



Research article

A constrained optimal control framework for vehicle platoons with delayed communication

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Abstract: Vehicle platooning using connected and automated vehicles (CAVs) has attracted considerable attention. In this paper, we address the problem of optimal coordination of CAV platoons at a highway on-ramp merging scenario. We present a single-level constrained optimal control framework that optimizes the fuel economy and travel time of the platoons while satisfying the state, control, and safety constraints. We also explore the effect of delayed communication among the CAV platoons and propose a robust coordination framework to enforce lateral and rear-end collision avoidance constraints in the presence of bounded delays. We provide a closed-form analytical solution to the optimal control problem with safety guarantees that can be implemented in real time. Finally, we validate the effectiveness of the proposed control framework using a high-fidelity commercial simulation environment.

Keywords: optimal control; platoon coordination; connected automated and vehicles; intelligent transportation systems; vehicle communication

Mathematics Subject Classification: 58F15, 58F17, 53C35

1. Introduction

1.1. Motivation

Traffic congestion has increased significantly over the last decade [66]. Traffic scenarios such as crossing urban intersections, merging roadways, highway on-ramps, roundabouts, and speed reduction zones along with the driver responses [35, 69] to various disturbances in the transportation network are the primary sources of traffic congestion [56]. Emerging mobility systems, e.g., connected and automated vehicles (CAVs), lay the foundation to improve safety and transportation efficiency at these scenarios by providing the users the opportunity to better monitor the transportation network conditions and make optimal decisions [24, 70, 78]. Having enhanced computational capabilities, CAVs can establish real-time communication with other vehicles and infrastructure to

increase the capacity of critical traffic corridors, decrease travel time, and improve fuel efficiency and safety [8, 14, 44, 49, 87]. However, the cyber-physical nature of emerging mobility systems, e.g., data and shared information through vehicle-to-vehicle (V2V) and vehicle-to-infrastructure (V2I) communication, is associated with significant technical challenges and gives rise to a new level of complexity [51] in modeling and control [27].

There have been two major approaches to utilizing the connectivity and automation of vehicles, namely, coordination and platooning. The concept of coordination through different traffic scenarios is enabled by the vehicle-to-everything communication protocol among the CAVs and the surrounding infrastructure. On the other hand, real-time computation and automation of CAVs enables safe and comfortable trajectories with extremely short headway in the form of CAV platoon, which consists of a string [11] of consecutive CAVs traveling together at a constant headway and speed.

In this paper, we employ the concepts of CAV coordination and platooning to address the problem of minimizing traffic congestion at the traffic scenarios in an energy-efficient manner. In particular, we aim at optimally coordinating platoons of CAVs at highway on-ramp merging [60] in the presence of bounded delays among platoon leaders while guaranteeing state, control, and safety constraints.

1.2. Literature review

CAV coordination has been explored to mitigate the speed variation of individual CAVs throughout the transportation network [59]. Early efforts [5, 39] considered a single string of vehicles that was coordinated through a merging roadway by employing a linear optimal regulator. In 1993, Varaiya [77] outlined the key features of an automated intelligent vehicle/highway system, and proposed a basic control system architecture. In 2004, Dresner and Stone [22] proposed the use of the reservation scheme to control a signal-free intersection of two roads. Since then, several research efforts [3, 6, 21, 23, 30] have extended this approach for coordination of CAVs at urban intersections. More recently, a decentralized optimal control framework was presented in [50, 53] for coordinating online CAVs at different traffic scenarios such as on-ramp merging roadways, roundabouts, speed reduction zones, and signal-free intersections. The framework uses a hierarchical structure consisting of an upper-level vehicle coordination problem to minimize travel time and a low-level energy minimization problem. The state and control constraints in the coordination problem has been addressed in [16, 17, 45, 47, 53, 54] by incorporating the constraints in the low-level optimization problem, and in [52, 55] by incorporating the constraints in the upper-level optimization problem. Detailed discussions of the research reported in the literature to date on the coordination of CAVs can be found in [64] and [28].

The aforementioned coordination approaches are vehicle-centric approaches focusing on control of individual CAVs within the network, whereas platooning can leverage the full potential of CAVs to enhance the current optimal coordination of CAVs. The concept of platoon formation gained momentum in the 1980s and 1990s as a system-level approach to address traffic congestion [63, 68, 77] and has been shown to have significant benefits [2, 38]. Shladover et al. [68] presented the concept of operating automated vehicles in closely spaced platoons as part of an automated highway system, and pioneered the California Partners for Advanced Transportation Technology (PATH) program to conduct heavy-duty truck platooning from 2001 to 2003. Rajamani et al. [63] discussed the lateral and longitudinal control of CAVs for the automated platoon formation. From a system point of view, platooning of vehicles yields additional mobility benefits. It has been

shown that capacity at a traffic bottleneck, such as an intersection, can be doubled or even tripled by platooning of vehicles [40]. Moreover, platooning improves the fuel efficiency of the vehicles due to the reduction of the aerodynamic drag within the platoon, especially at high cruising speeds [1, 9, 67, 73]. Various research efforts in the literature have addressed vehicle platooning at highways to increase fuel efficiency, traffic flow, driver comfort, and safety. To date, there has been a rich body of research focusing on exploring several methods of forming and/or utilizing platoons to improve transportation efficiency [4, 12, 31, 33, 46, 62, 76, 79, 82, 85]. A detailed discussion on different approaches in vehicle platooning systems at highways can be found in [10, 34, 86].

1.3. Objectives and contributions of the paper

In this paper, we address the problem of coordinating CAV platoons at a highway on-ramp merging. The main objective is to leverage the key concepts of CAV coordination and platooning, and establish a control framework for platoon coordination aimed at improving network performance while guaranteeing safety.

The key contributions of this paper are (i) the development of a mathematically rigorous optimal control framework for platoon coordination that completely eliminates stop-and-go driving behavior, and improves fuel economy and traffic throughput of the network, (ii) the derivation and implementation of the optimal control input in real time that satisfies the state, control, and safety constraints subject to bounded delayed communication, and (iii) the validation of the proposed control framework using a commercial traffic simulator by evaluating its performance compared to a baseline scenario.

1.4. Comparison with related work

To the best of our knowledge, this paper is the first attempt to establish a rigorous constrained optimal control framework for coordination of vehicular platoons at a highway on-ramp merging in the presence of bounded inter-platoon delays. This paper advances the state of the art in the following ways. First, in contrast to other efforts that neglected state/control constraints [36, 37, 71], our framework guarantees satisfaction of all of the state, control, and safety constraints in the system. Second, our framework unlike the several efforts in the literature at highway on-ramp merging scenario [61, 65, 81, 83] does not impose a strict first-in-first-out queuing policy to ensure lateral safety. Third, in this paper, we consider the bounded delay in the inter-platoon communication, which most of the studies in the coordination of vehicular platoons neglect [20, 29, 36]. Finally, our framework yields a closed-form analytical solution while satisfying all of the system constraints, and thus it is appropriate for real-time implementation on-board the CAVs [41].

1.5. Organization of the paper

The remainder of the paper is organized as follows. In Section 2, we present the modeling framework and formulate the problem. In Section 3, we provide a detailed exposition of the optimal control framework and the algorithm to implement the closed-form analytical solution to the constrained optimal control problem. In Section 4, we evaluate the effectiveness of the proposed approach in a simulation environment. Finally, we draw conclusions and discuss the next steps in Section 5.

2. Modeling framework

We consider the problem of coordinating platoons of CAVs in a scenario of highway on-ramp merging (Figure 1). Although our analysis can be applied to any traffic scenario, e.g., signal-free intersections, roundabouts, and speed reduction zones, we use a highway on-ramp as a reference to present the fundamental ideas and results of this paper.

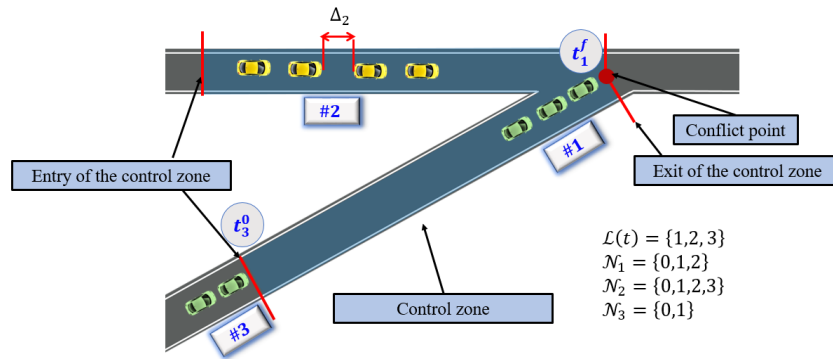


Figure 1. On-ramp merging with a single conflict point for platoons of CAVs. The control zone is highlighted in light blue color, the entry time and exit time to the control zone are depicted with circles, and example sets of platoon leaders and followers are shown. The structure of the set of platoons $\mathcal{L}(t)$ and the set of CAVs \mathcal{N}_i for each platoon $i \in \mathcal{L}(t)$ are shown, where the formal definitions are given in Definitions 2.1 and 2.2.

The on-ramp merging scenario includes a control zone that spans a finite area of both the main road and the ramp road, within which the platoons of CAVs are coordinated using an optimal control framework. Inside the control zone, the leader of each platoon can communicate with the coordinator. The coordinator does not make any decisions for the CAVs and only acts as a database for the CAVs. The paths of the main road and the ramp road intersect at a point called conflict point, indexed by $n \in \mathbb{N}$, at which lateral collision may occur. We consider that CAVs have already formed platoons upstream of the control zone. We refer interested readers to [42, 46, 48, 74] for further details on platoon formation.

2.1. Network topology and communication

In our modeling framework, we impose the following communication structure based on the standard V2V and V2I communication protocol as shown in Figure 2.

1. Bidirectional inter-platoon communication: The leaders of each platoon can exchange information with each other via the coordinator through a V2I communication protocol. The flow of information is bidirectional.
2. Unidirectional intra-platoon communication: The following CAVs of each platoon can subscribe to the platoon leader's state and control information. The flow of information is unidirectional from the platoon leader to the following CAVs within that platoon. Note that the unidirectional communication is sufficient for our coordination framework and it is more restrictive. However, our framework could also seamlessly incorporate bi-directional intra-platoon communication.

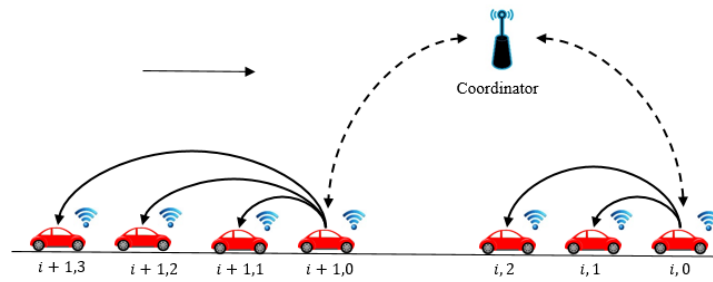


Figure 2. Network topology for information flow: (i) bidirectional inter-platoon communication (dashed double-headed arrow) between the platoon leaders via the coordinator, and (ii) unidirectional intra-platoon communication (solid single-headed arrow) from platoon leader to the platoon followers.

When a platoon leader enters the control zone, it subscribes to the bidirectional inter-platoon communication protocol to connect with the coordinator and access the information of platoons that are already in the control zone. After obtaining this information, the leader derives its optimal control input (acceleration/deceleration) to cross the control zone without any lateral or rear-end collision with the other CAVs, and without violating any of the state and control constraints. The leader then communicates its derived control input and trajectory information to its followers using the unidirectional intra-platoon communication protocol so that the following CAVs can compute their control input. Finally, the platoon leader transmits its information to the coordinator so that the subsequent platoon leaders can plan their trajectories accordingly.

In this paper, we enhance our framework to consider delayed transmission during the inter-platoon communication protocol due to the physical distance among the platoons. On the other hand, since the CAVs within each platoon are closely spaced, we consider that there is an instantaneous flow of information within the intra-platoon communication protocol. In our modeling framework, we make the following assumption regarding the nature of delay during the inter-platoon communication protocol.

Assumption 1. The communication delay during the bidirectional inter-platoon communication between each platoon leader and the coordinator is bounded and known a priori.

Assumption 1 enables the determination of upper bounds on the state uncertainties as a result of sensing or communication errors and delays, and incorporates these into more conservative safety constraints, the exposition of which we provide in Section 3.3. This is a reasonable assumption since the boundedness of the delay is necessary to make the coordination framework robust [72, 84]. Even if the delay is not bounded, we can always construct a probabilistic bound and prove the robustness with predefined probability. Furthermore, we can either learn about the delay in the communication online by collecting data before arriving at the control zone or use some prior estimate.

2.2. Dynamics and constraints

Next, we provide some definitions that are necessary in our exposition.

Definition 2.1. The queue that designates the order in which each platoon leader entered the control zone is given by $\mathcal{L}(t) = \{1, \dots, L(t)\}$, where $L(t) \in \mathbb{N}$ is the total number of platoons that are inside the

control zone at time $t \in \mathbb{R}_{\geq 0}$. When a platoon exits the control zone, its index is removed from $\mathcal{L}(t)$.

Definition 2.2. CAVs within platoon $i \in \mathcal{L}(t)$ are indexed with set $\mathcal{N}_i = \{0, 1, \dots, m_i\}$, where 0 and $m_i \in \mathbb{N}$ denote the leader and last CAV of the platoon i , respectively. The size of each platoon $i \in \mathcal{L}(t)$ is thus the cardinality of set \mathcal{N}_i , and denoted by $M_i := m_i + 1$.

In our analysis, we consider that the dynamics of each CAV $j \in \mathcal{N}_i$ in platoon $i \in \mathcal{L}(t)$ is governed by a double integrator,

$$\begin{aligned}\dot{p}_{i,j}(t) &= v_{i,j}(t), \\ \dot{v}_{i,j}(t) &= u_{i,j}(t),\end{aligned}\tag{2.1}$$

where $p_{i,j}(t) \in \mathcal{P}$, $v_{i,j}(t) \in \mathcal{V}$, and $u_{i,j}(t) \in \mathcal{U}$ denote position, speed, and control input at $t \in \mathbb{R}_{\geq 0}$, respectively. The sets \mathcal{P} , \mathcal{V} , and \mathcal{U} , are compact subsets of \mathbb{R} . Inside the control zone, the control input $u_{i,j}(t)$ of each CAV $j \in \mathcal{N}_i$, $i \in \mathcal{L}(t)$ is computed based on the optimal coordination framework proposed in Section 3. Outside the control zone, the CAVs are controlled by the Wiedemann car-following model [80] adopted by VISSIM [25]. The discussion on the impact of car-following model and the associated uncertainties at the upstream and downstream of the control zone falls outside the scope of this paper.

Remark 1. In what follows, to simplify the notations, we use the subscript i instead of $i, 0$ to denote the leader of platoon $i \in \mathcal{L}(t)$.

Let $t_{i,0}^0 = t_i^0 \in \mathbb{R}_{\geq 0}$ be the time that the leader of platoon $i \in \mathcal{N}(t)$ enters the control zone, and

$$t_{i,0}^f = t_i^f > t_i^0 \in \mathbb{R}_{\geq 0}$$

be the time that the leader of platoon i exits the control zone. Since each CAV $j \in \mathcal{N}_i$, $i \in \mathcal{L}(t)$, has already formed a platoon upstream of the control zone, when the leader enters the control zone at time t_i^0 , we have

$$v_{i,j-1}(t_i^0) - v_{i,j}(t_i^0) = 0$$

and

$$p_{i,j-1}(t_i^0) - p_{i,j}(t_i^0) - l_c = \Delta_i,$$

where l_c denote the length of each CAV j , and Δ_i is the safe bumper-to-bumper inter-vehicle gap between CAVs $j, j-1 \in \mathcal{N}_i$ within each platoon $i \in \mathcal{L}(t)$. This bumper-to-bumper inter-vehicle gap is imposed by the platoon forming control in platooning zone upstream of the control zone. After exiting the control zone at t_i^f , the leader of platoon i cruises with constant speed $v_i(t_i^f)$ until the last follower in the platoon exits the control zone. Afterwards, each platoon member $j \in \mathcal{N}_i$, $i \in \mathcal{L}(t)$ is controlled by a suitable car-following model [80] which ensures satisfying rear-end safety constraint.

For each CAV $j \in \mathcal{N}_i$ in platoon $i \in \mathcal{L}(t)$ the control input and speed are bounded by

$$u_{\min} \leq u_{i,j}(t) \leq u_{\max},\tag{2.2}$$

$$0 < v_{\min} \leq v_{i,j}(t) \leq v_{\max},\tag{2.3}$$

where u_{\min}, u_{\max} are the minimum and maximum control inputs and v_{\min}, v_{\max} are the minimum and maximum speed limit, respectively.

To ensure rear-end safety between platoon $i \in \mathcal{L}(t)$ and preceding platoon $k \in \mathcal{L}(t)$, we have

$$p_{k,m_k}(t) - p_i(t) \geq \delta_i(t) = \gamma + \varphi \cdot v_i(t), \quad (2.4)$$

where m_k is the last follower in the platoon k physically located in front of platoon i and $\delta_i(t)$ is the safe speed-dependent distance, while γ and $\varphi \in \mathbb{R}_{>0}$ are the standstill distance and reaction time, respectively.

Similarly, to guarantee rear-end safety within CAVs inside each platoon $i \in \mathcal{L}(t)$, we enforce

$$p_{i,j-1}(t) - p_{i,j}(t) \geq \Delta_i + l_c, \quad \forall j \in \{1, \dots, m_i\}. \quad (2.5)$$

Finally, let $k \in \mathcal{L}(t)$ correspond to another platoon that has already entered the control zone and may have a lateral collision with platoon $i \in \mathcal{L}(t)$ at the conflict point.

For the first case in which platoon i reaches the conflict point after platoon k , we have

$$t_i^f - t_{k,m_k}^f \geq t_h, \quad (2.6)$$

where $t_h \in \mathbb{R}_{>0}$ is the minimum time headway between any two CAVs entering the conflict point that guarantees safety, t_i^f is the time that the leader of platoon i exits the control zone (recall that the conflict point is located at the exit of the control zone), and t_{k,m_k}^f is the time that the last CAV of the platoon k exits the control zone. Likewise, for the second case in which platoon i reaches the conflict point before platoon k , we have

$$t_k^f - t_{i,m_i}^f \geq t_h. \quad (2.7)$$

Remark 2. Since $0 < v_{\min} \leq v_{i,j}(t)$, the position $p_{i,j}(t)$ is a strictly increasing function. Thus, the inverse of $t_i(\cdot) = p_{i,j}^{-1}(\cdot)$ exists and it is called the *time trajectory* of CAV j in platoon i [52]. Therefore, for each candidate path of platoon i , there exists a unique time trajectory which can be evaluated at the conflict point to find the time t_i^f that platoon $i \in \mathcal{L}(t)$ reaches the conflict point.

Remark 3. Given the time t_i^f that the platoon leader of platoon $i \in \mathcal{L}(t)$ exits the control zone, we compute the time t_{i,m_i}^f that the last platoon member $m_i \in \mathcal{N}_i$ exits the control zone as

$$t_{i,m_i}^f = t_i^f + \frac{(M_i - 1)(\Delta_i + l_c)}{v_i(t_i^f)}. \quad (2.8)$$

To guarantee lateral safety between platoon i and platoon k at the conflict point, either (2.6) or (2.7) must be satisfied. Therefore, we impose the following lateral safety constraint on platoon i ,

$$\min \left\{ t_h - (t_i^f - t_{k,m_k}^f), t_h - (t_k^f - t_{i,m_i}^f) \right\} \leq 0. \quad (2.9)$$

With the state, control and safety constraints defined above, we now impose the following assumption:

Assumption 2. Upon entering the control zone, the initial state of each CAV $j \in \mathcal{N}_i(t)$, $i \in \mathcal{L}(t)$, is feasible, that is, none of the speed or safety constraints are violated.

This is a reasonable assumption since CAVs are automated; therefore, there is no compelling reason for them to violate any of the constraints by the time they enter the control zone.

2.3. Information structure

In this section, we formalize the information structure that is communicated between the CAV leaders and the coordinator inside the control zone.

Definition 2.3. Let ϕ_i be the vector containing the parameters of the optimal control policy (formally defined in Section 3.1) of the leader of platoon $i \in \mathcal{L}(t_i^0)$. Then, the *platoon information set* \mathcal{I}_i that the leader of platoon i can obtain from the coordinator after entering the control zone at time $t = t_i^0$ is

$$\mathcal{I}_i = \{\phi_{1:L(t_i^0)-1}, M_{1:L(t_i^0)}, t_{1:L(t_i^0)}^0, t_{1:L(t_i^0)-1}^f\}, \quad (2.10)$$

where $\phi_{1:L(t_i^0)} := [\phi_1, \dots, \phi_{L(t_i^0)-1}]^T$, $M_{1:L(t_i^0)} := [M_1, \dots, M_{L(t_i^0)}]^T$, $t_{1:L(t_i^0)}^f := [t_1^0, \dots, t_{L(t_i^0)}^0]^T$ and $t_{1:L(t_i^0)-1}^f := [t_1^f, \dots, t_{L(t_i^0)-1}^f]^T$.

Remark 4. The information structure \mathcal{I}_i for each platoon $i \in \mathcal{L}(t_i^0)$ indicates that the control policy, entry time to the control zone t_j^0 , exit time of the control zone t_j^f , and the platoon size M_j of each platoon $j \in \mathcal{L}(t_i^0) \setminus \{i\}$ already existing within the control zone is available to the leader of platoon i through the coordinator. Note that, although the leader of platoon i knows the endogenous information t_i^0 and M_i , it needs to compute the vector of its own optimal control input parameters ϕ_i and the merging time t_i^f , which we discuss in section 3.

Definition 2.4. The *member information set* $\mathcal{I}_{i,j}(t)$ that each platoon member $j \in \mathcal{N}_i \setminus \{0\}$ belonging to each platoon $i \in \mathcal{L}(t)$ at time $t \in [t_i^0, t_i^f]$ can obtain is

$$\mathcal{I}_{i,j} = \{p_{i,0}(t), v_{i,0}(t), u_{i,0}(t)\}. \quad (2.11)$$

Remark 5. The unidirectional intra-platoon communication protocol allows each platoon member $j \in \mathcal{N}_i \setminus \{0\}$ belonging to platoon $i \in \mathcal{L}(t)$ to access the state and control input information of its platoon leader in the form of $\mathcal{I}_{i,j}$ at each time $t \in [t_i^0, t_i^f]$. The set $\mathcal{I}_{i,j}$ is subsequently used to derive the optimal control input $u_{i,j}^*(t)$ of each platoon member j , which we discuss in detail in Section 3.2.

3. Optimal coordination framework

In what follows, we introduce our coordination framework which consists of two optimal control problems. The first problem is to develop an energy-optimal control strategy for the platoon leaders to minimize their travel time while guaranteeing that none of their state, control, and safety constraints becomes active. The second problem is concerned with the optimal control of followers within each platoon in order to maintain the platoon formation while ensuring safety and string stability.

3.1. Optimal control of platoon leaders

In this section, we extend the single-level optimization framework we developed earlier for coordination of CAVs in [52] to establish a framework for coordinating platoons of CAVs. Upon entrance to the control zone, the leader of platoon $i \in \mathcal{L}(t)$ must determine the exit time t_i^f (recall that based on Remark 1, this is the time that the leader of platoon i exits the control zone). The exit time t_i^f corresponds to the unconstrained energy optimal trajectory for the platoon leader ensuring that the

resulting trajectory does not activate any of Eqs (2.1)–(2.4) and Eq (2.9). The unconstrained solution of the leader of platoon i is given by [52]

$$\begin{aligned} u_i(t) &= 6a_it + 2b_i, \\ v_i(t) &= 3a_it^2 + 2b_it + c_i, \\ p_i(t) &= a_it^3 + b_it^2 + c_it + d_i, \end{aligned} \quad (3.1)$$

where a_i, b_i, c_i, d_i are constants of integration. The leader of platoon i must also satisfy the boundary conditions

$$p_i(t_i^0) = p_i^0, \quad v_i(t_i^0) = v_i^0, \quad (3.2)$$

$$p_i(t_i^f) = p_i^f, \quad u_i(t_i^f) = 0, \quad (3.3)$$

where p_i is known at t_i^0 and t_i^f by the geometry of the road, and v_i^0 is the speed at which the leaders of platoon i enters the control zone. The final boundary condition, $u_i(t_i^f) = 0$, results from $v_i(t_i^f)$ being left unspecified [13]. There are five unknown variables that determine the optimal trajectory of the leader of the platoon i , four constants of integration from Eq (3.1), and the unknown exit time t_i^f . The value of t_i^f guarantees that the unconstrained trajectories in Eq (3.1) satisfy all the state, control, and safety constraints in Eqs (2.2)–(2.4), respectively, and the boundary conditions in Eq (3.3). In practice, for the leader of each platoon $i \in \mathcal{L}(t)$, the coordinator stores the optimal exit time t_i^f and the corresponding coefficients a_i, b_i, c_i, d_i . We denote the coefficients of the optimal control policy for the leader of platoon $i \in \mathcal{L}(t)$ by vector $\phi_i = [a_i, b_i, c_i, d_i]^T$, which is an element of platoon information set for the leader of platoon $i \in \mathcal{L}(t)$ (Definition 2.3).

We formally define our single-level optimization framework for platoon leaders as follows.

Problem 1. *Upon entering the control zone, each leader of platoon $i \in \mathcal{L}(t)$ accesses the information set \mathcal{I}_i and solves the following optimization problem at t_i^0*

$$\min_{t_i^f \in \mathcal{T}_i(t_i^0)} t_i^f \quad (3.4)$$

subject to:

$$(2.4), (2.9), (3.1),$$

where the compact set $\mathcal{T}_i(t_i^0) = [t_i^f, \bar{t}_i^f]$ is the set of feasible solution of the leader of platoon $i \in \mathcal{N}(t)$ for the exit time that satisfy the boundary conditions without activating the constraints, while t_i^f and \bar{t}_i^f denote the minimum and maximum feasible exit time computed at t_i^0 .

Remark 6. We can derive the optimal control input of the platoon leaders using the solution of Problem 1, t_i^f , the boundary conditions Eqs (3.2)–(3.3) and Eq (3.1).

In what follows, we continue our exposition by briefly reviewing the process to compute the compact set $\mathcal{T}_i(t_i^0)$ at time t_i^0 using the speed and control input constraints Eqs (2.2)–(2.3), initial

condition Eq (3.2), and final condition Eq (3.3). Details regarding the derivation of the compact set $\mathcal{T}_i(t_i^0)$ can be found in [15].

The lower-bound t_i^f of $\mathcal{T}_i(t_i^0)$ can be computed by considering the state and control constraints and boundary conditions as

$$t_i^f = \min \{t_{i,u_{\max}}^f, t_{i,v_{\max}}^f\}, \quad (3.5)$$

where,

$$t_{i,v_{\max}}^f = \frac{3(p_i(t_i^f) - p_i(t_i^0))}{v_i(t_i^0) + 2v_{\max}},$$

$$t_{i,u_{\max}}^f = \frac{\sqrt{9v_i(t_i^0)^2 + 12(p_i(t_i^f) - p_i(t_i^0))u_{\max}} - 3v_i(t_i^0)}{2u_{\max}}.$$

Here, $t_{i,v_{\max}}^f$ and $t_{i,u_{\max}}^f$ are the times which the leader of platoon $i \in \mathcal{L}(t)$ achieves its maximum speed at the end of control zone and its maximum control input at the entry of the control zone, respectively. Similarly, we derive the upper-bound \bar{t}_i^f as

$$\bar{t}_i^f = \begin{cases} t_{i,v_{\min}}^f, & \text{if } 9v_i(t_i^0)^2 + 12(p_i(t_i^f) - p_i(t_i^0))u_{\min} < 0, \\ \max\{t_{i,u_{\min}}^f, t_{i,v_{\min}}^f\}, & \text{otherwise,} \end{cases} \quad (3.6)$$

where

$$t_{i,v_{\min}}^f = \frac{3(p_i(t_i^f) - p_i(t_i^0))}{v_i(t_i^0) + 2v_{\min}},$$

$$t_{i,u_{\min}}^f = \frac{\sqrt{9v_i(t_i^0)^2 + 12(p_i(t_i^f) - p_i(t_i^0))u_{\min}} - 3v_i(t_i^0)}{2u_{\min}}.$$

Similar to the previous case, $t_{i,v_{\min}}^f$ and $t_{i,u_{\min}}^f$ are the times at which the leader of platoon $i \in \mathcal{L}(t)$ achieves its minimum speed at the end of control zone and its minimum control input at the entry of the control zone, respectively.

Note that, the solution to the optimal control problem 1 yields the optimal control input $u_i^*(t)$ for each platoon leader $i \in \mathcal{L}(t)$ for $t \in [t_i^0, t_i^f]$. However, the solution to this problem does not consider the stability criteria of the platoon [58], which is essential to guarantee safety within the platooning CAVs. In the following section, we first introduce the notion of stability during platoon coordination, and then propose a control structure $u_{i,j}(t)$ for each platoon member $j \in \mathcal{N}_i \setminus \{0\}$, $i \in \mathcal{L}(t)$ that is optimal subject to constraints, and satisfies the stability properties.

3.2. Optimal control of followers within each platoon

Stability properties of the platoon system are well discussed in the literature [26,57,58]. In general, there are two types of stability: (a) local stability, which describes the ability of each platoon member to converge to a given trajectory, and (b) string stability, where any bounded disturbance introduced into the platoon is not amplified while propagating downstream along the vehicle string. In this paper, we adopt the following definition of platoon stability that encompasses the above stability notions [26].

Definition 3.1. A platoon $i \in \mathcal{L}(t)$ is stable if, for any bounded initial disturbances to all the CAVs $j \in \mathcal{N}_i$, the position fluctuations of all the CAVs remain bounded (string stability) and approach zero as time goes to infinity (local stability).

With the stability properties Definition 3.1, we introduce the control problem of each platoon member $j \in \mathcal{N}_i \setminus \{i\}$, $i \in \mathcal{L}(t)$.

Problem 2. Each platoon member $j \in \mathcal{N}_i \setminus \{0\}$, $i \in \mathcal{L}(t)$ needs to derive its control input $u_{i,j}(t)$ for all $t \in [t_i^0, t_i^f]$ that

1. is energy and time-optimal subject to the state and control constraints in Eqs (2.2)–(2.3), and rear-end collision avoidance constraint in Eq (2.5), and
2. satisfying the stability properties according to Definition 3.1.

We provide the following proposition that addresses the Problem 2.

Proposition 1. For each platoon member $j \in \mathcal{N}_i \setminus \{0\}$ in the platoon $i \in \mathcal{L}(t)$, the optimal control input $u_{i,j}(t) = u_{i,0}^*(t)$, where $u_{i,0}^*(t)$ is the solution to Problem 1, is an optimal solution to Problem 2.

Next, we provide the proof of Proposition 1 using the following Lemmas.

Lemma 3.2. For each platoon member $j \in \mathcal{N}_i \setminus \{0\}$ in each platoon $i \in \mathcal{L}(t)$, the control input $u_{i,j}(t) = u_{i,0}^*(t)$ for all $t \in [t_i^0, t_i^f]$ is energy- and time-optimal subject to the control Eq (2.2), state Eq (2.3) and safety constraint Eq (2.5).

Proof. (a) Optimality: We derive the control input $u_{i,0}^*(t)$ of the leading CAV $i \in \mathcal{L}(t)$ by solving Problem 1. The optimal trajectory of the leader of platoon $i \in \mathcal{L}(t)$ is given by Eq (3.1) for all $t \in [t_i^0, t_i^f]$. Thus, for each platoon member $j \in \mathcal{N}_i \setminus \{0\}$, the control input $u_{i,j}(t)$ such that $u_{i,j}(t) = u_{i,0}^*(t)$ also generates optimal linear control, quadratic speed and cubic position trajectories as in Eq (3.1).

(b) Constraint satisfaction: Since $u_{i,j}(t) = u_{i,0}^*(t)$, for each platoon member $j \in \mathcal{N}_i \setminus \{0\}$, $i \in \mathcal{L}(t)$, we have $v_{i,j}(t) = v_{i,0}^*(t)$ for all $t \in [t_i^0, t_i^f]$. The trajectories $v_{i,0}^*(t)$ and $u_{i,0}^*(t)$ do not violate any constraints in Eqs (2.2)–(2.3) since they are derived by solving Problem 1. Therefore, the trajectories $v_{i,j}(t)$ and $u_{i,j}(t)$ of each platoon member j are ensured to satisfy constraints in Eqs (2.2)–(2.3). Additionally, if $u_{i,j}(t) = u_{i,0}^*(t)$, the inter-vehicle gap $p_{i,j-1}(t) - p_{i,j}(t) - l_c$ between two consecutive platoon members $j, j-1 \in \mathcal{N}_i$, $i \in \mathcal{L}(t)$ is time invariant, and equal to Δ_i . Thus, the rear-end safety within CAVs within the platoon i in Eq (2.5) is guaranteed to be satisfied.

Lemma 3.3. Each platoon member $j \in \mathcal{N}_i \setminus \{0\}$ for each platoon $i \in \mathcal{L}(t)$ with the control input $u_{i,j}(t) = u_{i,0}^*(t)$ for all $t \in [t_i^0, t_i^f]$ is locally stable, and the resulting platoon i is string stable.

Proof. (a) Local stability: Since $u_{i,j}(t) = u_{i,0}^*(t)$, for each platoon member $j \in \mathcal{N}_i \setminus \{0\}$, $i \in \mathcal{L}(t)$, we have $v_{i,j}(t) = v_{i,0}^*(t)$ for all $t \in [t_i^0, t_i^f]$. Since there is no communication delay within the unidirectional intra-platoon communication protocol, the speed of each platoon member j converges instantaneously to the speed of the platoon leader $v_{i,0}^*(t)$, which implies local stability.

(b) String stability: A sufficient condition for the string stability of a platoon $i \in \mathcal{L}(t)$ containing CAVs $j \in \mathcal{N}_i$ is $\|\frac{u_{i,j}(s)}{u_{i,j-1}(s)}\|_\infty \leq 1$ [57], where $u_{i,j}(s)$ is the Laplace transform of the control input $u_{i,j}(t)$. Since $u_{i,j}(t) = u_{i,0}^*(t)$ for all $t \in [t_i^0, t_i^f]$, we have $u_{i,j}(s) = u_{i,0}^*(s)$ for all $j \in \mathcal{N}_i \setminus \{0\}$, which yields $\|\frac{u_{i,j}(s)}{u_{i,j-1}(s)}\|_\infty = 1$. Thus, each platoon $i \in \mathcal{L}(t)$ is string stable.

3.3. Delay in platoon communication

In this section, we enhance our framework to include delay in the bi-directional inter-platoon communication. From Assumption 1, we know that delay is bounded and this bound is known a priori. In particular, suppose the delay in bi-direction communication of platoon leaders takes values in $[\tau_{\min}, \tau_{\max}]$, where $\tau_{\min} \in \mathbb{R}_{\geq 0}$ and $\tau_{\max} \in \mathbb{R}_{\geq 0}$ correspond to the minimum and maximum communication delay, respectively. To account for the effects of communication delays in our framework, we consider the worst-case scenario. Namely, we consider that it takes $0.5 \tau_{\max}$ until the coordinator receives the request from the platoon leader, and it takes an extra $0.5 \tau_{\max}$ for the leader of platoon $i \in \mathcal{L}(t)$ to receive the platoon information \mathcal{I}_i . Thus, the leader needs to cruise with the constant speed that it entered the control zone for τ_{\max} until it receives the platoon information \mathcal{I}_i to plan its optimal trajectory. After receiving this information, the platoon leader computes the compact set of the feasible solution \mathcal{T}_i at time $t_i^0 + \tau_{\max}$ with initial condition $v_i(t_i^0 + \tau_{\max})$ and $p_i(t_i^0 + \tau_{\max})$. Using the compact set $\mathcal{T}_i(t_i^0 + \tau_{\max})$ of the feasible solution, the leader derives its optimal control policy by solving Problem 1. Then, it sends the computed trajectory at time $t_i^0 + \tau_{\max}$ to the coordinator. In the worst-case scenario, the coordinator receives this information after $0.5\tau_{\max}$ at $t_i^0 + 1.5\tau_{\max}$. To ensure that new arriving platoons have access to this information, we need to have the following constraint on the initial conditions of platoons upon entrance the control zone.

Proposition 2. *Let platoons i and j , $i, j \in \mathcal{L}(t)$, enter the control zone at time t_i^0 and $t_j^0 > t_i^0$, respectively. In the presence of a bi-directional inter-platoon communication delay, which takes value in $[\tau_{\min}, \tau_{\max}]$, the optimal trajectory of platoon i is accessible to platoon j , if $t_j^0 - t_i^0 \geq \tau_{\max}$.*

Proof. Platoon i computes its optimal trajectory at $t_i^0 + \tau_{\max}$, but in the worst-case scenario, due to delay in communication, this information becomes available to the coordinator at $t_i^0 + 1.5 \tau_{\max}$. On the other hand, upon entrance the control zone, platoon j sends a request to the coordinator to receive platoon information \mathcal{I}_j . However, the coordinator receives this request at $t_j^0 + 0.5 \tau_{\max}$. In order to have the optimal trajectory of platoon i accessible to platoon j we need to have $t_j^0 + 0.5 \tau_{\max} \geq t_i^0 + 1.5 \tau_{\max}$, and the result follows.

Remark 7. We can ensure that the condition in Proposition 2 holds by using an appropriate controller in the platooning zone upstream of the control zone.

3.4. Implementation of the optimal coordination framework

In Sections 3.1, 3.2 and 3.3, we provided the exposition of the intricacies of our proposed control framework for optimal platoon coordination. In this section, we introduce the approach that can be applied to implement this framework in real time.

While entering the control zone at time t_i^0 , platoon leader $i \in \mathcal{L}(t)$ obtains the platoon information \mathcal{I}_i from the coordinator and solves the optimization problem (1) by constructing the feasible set $\mathcal{T}_i(t)$ and iteratively checking the safety constraint. The resulting optimal exit time t_i^f is then used along with the initial Eq (3.2) and boundary Eq (3.3) conditions to derive the vector of control input coefficients ϕ_i using Eq (3.1). Subsequently, each CAV $j \in \mathcal{N}_i$ in platoon $i \in \mathcal{L}(t)$ computes its optimal control input $u_{i,j}(t)$ at each time instance $t \in [t_i^0, t_i^f]$ using ϕ_i . In what follows, we provide an algorithm that delineates the step-by-step implementation of the proposed optimal platoon coordination framework.

Algorithm 1 Vehicular Platoons Coordination Algorithm

```

1: for  $i \in \mathcal{L}(t)$  do
2:   for  $j \in \mathcal{N}_i$  do
3:     if  $j=0$  then
4:        $u_{i,j} = 0 \quad \forall t \in [t_i^0, t_i^0 + \tau_{\max})$ 
5:       Compute  $\mathcal{T}_i(t_i^0 + \tau_{\max})$ 
6:        $t_i^f, \phi_i \leftarrow \text{Platoon Leader Control}()$ 
7:        $[a_i, b_i, c_i, d_i] \leftarrow \phi_i$ 
8:        $u_{i,j}(t) \leftarrow 6a_it + 2b_i$ 
9:     else
10:       $u_{i,j}(t) = u_{i,0}(t)$ 
11:    end if
12:  end for
13: end for

```

▶ Platoon leader
 ▶ Cruise with constant speed
 ▶ Based on Eqs (3.5)–(3.6)
 ▶ Algorithm 2
 ▶ $\forall t \in [t_i^0 + \tau_{\max}, t_i^f]$
 ▶ Platoon followers

Algorithm 2 Platoon Leader Control

Input: Platoon Information set \mathcal{I}_i , Compact feasible set $\mathcal{T}_i(t_i^0 + \tau_i) = [\underline{t}_i^f, \bar{t}_i^f]$

Output: Exit time t_i^f , Coefficients of the optimal control policy ϕ_i

```

1:  $\underline{t}_i^f \leftarrow \underline{t}_i^f$ 
2:  $k \leftarrow$  platoon physically located in front of platoon  $i$ 
3:  $p_{k,m_k}(t) \leftarrow p_k(t) - (M_k - 1)(\Delta_k + l_c)$ 
4: while  $p_{k,m_k}(t) - p_i(t) < \delta_i(t)$  do
5:    $\underline{t}_i^f \leftarrow \underline{t}_i^f + dt$ 
6: end while
7:  $lateral \leftarrow$  list of all platoons  $j < i$  from the other road
8: for  $j \in lateral$  do
9:   Compute  $t_{j,m_j}^f$  and  $t_{i,m_i}^f$  from (2.8)
10:  while  $t_i^f - t_{j,m_j}^f < t_h$  AND  $t_j^f - t_{i,m_i}^f < t_h$  do
11:     $\underline{t}_i^f \leftarrow \underline{t}_i^f + dt$ 
12:  end while
13: end for
14: Compute  $\phi_i$ 

```

▶ Position of the last follower
 ▶ Rear-end safety
 ▶ Lateral safety
 ▶ From Eqs (3.1)–(3.3)

4. Simulation example

4.1. Simulation setup

To evaluate and validate the performance of our proposed optimal platoon coordination framework, we employ the microscopic traffic simulation software VISSIM v11.0 [25]. We create a simulation environment with a highway on-ramp merging, which has a control zone of length 560 m. In our simulation framework, we use VISSIM's component object model (COM) interface with Python 2.7 to generate platoons of CAVs on the main road and the on-ramp at different time intervals. The time interval between two consecutive platoon generations is randomized with a uniform probability distribution, and the bounds can be controlled to increase or decrease the traffic volume in each roadway. The length of each platoon is also randomly selected from a set of 2 to 4 vehicles with equal probability. The maximum speed limit, v_{\max} , of each roadway is set to be 16.67 m/s, and the maximum and minimum acceleration limit is 3 m/s^2 and -3 m/s^2 , respectively. Vehicles enter the main road and the on-ramp with a traffic volume of 700 and 650 vehicle per hour per lane with random initial speed uniformly chosen from a set of 13.89 to 16.67 m/s. Videos of the experiment can be found at the supplemental site (<https://sites.google.com/view/ud-ids-lab/CAVPLT>).

To evaluate the performance of the proposed optimal control framework, we simulate the following control cases.

(a) Baseline 1: All vehicles in the network are human-driven vehicles. In this scenario, the Wiedemann car-following model [80] adopted by VISSIM [25] is applied. The conflict point of the on-ramp merging scenario has a priority mechanism, where the vehicles on the ramp road are required to yield to the vehicle on the main road within a certain look-ahead distance. Vehicles enter the network individually without forming any platoons.

(b) Baseline 2: Similar to the above case, all the vehicles are human-driven vehicles integrated with the Wiedemann car-following model and follow the priority mechanism set at the conflict point of the on-ramp merging scenario. The difference is that, when vehicles enter the network, they have already formed platoons. Note, we consider this case to simulate the same initial condition of the optimal coordination case, which we will discuss next.

(c) Optimal Coordination: All the vehicles present in the network are connected and automated. They enter the network forming platoons of different sizes and optimize their trajectories based on the optimal coordination framework presented in Section 3.

We use the COM application programming interface to interact with the VISSIM simulator externally and implement the proposed optimal coordination framework. At each simulation time step, we use the VISSIM-COM interface to collect the required vehicle attributes from the simulation environment and pass them to the external python script. The external python script implements the proposed single-level optimal control algorithm (Section 3.4) to compute the optimal control input of each CAV within the control zone. Finally, the speed of each platooning CAV is updated in VISSIM traffic simulator in real-time using the COM interface.

4.2. Results and discussion

In Figure 3, the path trajectories of the optimal coordinated CAV platoons traveling through the main road and the ramp road of the considered on-ramp merging scenario are shown. The spatial gaps

between the trajectory paths indicate that our framework satisfies the rear-end collision avoidance constraint without any violation. Moreover, note that the path trajectories in Figure 3 are not parallel. The coefficients ϕ_i in Eq (3.1) can vary depending on the activation of state, control and safety constraints, and different initial and terminal boundary conditions. Thus the optimal solution Eq (3.1) can lead to non-parallel path trajectories.

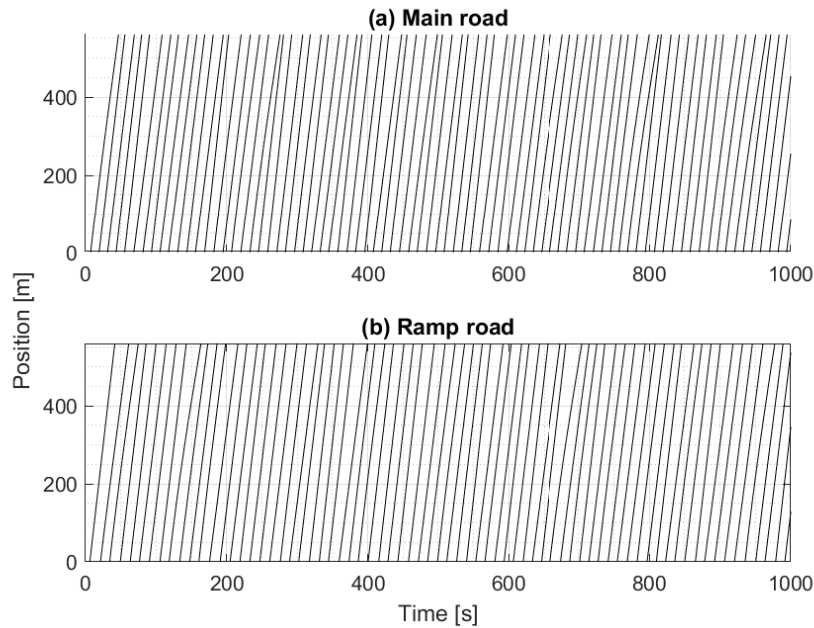


Figure 3. The position trajectories of the optimally coordinated CAV platoons at the (a) main road and (b) ramp road are shown.

To visualize the performance of the proposed coordination framework in comparison with the baseline cases, we focus on Figures 4 and 5. In Figure 4, the speed trajectories of all the vehicles in the network are shown. In Figure 4 a–b, both baseline cases show stop-and-go driving behavior close to the conflict point of the on-ramp merging scenario. In contrast, with the optimal coordination framework, we are able to completely eliminate stop-and-go driving behavior, as shown in Figure 4c. The elimination of the stop-and-go driving behavior has associated benefits, namely, the minimization of transient engine operation and travel time, as shown in Figure 5. In Figure 5a, the baseline case with vehicle platoons (red) shows sudden increase in fuel consumption near the conflict point due to the transient engine operation induced by the stop-and-go driving behavior. In contrast, the cumulative fuel consumption trajectories of the optimally coordinated CAVs (black) remain steady throughout their path. Note that, we use the polynomial metamodel proposed in [32] to compute the fuel consumption of each vehicle. In Figure 5b, we illustrate the distribution of total travel time of the vehicles for the baseline (maroon) and the optimal coordination (blue) framework. The high variance of the travel time for the baseline case compared to the optimal coordination approach indicates increased traffic throughput of the network.

Finally, we provide the summary of the performance metrics in Table 1. Based on the simulation, the optimal coordination framework shows significant improvement over the baseline cases in terms of average travel time and fuel consumption.

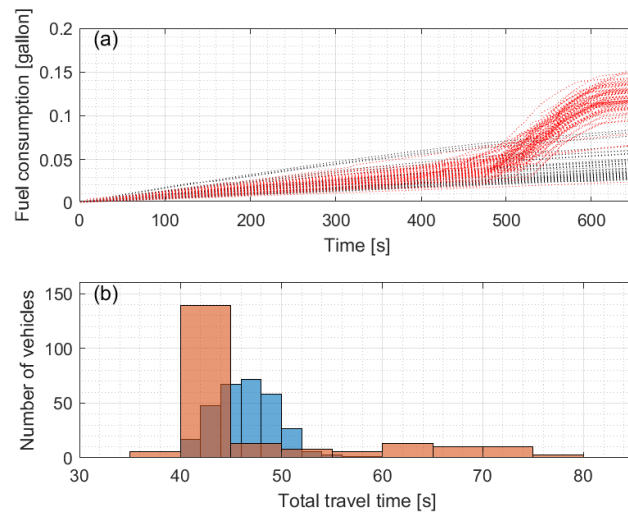


Figure 4. Comparison of performance metrics: (a) cumulative fuel consumption of optimal coordination (black) vs. baseline with platooning (red), and (b) total travel time distribution of optimal coordination (blue) vs. baseline with coordination (maroon).

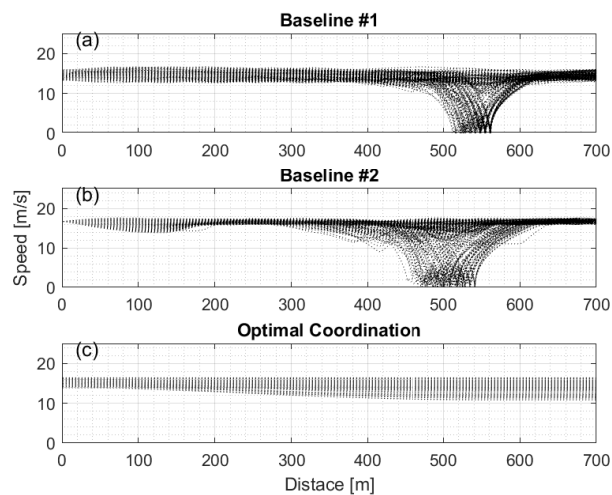


Figure 5. Speed profiles of 400 vehicles traveling through the on-ramp merging scenario for three cases: (a) baseline without platooning, (b) baseline with platooning and (c) optimal platoon coordination.

Table 1. Summary of performance metrics.

Performance Metrics	Avg. travel time [s]	Avg. fuel consumption [gallon]
Baseline 1	57.33	0.042
Baseline 2	52.79	0.05
Optimal Coordination	46.1	0.022
Improvement (baseline 1) [%]	19.6	46.9
Improvement (baseline 2) [%]	12.7	38.2

5. Concluding remarks

In this paper, we leveraged the key concepts of CAV coordination and platooning, and established a rigorous optimal platoon coordination framework for CAVs that improves fuel efficiency and traffic throughput of the network. We presented a single-level optimal control framework that simultaneously optimizes both fuel economy and travel time of the platoons while satisfying the state, control, and safety constraints. We developed a robust coordination framework considering the effect of delayed inter-platoon communication and derived a closed-form analytical solution of the optimal control problem using standard Hamiltonian analysis that can be implemented in real time using leader-follower unidirectional communication topology. Finally, we validated the proposed control framework using a commercial simulation environment by evaluating its performance. Our proposed optimal coordination framework shows significant benefits in terms of fuel consumption and travel time compared to the baseline cases.

Ongoing work addresses the delay in intra-platoon communication and its implications on platoon stability. A potential direction for future research includes relaxing the assumption of 100% CAV penetration considering the inclusion of human-driven vehicles [43, 75]. Moreover, the single-level optimal control framework for coordinating CAVs at traffic scenarios such as single intersections [18, 19, 52], multi-lane roundabouts [15], and network of intersections [7] have been explored. Future research should extend this framework to include platoons of CAVs.

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