{\max}]$. We then use the prospect-theoretic S-shaped value function $v(c_e(f_e)) : \mathbb{R}_{\geq 0} \to \mathbb{R}$ to capture how players may perceive such costs. Hence, we have

$$v(c_e(f_e)) = \begin{cases} \lambda(c_e^0 - c_e(f_e))^{\beta}, & \text{if } c_e(f_e) \leq c_e^0, \\ -(c_e(f_e) - c_e^0)^{\beta}, & \text{if } c_e(f_e) > c_e^0, \end{cases} \tag{4.73}$$

where the reference dependence is represented by $c_e^0 = c_e(f_e^{CRT})$, $\beta_1 = \beta_2 = \beta \in (0, 1)$, and for each $e \in \mathcal{E}$, we have $\pi_e \in [0, 1]$. We can justify $\beta_1 = \beta_2$ in the above definition as it has been verified to produce extremely good results, and the outcomes are consistent with the original data [238]. We define

$$\tilde{c}_e(f_e) = \begin{cases} c_e^0 - c_e(f_e), & \text{if } c_e(f_e) < c_e^0, \\ c_e(f_e) - c_e^0, & \text{if } c_e(f_e) > c_e^0. \end{cases} \tag{4.74}$$

Remark 4.2.6. *It is easy to note that our prospect-theoretic value function is "reversed," capturing the way the player will perceive the gains in travel time through a cost function. Using as a reference point the critical traffic flow on edge e, we can pinpoint the exact point that any more delays become socially unacceptable, i.e., a higher flow causes a higher travel time that the traveler will not tolerate.*

Under prospect theory, the new "cost function" is given by

$$c_e^{\mathrm{PT}}(f_e) = w(\pi_e) \cdot \lambda \cdot [\tilde{c}_e(f_e \,|\, c_e(f_e) < c_e^0)]^\beta - w(1 - \pi_e) \cdot [\tilde{c}_e(f_e \,|\, c_e(f_e) > c_e^0)]^\beta. \tag{4.75}$$

The total cost on some route r_i for player i under prospect theory is

$$c_{r_i}^{\mathrm{PT}}(x) = \sum_{e \in r_i} c_e^{\mathrm{PT}}(f_e). \tag{4.76}$$

Now, the total cost of some player i is given by

$$c_i^{\mathrm{PT}}(x) = \sum_{r_i \in \mathcal{R}} \sum_{e \in r_i} c_e^{\mathrm{PT}}(f_e). \tag{4.77}$$

Note, however, that in this case, the prospect-theoretic cost is capturing the gains and losses of a traveler. Thus, the aim is to maximize this function to maximize the gains. In other words, by minimizing the actual cost, we maximize the perceived gains in travel time by the traveler.

What we observe in (4.77) is that it is rather cumbersome to analyze it in game theory as issues in the smoothness of the function arise quickly. The key problem in analyzing such a function is that the exponent takes values in $(0, 1)$. However, we propose a new function that approximates the prospect-theoretic function and, most importantly, can be shown to have useful properties. Hence, we define the following function

$$\sigma(f_e) = \frac{\delta_1}{1 + \exp\left(\frac{\delta_2 - f_e}{\delta_3}\right)} + \delta_4, \tag{4.78}$$

where $\delta_1, \delta_2, \delta_3, \delta_4 \in \mathbb{R}$, and $f_e \in [0, f_e^{\max}]$. So we can write

$$c_i^{\mathrm{PT}}(x) = \sum_{r_i \in \mathcal{R}} \sum_{e \in r_i} \sigma(f_e). \tag{4.79}$$

4.2.3 Analysis and Properties of the Game

In this section, we provide a formal analysis of the properties of our proposed modeling framework and show that our game admits a NE in pure strategies.

Lemma 4.2.7. *The strategy space of the game $\mathcal{M}$ is non-empty, compact, and convex.*

Proof. To show that the set $\mathcal{X}_i = \{x_i \in \mathbb{R}_{\geq 0}^{|\mathcal{R}|} : \sum_{r_i \in \mathcal{R}} x_i^{r_i} = \bar{x}_i\}$ is non-empty, compact, and convex, we need to examine each property individually. (i) Non-empty: Consider the strategy where player i allocates all of their traffic, $\bar{x}_i$, to a single route, say r_1, and allocates zero traffic traffic to the remaining routes. Then, we have $x_i = (\bar{x}_i, 0, \ldots, 0) \in \mathbb{R}_{\geq 0}^{|\mathcal{R}|}$, which satisfies the constraint: $\sum_{r_i \in \mathcal{R}} x_i^{r_i} = \bar{x}_i$. Thus, $\mathcal{X}_i$ is non-empty for every player $i \in \mathcal{I}$. (ii) Compact: To show that $\mathcal{X}_i$ is bounded and closed, we only need to note that for each $x_i \in \mathcal{X}_i$, we must have $0 \leq x_i^{r_i} \leq \bar{x}_i$ for any route r_i. (iii) Convex: We need to show that for each player $i \in \mathcal{I}$, the set $\mathcal{X}_i$ is convex, i.e., for any $x_i, y_i \in \mathcal{X}_i$ and any $\mu \in [0, 1]$, we have $\mu x_i + (1 - \mu) y_i \in \mathcal{X}_i$. Let $x_i, y_i \in \mathcal{X}_i$ and $\lambda \in [0, 1]$. We want to show that $z_i = \lambda x_i + (1-\lambda)y_i \in \mathcal{X}_i$. We know that $x_i, y_i \in \mathcal{X}_i$ satisfy the constraint: $\sum_{r_i \in \mathcal{R}} x_i^{r_i} = \bar{x}_i$ and $\sum_{r_i \in \mathcal{R}} y_i^{r_i} = \bar{x}_i$. Now, consider the weighted sum:

$$\begin{aligned}\sum_{r_i \in \mathcal{R}} z_i^{r_i} &= \sum_{r_i \in \mathcal{R}} (\mu x_i^{r_i} + (1-\mu) y_i^{r_i}) \\ &= \mu \sum_{r_i \in \mathcal{R}} x_i^{r_i} + (1-\mu) \sum_{r_i \in \mathcal{R}} y_i^{r_i} \\ &= \mu \bar{x}_i + (1-\mu)\bar{x}_i = \bar{x}_i. \end{aligned} \tag{4.80}$$

Since z_i also satisfies the constraint, we have $z_i \in \mathcal{X}_i$. Therefore, $\mathcal{X}_i$ is convex for each player $i \in \mathcal{I}$. In conclusion, for each player $i \in \mathcal{I}$, the strategy space $\mathcal{X}_i$ is non-empty, compact, and convex. □

Next, we characterize the coefficients of σ function.

Lemma 4.2.8. *The approximation function given by (4.78) in the interval $[0, \kappa]$, $\kappa < f_e^{\max}$, is strictly concave with respect to f_e when $\delta_3 > 0$, $\delta_4 \in \mathbb{R}$, and (i) $\delta_1 > 0$, $\delta_2 > f_e$, or alternatively (ii) $\delta_1 < 0$, $\delta_2 < f_e$.*

Proof. Given that $f_e \geq 0$, we analyze the second-order derivative of the function $\sigma(f_e) = \frac{\delta_1}{1+\exp\left(\frac{\delta_2 - f_e}{\delta_3}\right)} + \delta_4$ to determine the conditions for strict concavity. First, let us find the first and second-order derivatives of σ with respect to f_e, i.e.,

$$\sigma'(f_e) = \frac{-\delta_1 \exp\left(\frac{\delta_2 - f_e}{\delta_3}\right)}{\delta_3 (1 + \exp\left(\frac{\delta_2 - f_e}{\delta_3}\right))^2}, \tag{4.81}$$

$$\sigma''(f_e) = \frac{2\delta_1 \exp\left(\frac{\delta_2 - f_e}{\delta_3}\right)(1 - \exp\left(\frac{\delta_2 - f_e}{\delta_3}\right))}{\delta_3^2 (1 + \exp\left(\frac{\delta_2 - f_e}{\delta_3}\right))^3}. \tag{4.82}$$

Now, we examine the conditions for $\sigma''(f_e) < 0$. First, δ_1 controls the sign of the second-order derivative as follows: if $\delta_1 < 0$ and $\delta_3 > 0$, $\sigma''(f_e)$ will be negative when $1 - \exp\left(\frac{\delta_2 - f_e}{\delta_3}\right) < 0$, which simplifies to $\delta_2 > f_e$. If $\delta_3 < 0$ in either of the cases, then the signs are reversed. We do require though that δ_3^2 is well-defined, so $\delta_3 \neq 0$. On greater detail, $1 - \exp\left(\frac{\delta_2 - f_e}{\delta_3}\right)$ determines the conditions for $\sigma''(f_e)$ to be negative. If $\delta_1 < 0$, we need $1 - \exp\left(\frac{\delta_2 - f_e}{\delta_3}\right) > 0$, which implies that $f_e > \delta_2 - \delta_3 \log(1)$ (since $f_e \geq 0$). If $\delta_1 < 0$, we need $1 - \exp\left(\frac{\delta_2 - f_e}{\delta_3}\right) < 0$, which implies that $f_e < \delta_2 - \delta_3 \log(1)$.

Combining these insights, we can conclude that the function

$$\sigma(f_e) = \frac{\delta_1}{1 + \exp\left(\frac{\delta_2 - f_e}{\delta_3}\right)} + \delta_4 \tag{4.83}$$

becomes strictly concave in the entire interval. So, it is strictly concave for $f_e \geq 0$ if: (i) $\delta_1 > 0, \delta_3 > 0$ and $f_e < \delta_2$; (ii) $\delta_1 < 0$, $\delta_3 > 0$, and $f_e > \delta_2$. If $\delta_3 < 0$, then the relation between f_e and δ_2 is naturally reversed. Note that the parameter δ_4 does not affect the convexity of the function, as it only shifts the function vertically. Therefore, we have derived the necessary conditions that ensure $\sigma''(f_e)$ is negative for all f_e, making $\sigma(f_e)$ strictly concave. □

It follows easily that it is strictly decreasing, continuous, and (continuously) differentiable with respect to the traffic flow $f_e \in [0, f_e^{\max}]$ for any edge $e \in \mathcal{E}$.

Now we provide a discussion of the error characterization of our approximation function. Let us define the error function Φ as the squared difference between $c_e^{\mathrm{PT}}(f_e)$ and $\sigma(f_e)$, integrated over the interval $[0, \kappa]$:

$$\Phi(\delta_1, \delta_2, \delta_3, \delta_4) = \int_0^{\kappa} \left(c_e^{\mathrm{PT}}(f_e) - \sigma(f_e)\right)^2 \mathrm{d}x. \tag{4.84}$$

The goal is to minimize Φ with respect to the parameters $\delta_1, \delta_2, \delta_3$, and δ_4. First, we find the critical points of Φ by setting its gradient to zero and solving the resulting system of equations: $\nabla\Phi(\delta_1, \delta_2, \delta_3, \delta_4) = 0$. This results in a system of equations involving the partial derivatives of Φ with respect to each of the parameters, i.e., $\frac{\partial \Phi}{\partial \delta_1} = 0$, $\frac{\partial \Phi}{\partial \delta_2} = 0, \frac{\partial \Phi}{\partial \delta_3} = 0$, and $\frac{\partial \Phi}{\partial \delta_4} = 0$. To compute these partial derivatives, we need to differentiate the integrand with respect to each parameter and then integrate it again, for example, $\frac{\partial \Phi}{\partial \delta_1} = \int_0^{\kappa} \frac{\partial}{\partial \delta_1} \left(c_e^{\mathrm{PT}}(f_e) - \sigma(f_e)\right)^2 \mathrm{d}x$. This process needs to be repeated for all parameters. However, due to the complexity of the function $c_e^{\mathrm{PT}}(f_e)$ (being a non-linear piecewise function), it is not possible to obtain an explicit analytical expression for these partial derivatives. For our purposes, we rely on numerical optimization techniques to find the exact best-fit parameters that minimize the error function, as these methods can easily handle complex and non-linear optimization. In the next subsection, we provide a numerical example that showcases the efficacy of the approximation function.

Theorem 4.2.9. *The error* $\phi(\cdot) = \left(c_e^{PT}(f_e) - \sigma(f_e)\right)$ *is upper bounded by* $\gamma + \varepsilon$*, where* γ *is some real number and* $\varepsilon > 0$*.*

Proof. For the purposes of this proof we assume that $\beta = 0.5$, $\lambda = 2$ and $c_e^0 = 13$ and $f_e^{\text{CRT}} = 1$ and $c_e^0 = 14.95$. We substitute now the known equations to get $\phi(\delta_1, \delta_2, \delta_3, \delta_4) = -w(1-\pi_e)\cdot[\tilde{c}_e(f_e \mid c_e(f_e) > c_e^0)]^\beta - \frac{\delta_1}{1+\exp\left(\frac{\delta_2 - f_e}{\delta_3}\right)} - \delta_4$. Using a straightforward computation of the second-order derivative, we can get the inflection point of ϕ, which will lie in $(1, 1+\varepsilon)$. This means that it is sufficient for us to compute ϕ at $f_e = 1$, and focus on ϕ for $f_e > 1$. Since σ is smooth and strictly concave in that interval, it approximates the worst c_e^{PT} around the inflection point. So, we have the following $\phi(\delta_1, \delta_2, \delta_3, \delta_4) = -w(1-\pi_e)(c_e(f_e) - c_e^0)^\beta - \frac{\delta_1}{1+\exp\left(\frac{\delta_2 - f_e}{\delta_3}\right)} - \delta_4$. This expression simplifies to

$$\phi = -w(1-\pi_e)\left[13\left(1 + \frac{3}{20}(f_e)^4\right) - 14.95\right]^\beta - \frac{\delta_1}{1+\exp\left(\frac{\delta_2 - f_e}{\delta_3}\right)} - \delta_4, \tag{4.85}$$

where we have $\delta_1 < 0$ and $\delta_2, \delta_3, \delta_4 > 0$, and $\delta_2 > f_e$. Since $w(1-\pi_e)$ is only a positive parameter constant, it is negligible, and so we drop it from our analysis. The first component simplifies to $\left[\frac{39}{20}\left(1 - (f_e)^4\right)\right]^\beta$, which is negative when we evaluate near the inflection point. Next, it follows that the second component is positive for small values of δ_2 and δ_3. We use the Taylor series expansion evaluated at $f_e = 1+\varepsilon$, where ε is a small positive number to get

$$-\left[\frac{39}{20}\left((f_e)^4 - 1\right)\right]^\beta = -\sqrt{\frac{39}{5}}\sqrt{\varepsilon} - \frac{3}{4}\frac{\sqrt{39}}{5}\varepsilon^{\frac{3}{2}} - \frac{7}{32}\frac{\sqrt{39}}{5}\varepsilon^{\frac{5}{2}} + O(\varepsilon^{\frac{7}{2}}), \tag{4.86}$$

which is clearly negative. For the second component, we use the Taylor series expansion again at the same point $f_e = 1 + \varepsilon$, and get the following

$$\frac{\delta_1}{1+\exp\left(\frac{\delta_2-(1+\varepsilon)}{\delta_3}\right)} = \frac{\delta_1}{1+\exp\left(\frac{\delta_2-1}{\delta_3}\right)} + \frac{\delta_1 \exp\left(\frac{\delta_2-1}{\delta_3}\right)}{\delta_3\left(1+\exp\left(\frac{\delta_2-1}{\delta_3}\right)\right)^2} \cdot \varepsilon$$
$$\frac{1}{2}\frac{\delta_1 \exp\left(\frac{1}{\delta_3}+\frac{\delta_2}{\delta_3}\right)\left(\exp\left(\frac{1}{\delta_3}\right)-\exp\left(\frac{\delta_2}{\delta_3}\right)\right)}{\delta_3^2\left(\exp\left(\frac{1}{\delta_3}\right)+\exp\left(\frac{\delta_2}{\delta_3}\right)\right)^3} \cdot \varepsilon^2 + O(\varepsilon^3) \quad (4.87)$$

We combine the expressions for the first and second components. From what we have established so far we get that the error is given by

$$\phi = -\sqrt{\frac{39}{5}}\sqrt{\varepsilon} - \frac{3}{4}\frac{\sqrt{39}}{5}\varepsilon^{\frac{3}{2}} - \frac{7}{32}\frac{\sqrt{39}}{5}\varepsilon^{\frac{5}{2}} + O(\varepsilon^{\frac{7}{2}})$$
$$+ \frac{\delta_1}{1+\exp\left(\frac{\delta_2-1}{\delta_3}\right)} + \frac{\delta_1 \exp\left(\frac{\delta_2-1}{\delta_3}\right)}{\delta_3\left(1+\exp\left(\frac{\delta_2-1}{\delta_3}\right)\right)^2} \cdot \varepsilon$$
$$+ \frac{1}{2}\frac{\delta_1 \exp\left(\frac{1}{\delta_3}+\frac{\delta_2}{\delta_3}\right)\left(\exp\left(\frac{1}{\delta_3}\right)-\exp\left(\frac{\delta_2}{\delta_3}\right)\right)}{\delta_3^2\left(\exp\left(\frac{1}{\delta_3}\right)+\exp\left(\frac{\delta_2}{\delta_3}\right)\right)^3} \cdot \varepsilon^2 + O(\varepsilon^3). \quad (4.88)$$

We want to find an upper bound for the error, which means we need to show that (4.88) is less than or equal to $\gamma + \varepsilon$ for some $\gamma \in \mathbb{R}$. Note that for any $a, b \in \mathbb{R}$ with $a < 0$ and $b > 0$, it is always true that $a + b \leq \max\{a, b\}$. Thus, we can write

$$\phi \leq \max\Bigg\{ -\sqrt{\frac{39}{5}}\sqrt{\varepsilon} - \frac{3}{4}\frac{\sqrt{39}}{5}\varepsilon^{\frac{3}{2}} - \frac{7}{32}\frac{\sqrt{39}}{5}\varepsilon^{\frac{5}{2}} + O(\varepsilon^{\frac{7}{2}}),$$
$$\frac{\delta_1}{1+\exp\left(\frac{\delta_2-1}{\delta_3}\right)} + \frac{\delta_1 \exp\left(\frac{\delta_2-1}{\delta_3}\right)}{\delta_3\left(1+\exp\left(\frac{\delta_2-1}{\delta_3}\right)\right)^2} \cdot \varepsilon$$
$$+ \frac{1}{2}\frac{\delta_1 \exp\left(\frac{1}{\delta_3}+\frac{\delta_2}{\delta_3}\right)\left(\exp\left(\frac{1}{\delta_3}\right)-\exp\left(\frac{\delta_2}{\delta_3}\right)\right)}{\delta_3^2\left(\exp\left(\frac{1}{\delta_3}\right)+\exp\left(\frac{\delta_2}{\delta_3}\right)\right)^3} \cdot \varepsilon^2 + O(\varepsilon^3)\Bigg\}. \quad (4.89)$$

As ε is positively small, we take the limit as $\varepsilon \to 0$. We note that the term $-\sqrt{\frac{39}{5}}\sqrt{\varepsilon}$ dominates as $\varepsilon \to 0$, and so the first component approaches $-\infty$ as $\varepsilon \to 0$. For the second component, the term $\frac{\delta_1}{1+\exp\left(\frac{\delta_2-1}{\delta_3}\right)}$ dominates as $\varepsilon \to 0$, and since $\delta_1 < 0$ and $\delta_2, \delta_3 > 0$, the second component is positive. Therefore, we can write

$$\lim_{\varepsilon \to 0} \phi \leq \lim_{\varepsilon \to 0} \max \left\{ -\sqrt{\frac{39}{5}}\sqrt{\varepsilon}, \frac{\delta_1}{1+\exp\left(\frac{\delta_2-1}{\delta_3}\right)} \right\}. \tag{4.90}$$

As $\varepsilon \to 0$, we have $-\sqrt{\frac{39}{5}}\sqrt{\varepsilon} \to -\infty$, hence

$$\begin{aligned} \lim_{\varepsilon \to 0} \phi &\leq \lim_{\varepsilon \to 0} \max \left\{ -\sqrt{\frac{39}{5}}\sqrt{\varepsilon}, \frac{\delta_1}{1+\exp\left(\frac{\delta_2-1}{\delta_3}\right)} \right\} \\ &= \frac{\delta_1}{1+\exp\left(\frac{\delta_2-1}{\delta_3}\right)}. \end{aligned} \tag{4.91}$$

Now, let $\gamma = \frac{\delta_1}{1+\exp\left(\frac{\delta_2-1}{\delta_3}\right)}$. Since the second component is positive, we have $\gamma > 0$, thus $\phi \leq \gamma + \varepsilon$. Therefore, we have shown that the error ϕ is upper bounded by $\gamma + \varepsilon$, where $\gamma \in \mathbb{R}$ and $\varepsilon > 0$. □

Lemma 4.2.10. *A traffic flow x^* is a NE if and only if, for all $x \in \mathcal{X}$, for all $i \in \mathcal{I}$, we have $\langle \nabla_i c_i(x_i^*, x_{-i}^*), x_i - x_i^* \rangle \geq 0$, where by the chain rule with respect to the $x_i^{r_i}$ we can take the partial derivatives on the cost function to get*

$$\left\langle \sum_{r_i \in \mathcal{R}} \sum_{e \in r_i} \sigma'(f_e^*), x_i - x_i^* \right\rangle \geq 0. \tag{4.92}$$

Proof. We apply the chain rule with respect to the $x_i^{r_i}$ and substituting the given cost function $c_i(x)$ and flow $f_e(x)$. The traffic flow x^* is a NE if and only if, for all $x \in \mathcal{X}$ and for all $i \in \mathcal{I}$, we have $\langle \nabla_i c_i(x_i^*, x_{-i}^*), x_i - x_i^* \rangle \geq 0$, where $c_i(x) = \sum_{r_i \in \mathcal{R}} \sum_{e \in r_i} \sigma(f_e)$ and $f_e(x) = \sum_i \sum_{r_i \ni e} x_i^{r_i}$. Now, we compute the gradient of the cost function with respect to x_i:

$$\nabla_i c_i(x_i^*, x_{-i}^*) = \sum_{r_i \in \mathcal{R}} \sum_{e \in r_i} \frac{\partial \sigma(f_e)}{\partial x_i^{r_i}}. \tag{4.93}$$

To compute the partial derivative $\frac{\partial \sigma(f_e)}{\partial x_i^{r_i}}$, we use the chain rule: $\frac{\partial \sigma(f_e)}{\partial x_i^{r_i}} = \sigma'(f_e) \cdot \frac{\partial f_e}{\partial x_i^{r_i}}$, where $\sigma'(f_e)$ is the derivative of the function σ with respect to f_e. Since $f_e(x) =$

$\sum_i \sum_{r_i \ni e} x_i^{r_i}$, we have $\frac{\partial f_e}{\partial x_i^{r_i}} = 1$. Now, we can substitute this back into the gradient expression $\nabla_i c_i(x_i^*, x_{-i}^*) = \sum_{r_i \in \mathcal{R}} \sum_{e \in r_i} \sigma'(f_e^*)$. Finally, we can rewrite the NE condition using the gradient

$$\left\langle \sum_{r_i \in \mathcal{R}} \sum_{e \in r_i} \sigma'(f_e^*), x_i - x_i^* \right\rangle \geq 0. \tag{4.94}$$

This is the necessary and sufficient first-order condition for x_i^* of any player i to be a minimum of cost function $c_i(\cdot)$. □

Theorem 4.2.11. *The game $\mathcal{M}$ admits at least one NE.*

Proof. We formally prove the existence of a NE in the prospect-theoretic routing game using Brouwer's fixed point theorem. Recall that for any player i, $c_i^{\mathrm{PT}}(x) = \sum_{r_i \in \mathcal{R}} \sum_{e \in r_i} \sigma(f_e(x))$, where σ is our smooth and monotonic approximation function. We define the best-response correspondence for each player i as: $b_i(x_{-i}) = \arg\max_{x_i} c_i^{\mathrm{PT}}(x)$. Smoothness in the approximation function σ implies that it is continuous and has continuous derivatives. This implies that we can estimate the utility function $c_i^{\mathrm{PT}}(x)$ continuously with respect to the traffic vector x. To show that the best-response correspondence $b_i(x_{-i})$ is continuous, we need the arg max operator to be continuous. Since the set of maximizers is compact, which actually follows from the compactness of the strategy space by Lemma 4.2.7. By Lemma 4.2.8, we have that σ is concave on a specific interval $[0, \kappa]$. This implies that we can estimate the utility function $c_i^{\mathrm{PT}}(x)$ within the interval $[0, \kappa]$ pointwise in a strictly decreasing and concave curve with respect to x_i for any player $i \in \mathcal{I}$. However, a strictly concave function has at most one unique maximum, which ensures the single-validness of the best-response correspondence $b_i(x_{-i})$. We now define the combined best-response correspondence $B(x) = (b_1(x_{-1}), b_2(x_{-2}), \ldots, b_n(x_{-n}))$. Since each $b_i(x_{-i})$ is continuous, $B(x)$ is also continuous, and thus it maps the strategy space to itself. Hence now we can apply Brouwer's fixed point theorem, which guarantees that there exists a fixed point $x^* = B(x^*)$; the result then follows. □

4.2.4 Simulation Results

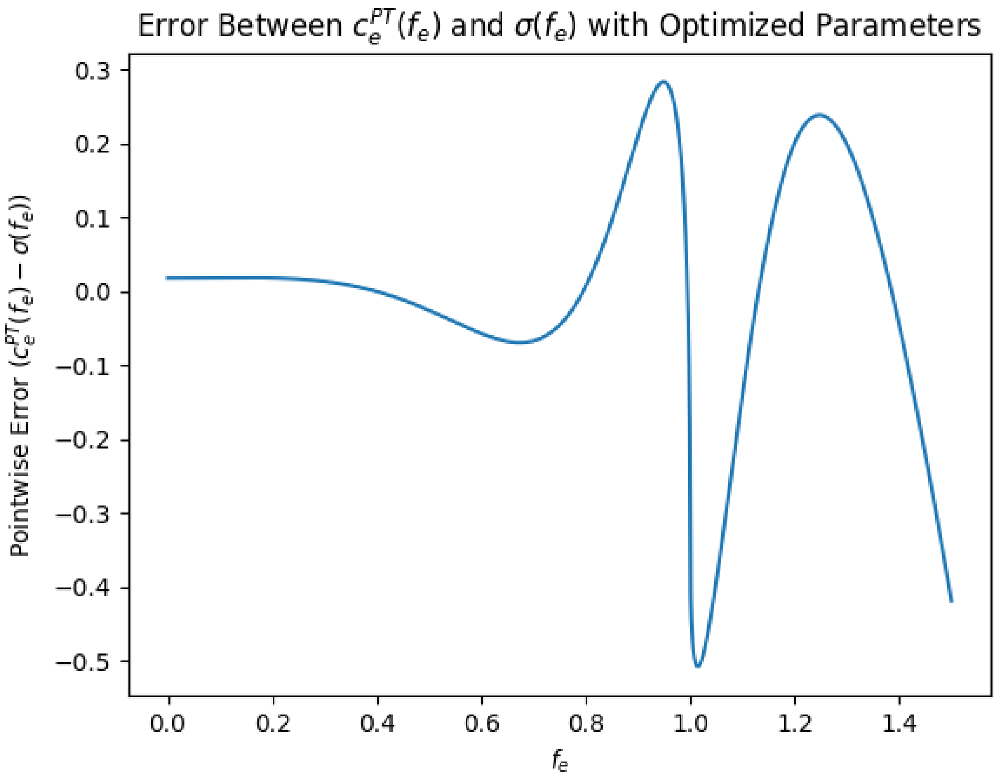

Figure 4.4: The plot of the errors.

In this section, we offer a numerical example to showcase the efficacy of our approach. We used the Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm to minimize the error between function $c_e^{\mathrm{PT}}(\cdot)$ and our approximation function $\sigma(\cdot)$ on the interval $[0, 3/2]$. We used the SciPy optimization function *minimize()* with the BFGS method to find the optimized parameters for $\sigma(\cdot)$ that minimize pointwise the smallest sum of squared errors. Table 4.2a shows the optimized parameters found by the BFGS algorithm, and Table 4.2b shows the error bound on the interval $[0, 3/2]$. The maximum error was 0.5072, the minimum error was 0, and the average error was 0.1043. These results provide an important summary of the performance of our approximation function and the best-fit parameters in estimating the prospect-theoretic function $c_e^{\mathrm{PT}}(\cdot)$.

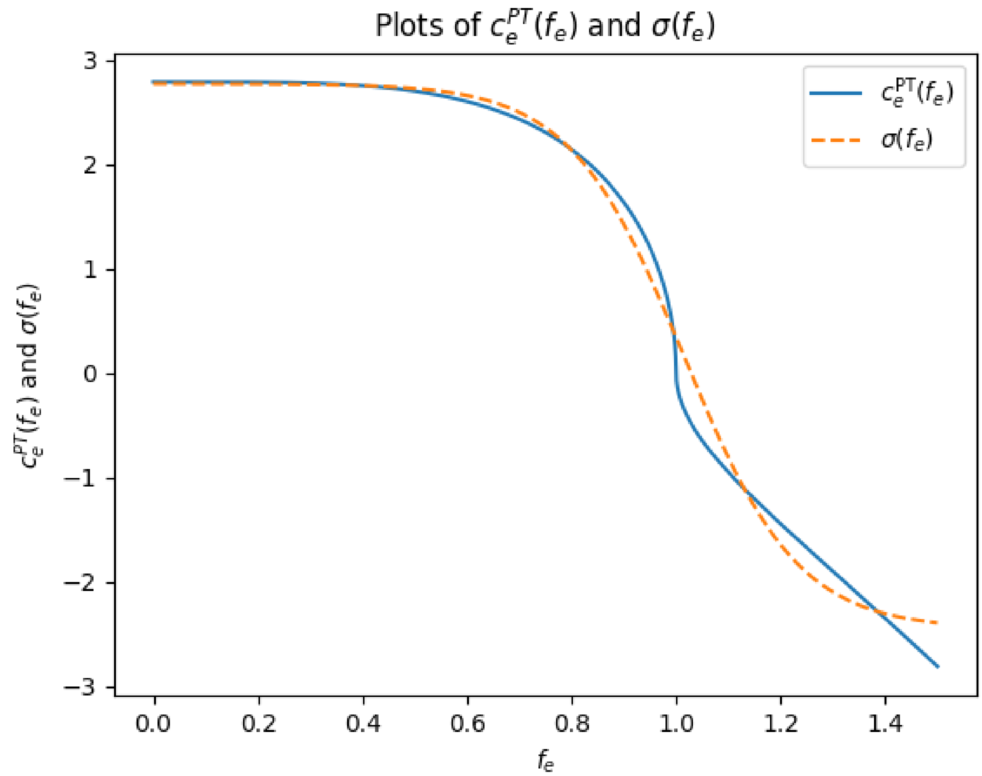

Figure 4.5: The plot of the original and approximation functions.

Parameter	Value
δ_1	-5.232
δ_2	1.015
δ_3	0.109
δ_4	2.776

(a) Optimized Parameters.

Error Measure	Value
Maximum Error	0.5072
Minimum Error	0.0000
Average Error	0.1043

(b) Error Bound on $[0, 3/2]$.

Table 4.2: Summarizing the parameters of the numerical example and the average/range of the error.

4.3 Summary

In this chapter, for the first part, we proposed a mobility game to study the behavioral interactions of travelers in a multimodal transportation network. First, we formulated a repeated non-cooperative routing game with a finite number of travelers.

In our first result, we showed that the mobility game admits a NE under the assumption of rationality. In our second result, we derived a bound for the PoA. Although we cannot have uniqueness at an equilibrium, our upper bound guarantees that the inefficiencies is as low as possible if the number of travelers is large enough (which is naturally expected in a mobility system). We also derived an upper bound for the PoS, showing that the greater the number of travelers, the close some NE can be to the social optimum. Next, we extended our game to consider the subjective behavior of travelers under prospect theory, and showed that our mobility game admits a NE.

In the second part of the chapter, we presented a prospect-theoretic game-theoretic modeling framework that incorporates an atomic splittable routing game with prospect theory to study travel behavior in mobility systems. We modeled the overestimation/underestimation of probabilities using Prelec's probability weighting function and we took into account the traffic uncertainties and travelers' perception of gains/losses in travel time using a prospect-theoretic S-shaped value function. We proposed an approximation function to address the non-linear and piecewise nature of the prospect-theoretic cost functions and showed that at least one NE exists. In addition, we derived an upper bound for the error. Lastly, we provided a numerical example and demonstrated the efficacy of our approximation for the routing game and summarized our results the best-fit parameters and range/average errors.

4.3.1 Implementation

In this subsection, we outline how our proposed mobility game can be potentially implemented. We consider a major metropolitan area with an extensive road and public transit infrastructure; a good example is Boston. Several key areas in Boston are connected by roads, buses, light rail, and bikes. These areas can serve as transport hubs from which travelers can utilize any of the available modes of transportation. We can apply the MaaS concept and offer on each transport hub travel services (e.g., navigation, location, booking, payment) to all passing travelers. Information can be shared among all travelers via a "mobility app," which allows travelers to access the services on the transport hubs. Using this app, travelers can pay for their travel needs and, at the same time, receive mobility payments. For example, a traveler who informs the app and uses a bike multiple times (per day or per week) can receive mobility payments. In addition, travelers travel multiple times and interact with each other more than once. So, travelers seek to move from one place to another while competing with many other travelers, use the transport hubs to access their preferred mode of transportation, and pay using a mobility app. Each mode of transportation offers different benefits in utility; for example, a car is more convenient than a bus and is expected to be in high demand. This naturally will lead to inefficiency and congestion.

The mobility game $\mathcal{M}$ with the utility structure defined in (4.4) captures the key factors that may play a role in a traveler's decision-making. It can be seen by Theorem 4.1.16 and Corollary 4.1.17 that an equilibrium exists and can be reached by the travelers without direct intervention from a central authority. The particular pricing mechanism we have proposed in (4.1) ensures all travelers with the computational power of their cellphone can quickly derive the NE strategy (route, transport hub, payment). This is important as we can avoid solving a mixed-integer nonlinear program for all the travelers in the mobility system. In addition, by Theorem 4.1.21 we can guarantee that the inefficiency of the mobility system stays low as long as the number of travelers remains large (something that is expected in a typical mobility system). We can guarantee that a NE will stay close to the social optimum as the number of travelers increases. Thus, even though we cannot guarantee the uniqueness of a NE, we can ensure that all NE are similarly efficient and nearly as efficient as the social optimum as long as the number of travelers is high.

Using prospect theory, we can also consider how a traveler can feel uncertain about whether they may receive mobility payments for choosing a more sustainable mode of transportation (e.g., bike). Under certain conditions, we show that indeed a NE exists (Theorem 4.1.25) and it can be reached by the travelers as they can travel from the hub that is nearest to their home to the hub that is nearest to their work (Corollary 4.1.26). Thus, our game $\mathcal{M}$ framework is proved to lead to a NE under two different behavioral models and capture the impact of the travelers' decision-making.

4.3.2 Technical Discussion and a Numerical Example

In this subsection, we discuss in more detail the technical implementation of our mobility game. So, we can compute a NE using the potential function, leveraging the fact that our game is a potential game with a finite set of travelers (as shown in Theorem 4.1.16). Note that this means that there exists a potential function (given by (4.7)) that maps each strategy profile to a real value. Intuitively, any change in any traveler's utility that unilaterally deviates from a strategy is equal to the change in our potential function. Hence, we can find the strategy profiles that simply maximize our potential function. One approach to achieve this for mixed-integer optimization problems is to use the branch-and-bound algorithm [196]. So, by Corollary 4.1.17, convergence to a pure strategy NE is guaranteed, thus we can find a NE, compute it, and use for our PoA analysis.

Typically, we solve numerically the optimization problem that arises from the routing game. We can either employ a gradient-based methods or a learning algorithm (e.g., fictitious play). The gradient-based method involves updating the travelers' strategies by moving in the direction of the gradient of the potential function. However, since the action sets are coupled and include route choices, the optional stop at a transport hub along the route, and the payment for the mobility service, the standard way solve this problem is as a mixed-integer nonlinear program (MINLP) (in such a case use the branch-and-bound or branch-and-cut algorithm). Alternatively, we can first use the specialized algorithm Dijkstra and find the best route and transport hub through the network based on criteria such as shortest distance and least time. Once we have this optimal route, we then use "fmincon" to compute the optimal value of the mobility payments along that route and transport hub.

We now offer a numerical example with a simple transportation network that has one unique origin-destination (OD) pair. In this network, there are three routes, namely route 1, route 2, and route 3. We assume that there are two mobility services: car and bike. The travel time on each route depends on the volume of traffic and the service. We have the following

$$\text{Route 1: } t_1(\text{car}) = 10 + x_1, \quad t_1(\text{bike}) = 8, \tag{4.95}$$

$$\text{Route 2: } t_2(\text{car}) = 5 + 2x_2, \quad t_2(\text{bike}) = 7, \tag{4.96}$$

$$\text{Route 3: } t_3(\text{car}) = 15, \quad t_3(\text{bike}) = 5 + x_3, \tag{4.97}$$

where x_k denotes the fraction of travelers choosing either of routes $k = 1, 2, 3$ using a car. We also consider that the pricing functions for the two modes: car and bike take the explicit form as $\tau(\text{car}) = 10$ and $\tau(\text{bike}) = -2$. So, travelers receive a \$2 incentive for choosing a bike. Suppose we have 50 travelers in total, with 30 travelers preferring route 1 and route 2 (we call this Group A), while the remaining 20 travelers prefer route 2 and route 3 (we call this Group B). Now, say that Group A chooses to utilize route 1 with a car, and Group B chooses to utilize route 3 using a car. We can now compute the utilities for each traveler:

$$\text{Route 1: } t_1(\text{car}) = 10 + 30 = 40, \quad \text{utility} = -40 - 10 = -50, \tag{4.98}$$

$$\text{Route 2: } t_2(\text{car}) = 5, \quad \text{utility} = -5 - 10 = -15, \tag{4.99}$$

$$\text{Route 3: } t_3(\text{car}) = 15, \quad \text{utility} = -15 - 10 = -25. \tag{4.100}$$

So, for Group A the utility for route 2 (car) is higher than route 1 (car), thus all travelers in Group A will deviate to route 2 (car). Similarly, in Group B the utility for route 3 (bike) is higher than Route 3 (car), and so all travelers will deviate to route 3 (bike). Let us now compute the utilities for any arbitrary traveler, i.e.,

$$\text{Route 1: } t_1(\text{car}) = 10, \quad u = -10 - 10 = -20, \tag{4.101}$$

$$\text{Route 1: } t_1(\text{bike}) = 8, \quad u = -8 + 2 = -6, \tag{4.102}$$

$$\text{Route 2: } t_2(\text{car}) = 5 + 2(30) = 65, \quad u = -65 - 10 = -75, \tag{4.103}$$

$$\text{Route 2: } t_2(\text{bike}) = 7, \quad u = -7 + 2 = -5, \tag{4.104}$$

$$\text{Route 3: } t_3(\text{car}) = 15, \quad u = -15 - 10 = -25, \tag{4.105}$$

$$\text{Route 3: } t_3(\text{bike}) = 5, \quad u = -5 + 2 = -3. \tag{4.106}$$

We continue our equilibrium analysis as follows: for Group A, the utility for route 1 (bike) is higher than Route 2 (car), so travelers will deviate to route 1 (bike). Next, Group B will not deviate as route 3 (bike) (already highest utility). Hence, we have reached the point in which no traveler can deviate and receive better utility; thus we have a NE. We notice that all travelers from Group A and Group B choose route 1 and route 3 utilizing bikes, respectively. We conclude that it is possible as we showed in our theoretical analysis for a NE to exist and it is easily converged to based on our pricing mechanism in this simple multimodal transportation network. Next, at this Ne, all travelers have chosen to utilize the mobility service: bike, which is a sustainable and environmentally-friendly mode of transportation. Our pricing mechanism naturally will favor such modes of transportation and provide incentives to travelers for better utilization. Thus, we can control travel demand and effectively reduce any inefficiencies that may arise from congestion or higher pollution levels caused by car usage. One last note: in this example, we used a similar method to the best-response dynamics approach and found the NE using only a few iterations. Although our example quickly leads to an equilibrium solution, for larger and more complex transportation networks, we cannot draw the same conclusion and thus, it remains future work to adopt more advanced optimization and algorithmic techniques in order to find and compute our mobility game's NE.

Chapter 5

CONCLUSIONS AND FUTURE WORK

In this dissertation, we have made significant contributions to the understanding of human behavior and the impact of selfish decision-making in mobility systems. Our research has broader implications for both society and academia, as it bridges the gap between behavioral economics, microeconomics, game theory, control, and transportation engineering. Our motivation stems from the desire to ensure accessibility and efficiency in the mobility systems of the future, as technological advancements make it easier for travelers to rely on cars. By examining traveler preferences, we aim to develop smart strategies for handling travel demand in a sustainable manner.

In Chapter 1, we identified research gaps in the existing literature and provided an overview of our research focus on social dilemmas in mobility decision-making. By doing so, we laid the foundation for our interdisciplinary approach, which combines insights from behavioral economics, game theory, control, and transportation engineering. In Chapter 2, we explored the social ramifications of human decision-making in connectivity and automation within a game-theoretic context. We investigated two methodologies for addressing the evolving social-mobility dilemma and derived conditions under which altruistic strategies emerge in the game. By investigating distinct methodologies, we offer novel insights into how mobility systems can handle travel demand while considering human preferences, ultimately promoting accessibility and efficiency in future transportation networks.

In Chapter 3, we delved into strategic traveler routing and mobility market design, examining various aspects of social resource allocation mechanism design in transportation networks. We proposed informationally decentralized mechanisms to efficiently allocate travel time and prevent congestion while achieving individual rationality, budget balance, and strong implementability. Furthermore, we explored a two-sided game for mobility systems and investigated the conditions for optimal, stable solutions in the presence of informational asymmetry. This interdisciplinary approach allows us to develop innovative solutions that address the challenges of congestion and resource allocation, paving the way for sustainable and efficient mobility systems in the future.

In Chapter 4, we investigated behavioral interactions in multimodal transportation networks using a mobility game. By incorporating prospect theory into our game-theoretic modeling framework, we examined travel behavior in mobility systems and provided insights into the existence of Nash equilibria under different behavioral assumptions. Our research contributes to the intellectual merit of the field by bridging theories from various disciplines, such as game theory, microeconomics, prospect theory, control, and transportation engineering, providing a comprehensive understanding of mobility systems and traveler routing. By incorporating these diverse perspectives, we provide a comprehensive understanding of travel behavior and routing decisions, ultimately informing the design of smart, efficient, and accessible mobility systems for the future.

In more detail the contributions in this dissertation can be characterized within the state of the art in the fields of mobility systems through the lens of game theory. The contributions of this dissertation are summarized as follows:

1. Provide a sociotechnical game-theoretic framework to study emerging mobility systems, setting up the stage for a holistic analysis of the social implications and consequences of CAVs while incorporating the social behavioral dynamics of travelers, passengers, and drivers (Chapter 2).

2. Comprehensive Mobility Market Design: advance our understanding of strategic

traveler routing and mobility market design by examining various aspects, such as two-sided games, shared mobility markets, and multi-modal mobility systems. It investigates the assignment of travelers to providers, the stability of such assignments, and the incorporation of informational asymmetry in mobility games. This research contributes to the development of economically sustainable, equitable, and efficient mobility markets by considering factors such as individual preferences, social welfare maximization, feasibility, stability, and budget fairness.(Chapter 3).

3. Behavioral Interactions in Multimodal Transportation Networks: This research contributes to the field by proposing a game-theoretic framework that incorporates both rational and subjective traveler behaviors within multimodal transportation networks. It demonstrates the existence of Nash equilibria under different behavioral assumptions and provides bounds for the Price of Anarchy and Price of Stability. By integrating prospect theory into the atomic splittable routing game, the research captures the nuances of traveler behavior in the face of traffic uncertainties and varying perceptions of travel time gains and losses, offering a more comprehensive understanding of travel behavior in modern mobility systems (Chapter 4).

In conclusion, this dissertation presents a significant contribution to the understanding of human behavior in mobility systems and offers valuable insights into the implications of selfish decision-making on the future of transportation. By bridging theories from different disciplines, our research provides a solid foundation for the development of innovative solutions that ensure accessibility, efficiency, and sustainability in the mobility systems of the future.

5.1 Future Work

In general, from an engineering standpoint, mechanism design is characterized by its trade-off between the design of optimal and efficient solutions that all agents will accept and realistic and system-wide properties such as simplicity, robustness, and computational trackability. As control problems are commonly dynamic, complex, and unpredictable [226], it still remains an open question to devise mechanisms are simple yet dynamic, robust, and trackable. At the same time, a key open question is to look at the intersection of mechanism design and machine learning allowing mechanisms with incentives that lead to efficient equilibria that can be learned in dynamic environments (i.e., extending the typical mechanism to address dynamic control problems). It is the authors' belief that engineering applications (e.g., communication networks, information systems, transportation networks) of large-scale systems, which are dynamic in nature, impose rather crucial challenges to the theoretical framework of mechanism design, and thus, inspire novel new mechanisms that will circumvent some of the limitations of the theory. The goal, of course, is to improve the applicability of these mechanisms in real-life problems and translate the usefulness of the theoretical insights into practice. Finally, game theory allows us to model the strategic interactions of systems consisted of multiple agents/players and who compete over resources. The theory of mechanism design allows us to adopt an objective-first approach and model the best possible game and its rules. As new developments and applications are continuous, it remains to be seen the next chapters of mechanism design in engineering and its true impact in solving the big problems.

One important limitation of our work in this dissertation is the assumption of complete information. Realistically, we cannot expect travelers to have accurate and complete knowledge of other travelers or the system's capabilities (network, road capacities). A potential direction for future research should relax this assumption by only allowing travelers to know their own actions and utilities. In the literature, attempts have been made to investigate the emergence of cooperation among selfish travelers and how to bound rationality/irrationality in travel-choice problems [74, 103, 118, 63]. A standard technique is Bayesian game-theoretic analyses, and recently, and techniques to learn representations of unknown information from observed data [12, 225, 136, 242, 57].

Another interesting direction for future research is to expand the current framework by explicitly designing the socially-efficient pricing functions to achieve the best possible equilibrium in the mobility system using techniques from mechanism design. Furthermore, to showcase the benefits of the proposed game-theoretic approach, a necessary extension of our work is using machine learning techniques [13, 183, 244, 131, 117] with real-life data.

Future work should also include conducting a simulation-based analysis under different traffic scenarios to showcase the practical implications of our work. An interesting research direction would involve to extend and enhance the traveler-behavioral model, motivated by a social-mobility survey. The objective with such a survey would be to observe any correlations between behavioral tendencies or attitudes of travelers and how they use shared vehicles (e.g., Uber, Lyft, taxicabs). One should address the following unanswered questions: (i) "will CAVs play a role and have a significant impact on the travelers' tendencies and behavior regarding mobility?" and (ii) "in emerging mobility systems, how likely will people share CAVs and will be the implications of a major mode of transportation shift?" We can provide asnwers to these question by including an extention and enhancement of the traveler-behavioral model, motivated by a social-mobility survey. Such an objective is to observe any correlations between behavioral tendencies or attitudes of travelers and their mode of transportation preference (including CAVs). For example, how likely are people to use CAVs instead of public transit? Will CAVs impact travelers' tendencies and behavior; if yes, then in what way? Answers can help us refine the proposed mobility market and improve our understanding of the socioeconomic impact of CAVs. Future research efforts will also focus on using methods, techniques, and insights from behavioral economics and mixed integer optimization theory to develop a holistic framework of the societal impact of connectivity and automation in mobility and provide socially-efficient, real-time solutions while tackling any potential rebound effects.

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