Stochastic Optimal Control for Series Hybrid Electric Vehicles

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Abstract— **Increasing demand for improving fuel economy and reducing emissions has stimulated significant research and investment in hybrid propulsion systems. In this paper, we address the problem of optimizing online the supervisory control in a series hybrid configuration by modeling its operation as a controlled Markov chain using the average cost criterion. We treat the stochastic optimal control problem as a dual constrained optimization problem. We show that the control policy that yields higher probability distribution to the states with low cost and lower probability distribution to the states with high cost is an optimal control policy, defined as an equilibrium control policy. We demonstrate the effectiveness of the efficiency of the proposed controller in a series hybrid configuration and compare it with a thermostat-type controller.**

I. INTRODUCTION

Widespread use of alternative hybrid powertrains is currently inevitable, and many opportunities for substantial progress remain [1]. The necessity for environmentally conscious vehicle designs, in conjunction with increasing concerns regarding US dependency on foreign oil and climate change, has led to significant investment in enhancing the propulsion portfolio with new technologies. Hybrid electric vehicles (HEVs) and plug-in HEVs have attracted considerable attention due to their potential to reduce petroleum consumption and greenhouse gas emissions in the transportation sector. A typical HEV powertrain configuration consists of the fuel converter (engine), the electric machines (motor and generator), the energy storage system (battery), the torque coupler, and the transmission. In series HEV configurations, the electric motor is the only means of providing the power demanded by the driver. The optimal split of the power demanded by the driver constitutes a challenging control problem defined as the supervisory control or energy management problem [2] and has been the object of intense study for the last decade.

The objective of the control problem formulation in the series hybrid configuration, in particular, is to operate the engine efficiently while maintaining the state-of-charge (SOC) of the energy storage system within acceptable limits. Jalil, Kheir, and Salman [3] proposed a rule-based control strategy

using a thermostat-type behavior to optimize the power management control in a series HEV. Brahma, Guezennec, and Rizzoni [4] implemented a deterministic dynamic programming (DP) solution to address this problem. Tate and Boyd [5] introduced the application of convex optimization to hybrid vehicle optimization. They posed the problem of finding optimal engine operation in a pure series hybrid vehicle over a fixed driving cycle subject to a number of constraints related to components, e.g., engine, battery, and motor. In their approach, the authors formulated the problem of optimizing fuel consumption as a nonlinear convex optimization problem, which was then approximated as a linear program. Mckain *et al.* [6] evaluated the emissions and fuel economy of series hybrid electric buses and compared their performance with that of conventional ones.

In later research efforts, Barsali, Miulli, and Possenti [7] proposed an online control algorithm for a series HEV relying on overall parameters characterizing the driving schedule and being easily adaptable to sudden changes in the driving regime. Pisu and Rizzoni [8] developed an equivalent consumption minimization strategy (ECMS) with the aim to optimize the energy management for a series hybrid electric heavy duty truck with two energy storage systems, i.e., battery and ultracapacitor. Konev, Lezhnev, and Kolmanovsky [9] implemented a control strategy for a series HEV that ensured gradual operation of the engine-generator unit along the steady-state optimal operating points. In the proposed approach, the HEV operation was modeled as a random process. The algorithm attempts to minimize the probability of discharging or overcharging the battery beyond a predetermined SOC target. The probability was estimated based on the statistics derived from the past history of the SOC. Pérez *et al.* [10] following [4] used DP to minimize fuel consumption with respect to power split between the engine and energy storage unit in a series HEV.

Wang, Li, and Xu [11] proposed a power management control strategy using support vector machines (SVMs). SVM is a technique in the field of statistical learning originally developed for classification problems. In this context, their approach includes setting up an operation database of different load sequences, initial SOC and vehicle speed; then applying a fast Fourier transform on load sequences to select characteristic features and generate a new database, and, finally, training SVM to produce the controller classifier.

Yoo *et al.* [12] presented another rule-based control strategy for a series HEV with three power sources, i.e., battery, super capacitor, and gen-set. Serrao and Rizzoni [13] proposed an analytical solution based on Pontryagin's minimum principle for the optimal power management control

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problem in a series hybrid electric refuse collection truck. The equivalence factors that allow for the transformation of electrical energy into future fuel consumption must be determined with optimization techniques and are related to the driving cycles that the vehicle follows.

More recently, Serrao, Onori, and Rizzoni [14] presented a formal analytical derivation of ECMS for energy management in a series HEV based on Pontryagin's minimum principle. The Hamiltonian equation was interpreted as the sum of the actual fuel consumption in the engine and of a term that has the same units and is related to the use of the battery power. This additional term represents the virtual consumption associated with the battery use and is related to the future fuel consumption due to the use of the battery at the present time as explained intuitively in the first papers on ECMS [15] and [16]. Finally, Ripaccioli *et al.* [17] illustrated the use of stochastic model predictive control (SMPC) for power management control in a series HEV. The power demanded from the driver was modeled as a Markov chain trained from a given family of driving cycles. A linear model was used by SMPC to derive an optimal control policy (engine power). Cairano *et al.* [18] proposed an energy management control strategy for a series HEV.

In this paper, we address the problem of optimizing online the supervisory control in a series HEV configuration. We model HEV operation as a controlled Markov chain using the average cost criterion. We treat the stochastic optimal control problem as a dual constrained optimization problem. We show that the control policy that yields higher probability distribution to the states with low cost and lower probability distribution to the states with high cost is an optimal control policy, defined as an equilibrium control policy. The contribution of this paper is the development of an online supervisory controller for a series HEV that, under certain conditions, can yield an optimal solution of the stochastic control problem.

The remainder of the paper proceeds as follows: In Section II, we introduce our notation and formulate the problem. In Section III, we present the equilibrium control policy and show that it minimizes the average cost criterion. In Section IV, we demonstrate the effectiveness of the efficiency of the proposed controller in a series HEV truck and compare it with a thermostat-type controller. Finally, we present concluding remarks in Section V.

II. Problem Formulation

A. Series Hybrid Electric Vehicle Configuration

In the series HEV configuration considered here and illustrated in Fig. 1, the motor provides all the power demanded by the driver. Thus we can operate the engine at any desired combination of engine torque and speed. The objective of the centralized controller is to maintain the SOC of the battery within a given range while operating the engine efficiently. So the optimal control policy of the centralized controller is the engine power at each instant of time with respect to the engine's speed that minimizes fuel consumption. To operate the engine under the condition designated by the

centralized controller, a PID controller regulates the engine torque through the generator as shown in Fig. 2. The optimal engine power is converted to electrical power through the generator and goes to the battery.
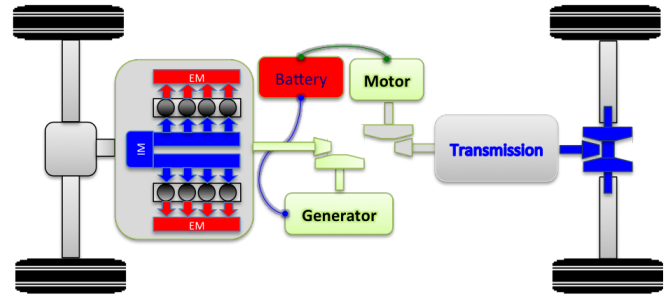

Fig. 1. The series HEV configuration.

Following the framework proposed in [19], [20], engine operation is modeled as a controlled Markov chain with a state space $\mathcal{S} \subset \mathbb{R}^n$, and a control space $\mathcal{U} \subset \mathbb{R}^m$ from which control actions are chosen. The state space $\mathcal{S}$ is the entire range of the engine speed and the control space $\mathcal{U}$ is the engine power range; thus both the state space $\mathcal{S}$ and control space $\mathcal{U}$ are compact spaces. The Markov chain is the evolution of the engine speed and the uncertainty is related to the power demanded by the driver through the battery SOC.
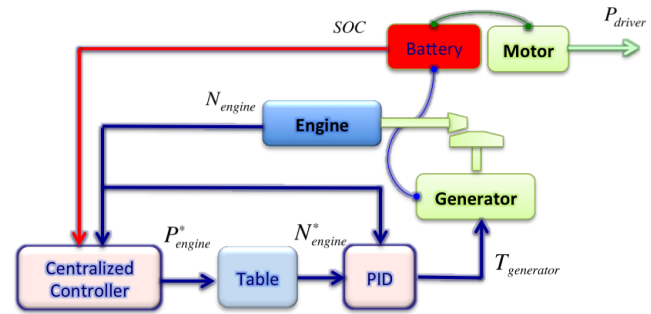

Fig. 2. The centralized control scheme.

B. Average Cost Criterion

A state-dependent constraint is incorporated in the formulation; that is, for each state (engine speed) $i \in \mathcal{S}$ a nonempty set $\mathcal{C}(i) \subset \mathcal{U}$ of admissible control actions (engine power) is given.

Definition 2.1: The set of admissible state/action pairs is defined as

$$\Gamma := \{(i, u) | i \in \mathcal{S} \text{ and } u \in \mathcal{C}(i)\},$$

where Γ is the intersection of a closed subset of $\mathbb{R}^n \times \mathbb{R}^m$ with the set $\mathcal{S} \times \mathcal{U}$. That is, Γ is closed with respect to the induced topology on $\mathcal{S} \times \mathcal{U}$. It follows that for each state $i \in \mathcal{S}$, $\mathcal{C}(i)$ is compact.

Definition 2.2: The set of Borel measurable functions is defined as $\Pi_i := \{\mu_i : \mathcal{S} \to \mathcal{U} | \mu_i$ is Borel measurable and $\mu_i \in \mathcal{C}(i)\}, \forall i \in \mathcal{S}$.

Let $\Pi\colon\ =\ \Pi_i^\infty$, $i \in \mathcal{S}$, be the set of all sets of all sequences $\pi = \{\mu_1, \mu_2, ..., \mu_n\}$. Each sequence in Π is called a *stationary control policy* and operates as follows. Associated with each state $i \in \mathcal{S}$ is the Borel measurable function $\mu_i \in \mathcal{C}(i)$. If at any time the centralized controller finds the engine in state i, then the controller always chooses the action μ_i.

The evolution of the engine occurs at each of a sequence of stages $t = 0, 1, ...$, and it is portrayed by the sequence of the random variables X_t and U_t corresponding to the system's state (engine speed) and control action (engine power). At each stage, the controller observes the system's state $X_t = i \in \mathcal{S}$, and executes an action $U_t = \mu_i$, from the feasible set of actions $\mu_i \in \mathcal{C}(i)$ at that state. At the next stage t, the system transits to the state $X_{t+1} = j \in \mathcal{S}$ imposed by the conditional probability $P(X_{t+1} = j | X_t = i, U_t = \mu_i)$, and a cost $k(X_t = i, U_t = \mu_i) = k(i, \mu_i)$ is incurred corresponding to fuel consumption. After the transition to the next state has occurred, a new action is selected and the process is repeated. We are concerned with deriving a stationary optimal control policy (sequence of engine power) to minimize the long-run average cost (average fuel consumption) per unit time, that is

$$J(\pi) = \min_{\pi \in \Pi} \lim_{T \to \infty} \frac{1}{T+1} E\left[\sum_0^T k(X_t, U_t)\right]. \quad (1)$$

For each policy $\pi \in \Pi$, $\mathbf{P}(\pi)$ is the transition probability matrix, the elements of which represent the conditional probability of moving from one state to another under the policy π, that is, $P(X_{t+1} = j | X_t = i, U_t = \mu_i)$. To guarantee that the limit in (1) exists, we assume that for each stationary control policy $\pi = \{\mu_1, \mu_2, ..., \mu_n\}$, the Markov chain $\{X_t | t = 1, 2, ...\}$ has a single ergodic class. That is, for each stationary policy $\pi \in \Pi$, there is a unique probability distribution (row vector) $\beta(\pi) = [\beta_1(\pi), \beta_2(\pi), ..., \beta_i(\pi), ...\beta_n(\pi)], \forall i \in \mathcal{S}$, such that $\beta(\pi) = \beta(\pi) \cdot \mathbf{P}(\pi)$, with $\sum_{i \in \mathcal{S}} \beta_i(\pi) = 1$. Under our assumption, it is known that

$$\lim_{T \to \infty} \frac{1}{T+1} \sum_{t=0}^T [\mathbf{P}(\pi)]^t = \mathbf{1} \cdot \beta(\pi), \quad (2)$$

where $\mathbf{1} = [1, 1, ..., 1]^T$ is the column vector whose elements are all unity. Substituting (2) into (1) shows that the long run average cost, $J(\pi)$, does not depend on the initial state and is given more simply as

$$J(\pi) = \beta(\pi) \cdot k(\pi), \quad (3)$$

where $k(\pi) = [k(1, \mu_1), k(2, \mu_2), ..., k(i, \mu_i), ...k(n, \mu_n)]^T$ is the column vector of the cost function. Since $\mathbf{P}(\pi)$ is assumed to be continuous, it follows from (2) that $\beta(\pi)$ is continuous. Thus, since the Borel measurable cost function $k\colon \Gamma \to \mathbb{R}$ is continuous, so is $J(\pi)$. Hence by compactness of $\mathcal{S} \times \mathcal{U}$ an optimal stationary control policy exists.

III. OPTIMAL CONTROL POLICY

A. Equilibrium Control Policy

In this section, we formulate the problem of deriving an optimal control policy that minimizes the average cost as a dual constrained optimization problem. The motivation behind this new formulation is the structure of the average cost as expressed in (3). In particular, the average cost depends on two vectors, i.e., the stationary probability distribution and the vector of the cost function. The Markov chain has a single ergodic class and since we permit the single ergodic class to depend on π, different control policies will yield different probability distributions for each state $i \in \mathcal{S}$. Consequently, we seek a solution ensuring that the control policy endows a stationary probability distribution that yields higher probability at the states with low cost, and lower probability at the states with high cost. Thus, we can formulate the problem as the following dual optimization problem:

$$\min_k \sup_\beta \beta(\pi) \cdot k(\pi), \text{or } \max_\beta \inf_k \beta(\pi) \cdot k(\pi) \quad (4)$$

Definition 4.1: A control policy $\bar{\pi} = \{\mu_1, \mu_2, ..., \mu_i, ..., \mu_N\}$ is an equilibrium control policy if it yields a solution for the above optimization problem corresponding to the following pair of vectors β^* and k^*

$$k^*(1, \mu_1) < k^*(2, \mu_2) < ... < k^*(i, \mu_i) < ... < k^*(N, \mu_N) \quad (5)$$

$$\beta_1^*(\bar{\pi}) > \beta_2^*(\bar{\pi}) > ... > \beta_i^*(\bar{\pi}) > ... > \beta_N^*(\bar{\pi}), \forall i \in \mathcal{S}, \quad (6)$$

where $N \in \mathbb{N}$ is the cardinality of $\mathcal{S}$.

Thus if an equilibrium control policy exists, it yields higher probability distribution to the states with low cost and lower probability distribution to the states with high cost.

Proposition 4.1 [21]: If the average cost $J\colon (\mathcal{S} \times \mathcal{U}) \to \mathbb{R}$ is a linear functional such that J is convex with respect to the measurable cost function k and its epigraph is closed for each β, then an equilibrium control policy exists.

B. Optimality Equation

Now, we formulate the optimality equation for the average cost criterion.

Theorem 4.1 [22]: Suppose the Markov chain has a single ergodic class and that the cost function, $k\colon \Gamma \to \mathbb{R}$, belongs to the set of all bounded, continuous, real-valued functions on $\mathcal{S}$. If $\pi \in \Pi$ is a control policy, then (C, J) is the solution of the following equation:

$$C + \mathbf{1} \cdot J = P(\pi) \cdot C + k(\pi), \quad (7)$$

where C is a column vector the elements of which belong to $\mathbb{R}$, $\mathbf{1} = [1, 1, ..., 1]^T$ is the column vector whose elements are all unity, J is the average cost, P is the stochastic kernel, and k is the cost function.

The principle of optimality suggests that the minimum average cost $J^* = \min_{\pi \in \Pi} J(\pi)$ would be given by the solution to

$$C + \mathbf{1} \cdot J^* = \min_{\pi \in \Pi}[P(\pi) \cdot C + k(\pi)]. \tag{8}$$

Theorem 4.2 [22]: Let $\pi \in \Pi$. If there exist (C, J^*) such that

$$C + \mathbf{1} \cdot J^* = \min_{\pi \in \Pi}[P(\pi) \cdot C + k(\pi)], \tag{9}$$

then π is the optimal control policy.

Theorem 4.3 : The equilibrium control policy is an optimal control policy.

Proof: Let π^* be the optimal control policy that minimizes the average cost $J^* = \min_{\pi \in \Pi}(J)$. Then from Theorem 4.2 there exists (C, J^*) such that

$$\begin{aligned} C + \mathbf{1} \cdot J^* &= \min_{\pi \in \Pi}[P(\pi) \cdot C + k(\pi)] \\ &= P(\pi^*) \cdot C + k^*(\pi^*), \end{aligned} \tag{10}$$

where $J^* = \beta(\pi^*) \cdot k^*(\pi^*)$ is the minimum average cost.

Let $\bar{\pi}$ be the equilibrium control policy. The optimal control policy π^* and the equilibrium control policy $\bar{\pi}$ yield the same cost-per-stage, $k^*(\bar{\pi}) = k^*(\pi^*)$ since the equilibrium control policy minimizes the average cost over cost-per-stage as shown in (4). The average cost for the equilibrium control policy is equal to $J(\bar{\pi}) = \beta^*(\bar{\pi}) \cdot k^*(\bar{\pi})$. Suppose the equilibrium control policy is not optimal. Then

$$C + \mathbf{1} \cdot J^* = P(\pi^*) \cdot C + k^*(\pi^*) \leqslant P(\bar{\pi}) \cdot C + k^*(\bar{\pi}). \tag{11}$$

Premultiplying (11) by the stationary probability distribution vector $\beta^*(\bar{\pi})$ corresponding to the equilibrium control policy gives

$$\beta^*(\bar{\pi}) \cdot C + J^* \leqslant \beta^*(\bar{\pi}) \cdot C + \beta^*(\bar{\pi}) \cdot k^*(\bar{\pi}), \tag{12}$$

since $\beta^*(\bar{\pi}) \cdot \mathbf{1} = 1$, $\sum_{i \in \mathcal{S}} \beta_i^*(\bar{\pi}) = 1$, and $\beta^*(\bar{\pi}) = \beta^*(\bar{\pi}) \cdot P(\bar{\pi})$.

Finally,

$$J^* \leqslant \beta^*(\bar{\pi}) \cdot k^*(\bar{\pi}), \tag{13}$$

or

$$\begin{aligned} &\beta_1(\pi^*) \cdot k_1^* + \beta_2(\pi^*) \cdot k_2^* + ... + \beta_N(\pi^*) \cdot k_N^* \\ &\leqslant \beta_1^*(\bar{\pi}) \cdot k_1^* + \beta_2^*(\bar{\pi}) \cdot k_2^* + ... + \beta_N^*(\bar{\pi}) \cdot k_N^*. \end{aligned} \tag{14}$$

The last equation implies that there will be some $i \in \mathcal{S}$ such that $\beta_i(\pi^*) \leqslant \beta_i^*(\bar{\pi})$. However, since the equilibrium control policy yields the maximum possible probability to each state, (6), and since $\sum_{i \in \mathcal{S}} \beta_i(\pi^*) = 1$, there will be some $j \in \mathcal{S}$ such that $\beta_j(\pi^*) \geq \beta_j^*(\bar{\pi})$, which is a contradiction since $\beta_j^*(\bar{\pi})$ is the maximum probability at state $j \in \mathcal{S}$. Hence

$$J^* = \beta^*(\bar{\pi}) \cdot k^*(\bar{\pi}). \tag{15}$$

∎

Conceptually, a solution is sought that ensures the control policy endows a stationary probability distribution yielding higher probability at the states with low cost and lower probability at the states with high cost. The equilibrium control policy yields the saddle point solution of the optimization problem (4), and it is an optimal control policy that minimizes the average cost J. The equilibrium control

policy exists if the average cost J is a linear functional such that J is convex with respect to the measurable cost function k and its epigraph is closed for each β.

The equilibrium control policy is essentially a characterization of the optimal solution of the average cost criterion. For practical situations with constraints consistent with those we study here, our results imply that recognition of the equilibrium control policy may be of value in deriving an optimal control policy in real time. Solving the original stochastic control problem, (3), for the series HEV configuration is computationally expensive and real-time implementation is not feasible; alternatively, the centralized controller can be designed to operate the engine under the equilibrium control policy, that is, operating the engine with a higher probability on the speed range corresponding to low fuel consumption, and with lower probability on the speed range corresponding to high fuel consumption.

IV. SUPERVISORY POWER MANAGEMENT CONTROL USING THE EQUILIBRIUM CONTROL POLICY

To validate the effectiveness of the centralized controller using the equilibrium control policy, we employed Autonomie [23]. A vehicle model from Autonomie's database representing a medium duty series HEV truck was used in this study. To identify the column vector of the cost function that is minimum for each state (engine speed) we plot the minimum brake specific fuel consumption (bsfc) of the engine for each engine speed as illustrated in Fig. 3. From this plot, we can choose the set of admissible state/action pairs Γ: $= \{(i, u) | i \in \mathcal{S}$ and $u \in \mathcal{C}(i)\}$. The criterion of selecting the set Γ is to be able to have a structure of the vector of the cost function, $k^*(\bar{\pi}) = [k(1, \mu_1), k(2, \mu_2), ..., k(i, \mu_i), ...k(n, \mu_n)]^T$, as designated by the equilibrium control policy $\bar{\pi} = \{\mu_1, \mu_2, ..., \mu_i, ..., \mu_N\}$; namely, $k^*(1, \mu_1) < k^*(2, \mu_2) < ... < k^*(i, \mu_i) < ... < k^*(N, \mu_N)$.
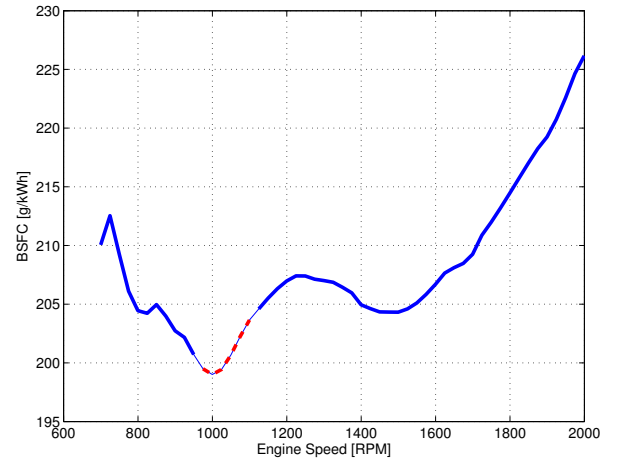
Fig. 3. The minimum brake specific fuel consumption values corresponding to the cost function with respect to engine speed.

The equilibrium control policy can be achieved by the

centralized controller, if the engine is operated at the speed range ensuring higher probability to the engine speed with lower bsfc values and lower probability to the engine speed with higher bsfc values $\beta_1^*(\bar{\pi}) > \beta_2^*(\bar{\pi}) > ... > \beta_i^*(\bar{\pi}) > ... > \beta_N^*(\bar{\pi}), \forall i \in \mathcal{S}$. However, the centralized controller needs to maintain the battery's SOC close to the target value (65% in this case). To achieve both objectives, we establish an one-on-one correlation, illustrated in Fig. 4, between SOC and a portion of the optimal engine power corresponding to red dashed line in Fig. 3. In previous research adopting the stochastic optimization framework described here, the SOC of the battery has been used as a component of the state. However, this may lead to a significantly large state space with implications for increasing the computational burden associated with the problem. In our approach, the SOC is treated as an additional uncertainty which is correlated to an additional power demand by means of one-on-one mapping. Depending on the SOC value, there is a corresponding amount of power P_{SOC} that needs to be provided to the battery to stay at the target SOC. This additional amount is added to the driver's power demand. The one-on-one mapping is a heuristic mapping that aims to provide an increasing power request, P_{SOC} as the SOC drops up to a certain maximum value. If the SOC is above the target value, then P_{SOC} is zero. Future research should investigate the optimal one-on-one correlation in conjunction of the battery's characteristics and properties.
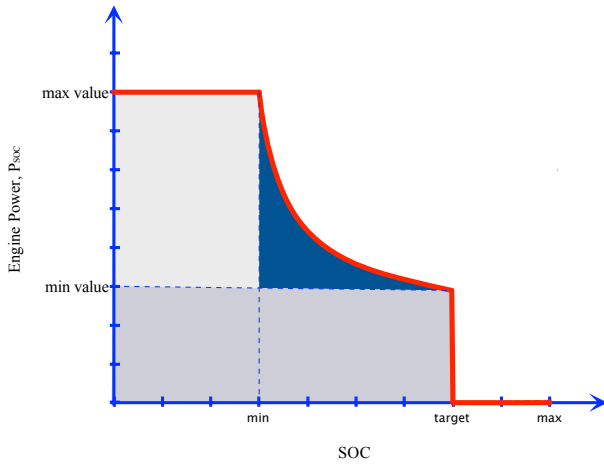

Fig. 4.   Engine power with respect to the state of charge of the battery.

The controller is set up to command the engine to provide the power corresponding to around 1,000 RPM, i.e., the minimum allowable value in the range of the engine power, whenever SOC is equal to 65% (target SOC) and gradually increase as SOC drops below 65% all the way down to the minimum allowable value (60% in this case), as illustrated in Fig. 4. Whenever the battery SOC is at 65%, the engine operates at its minimum possible bsfc value and gradually increases to the value corresponding to the maximum value in the set Γ.

To ensure that the probability distribution of the engine speed is correlated to the values specified by the equilibrium control policy, we imposed condition so that the engine is operated as specified by the probability distribution of the equilibrium control policy. In particular, we assign a given probability to each state (engine speed) belonging to the set Γ, such that

$$\beta_1^*(\bar{\pi}) > \beta_2^*(\bar{\pi}) > ... > \beta_i^*(\bar{\pi}) > ... > \beta_N^*(\bar{\pi}), \forall i \in \mathcal{S}, \tag{16}$$

with $\sum_{i \in \mathcal{S}} \beta_i(\pi) = 1$. Then we correlate the engine power to this stationary probability distribution. The transition probability matrix of the state space is computed from a computational learning model presented earlier [19] that converges to the stationary probability distribution of the Markov chain [20]. Although the centralized controller needs to maintain the battery's SOC within the target value as indicated by P_{SOC}, the engine should be operated optimally and the probability distribution of the engine speed should be constrained by (16).

To validate the effectiveness of the equilibrium control policy, we compared it to a thermostat-type controller as the latter has been a popular controller for series HEVs in the literature [7]. However, future research should investigate the benefits of the proposed controller by comparing it with other supervisory controllers proposed in the literature and under different driving cycles to be able to draw solid conclusions.

The thermostat-type controller was also set up to operate the engine within the maximum and minimum value of the engine power corresponding to red dashed line in Fig. 3. Both HEVs, the one having the thermostat-type controller and the one with centralized controller using the equilibrium control policy, were simulated over the CSHVR driving cycle, deemed characteristic for medium- and heavy-duty trucks, in Autonomie. An inherent algorithm in Autonomie run both HEV models over the same driving cycle multiple times until the initial and final SOC becomes the same. As a result the models ended up with different initial SOC values. The SOC plots for both models are shown in Fig. 5.
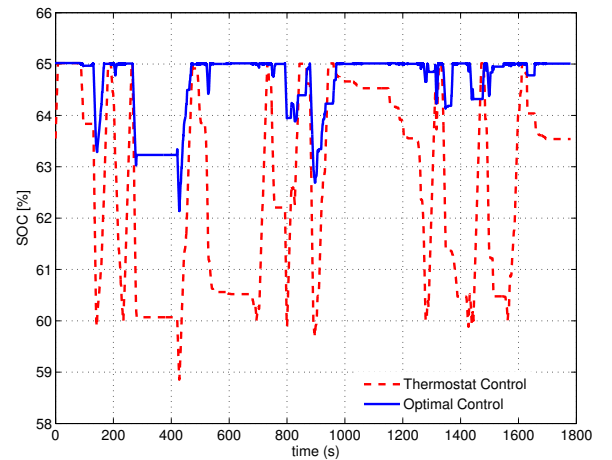

Fig. 5.   State of charge of the battery for the series hybrid electric truck with the thermostat-type controller and the centralized controller employing the equilibrium control policy.

The HEV model with the thermostat-type supervisory controller ended up with initial and final SOCs of 63.5%, whereas the HEV model with the supervisory controller using the equilibrium control policy ended up with initial and final SOCs of 65%. As the SOC drops below the target value, the controller increases the engine power taking values from the feasible set, Γ, with the intention to yield the stationary probability distribution and cost function corresponding to the equilibrium control policy, (5) and (6). As a result, a 6.6% fuel consumption improvement was achieved, as shown in Fig. 6.
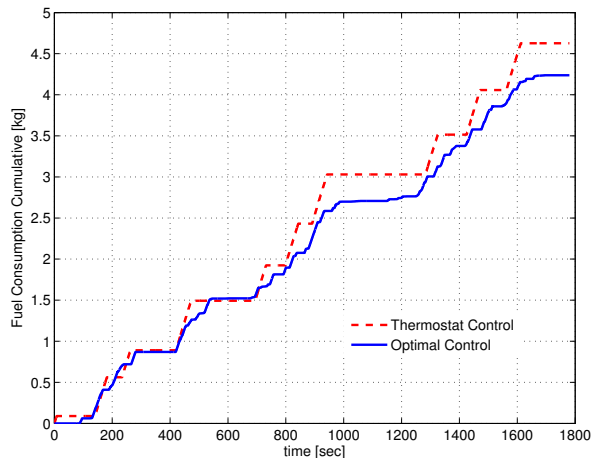

Fig. 6. Cumulative fuel consumption for the series hybrid electric truck with the thermostat-type controller and the centralized controller using the equilibrium control policy.

V. CONCLUDING REMARKS

The results presented here address the problem of online optimization of the supervisory control in a series HEV configuration. We modeled HEV operation as a controlled Markov chain using the average cost criterion, and we treated the stochastic optimal control problem as a dual constrained optimization problem. We showed that the equilibrium control policy is an optimal control policy operating the engine with a higher probability on the speed range corresponding to low fuel consumption and with lower probability on the speed range corresponding to high fuel consumption. The effectiveness of the centralized controller was validated through simulation in a series HEV truck. The supervisory controller yielded a 6.6% improvement in fuel consumption compared to a thermostat-type controller. Future research should compare the proposed supervisory controller with other supervisory controllers proposed in the literature and under different driving cycles. Future research should also investigate the optimal one-on-one correlation between the SOC and the engine power in conjunction of the battery's characteristics and properties.

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